The Shape of Reality: Recognition, Records and Forced Structure

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Abstract

A physical description must preserve distinguishable occurrences, the information needed to compare them, and records available to later interactions. We examine what these requirements force within Recognition Science, keeping the premises of each implication explicit. Finite record constructions give arithmetic with a ledger of logical commitments. A specified comparison apparatus, continuous reporting and reference calibration fix the reciprocal cost, while a positive additive reading and a first costly join force golden scaling. A linking requirement selects three dimensions within the stated spherical spatial class; a complete one-bit cyclic pass through three binary coordinates then has minimum length eight. Factorization, order and calibration determine the corresponding recognition weight. In a finite quantum carrier, preservation of calibrated sharp-decision optima forces exact state recovery, including correlations with a retained reference. We also identify what these mathematical results require of a physical realization: acquisition of the supplier state, complete resource accounting, attachment of controller and record dynamics, turnoff, and record retention. Ethical and economic effects act on the same physical bond ledger. The experiential interpretation requires a further admissibility law and a distinct subject-composition condition. The result is a connected account of forced structure and its physical premises, with explicit boundaries between a uniqueness theorem, a constructed representation and an enacted process.

Keywords: Recognition Science; physical records; reciprocal cost; quantum recovery; foundations of physics; consciousness

1 The question is what a physical account must preserve

An occurrence can differ from another occurrence. A comparison requires information about both. Examining an earlier record is another occurrence. These operational facts are the starting point. They do not assume a permanently surviving memory, an observer outside the

world, or a completed infinite future. A record need only remain available for the comparison in which it participates. A later inspection lies beyond the earlier inspection's cutoff.

The question is how much structure these facts and their physical realization require. There are two connected tasks. One is a forcing argument: determine what any description satisfying specified demands must contain. The other is a construction: identify the physical states, operations, resources and records that enact those demands. An existence proof for a mathematical representation does not acquire a supplier state. A counting theorem does not build a clock. The same distinction lets us credit a forced result while locating the next physical obligation precisely.

The labels in this paper name the architecture: the floor and statability at T−1, arithmetic at T0, the meta-principle at T1, stable distinction at T2, ledgers at T3, recognition at T4, cost at T5, golden scaling at T6, complete clocks at T7, spatial linking at T8, measure at T9 and absolute physical scale at T10. Spatial linking is discussed before clocks because the dimension fixes the number of binary coordinates. T−2 supplies a witness to the floor; it does not eliminate the mathematical language in which the witness is stated.

1.1 Relation to existing work and the contribution

Operational reconstructions ask which information-processing requirements select a physical theory. Chiribella, D'Ariano and Perinotti derive quantum theory from informational principles, with purification distinguishing it within their class [1]. Here the finite quantum carrier is supplied when recovery is studied. The question at that stage is which participant consequences an adequate report must preserve; the recovery argument does not independently derive the quantum carrier.

The arithmetic construction develops the distinction-generated record account of Washburn and Zlatanović [2]. The reciprocal cost has an earlier uniqueness characterization through a composition law and local calibration [3]. The reporting argument here uses a different stated premise: the energy ratio literally produced by a specified apparatus. These routes have different hypotheses, so one cannot replace the apparatus premise by reciprocal symmetry alone.

Several mathematical components are established tools. The complement-homology result used to express spatial linking is an instance of Alexander duality [4, Corollary 3.45]; the argument below supplies a direct open-cover proof in the class needed here. The attaining clock is a binary reflected Gray cycle, a standard combinatorial construction [5]. The scalar Kraus-overlap criterion reached by the decision argument is the Knill–Laflamme condition for exact quantum correction [6]. Those results are credited at their existing scope. Their physical interpretation still needs the particular spatial, clock and operational premises stated here.

The contribution is to connect these implications through explicit comparisons and retained records, and to follow the connection far enough to expose its physical construction requirements. The native reporting calculation fixes what the apparatus reports. The finite recovery argument passes from achievable calibrated decision optima to a channel recovering the entire state and every finite retained reference. The field and resource analysis then locates the concrete acquisition, attachment and retention equations that an enacted account must satisfy.

These are distinct conclusions; their hypotheses cannot be combined into an unconditional derivation by omitting the intermediate physical identifications.

1.2 Mathematical implications and physical realization

The mathematical arguments are given in the article, with the hypotheses of each implication stated explicitly. Identifying a mathematical carrier with a physical apparatus requires the corresponding preparation, interaction and record conditions. The unresolved construction equations are stated separately from the results they would realize.

2 T−2, T−1 and T0: a floor on which a claim can be stated

2.1 The extremal distinction

Let the empty type have no inhabitants and the unit type have one. They cannot be equal: transporting the unit inhabitant across such an equality would produce an inhabitant of the empty type. Thus there are two distinct types. This is an exact extremal witness to distinguishability. The type-forming and equality rules used to state the witness remain part of the mathematical language.

Here is the complete carrier-level equivalence. On a nonempty carrier K, call a specification nontrivial when it consists of a predicate P together with an inside witness x satisfying $P(x)$ and an outside witness y satisfying $\neg P(y)$. The absolute-floor package asks for such a specification and two distinguishable propositions in the surrounding language.

Proposition 2.1. *For a nonempty carrier K, the absolute-floor package exists if and only if K has two distinct elements.*

Proof. Given the package, choose its inside and outside witnesses x, y. If $x = y$, transport the proof of $P(x)$ to a proof of $P(y)$, contradicting the outside witness. Hence $x \neq y$.

Conversely, given $x \neq y$, take the predicate $P(z)$ to mean $z = x$. Reflexivity puts x inside, while symmetry of equality and $x \neq y$ put y outside. For the two propositions, use truth and falsehood. If those propositions were equal, transporting the proof of truth would give a proof of falsehood. These witnesses supply both fields of the absolute-floor package. □

In particular, if a carrier has no nontrivial specification, it has at most one element: any proposed pair of distinct elements would construct the forbidden equality predicate. This is the exact floor established here. It does not derive the language of types, predicates, equality and transport from an absence of all structure. Nor does the empty type, by its mathematical definition alone, identify a physical state of nothingness.

2.2 The licensed language of records

The arithmetic result starts with a finite syntax and its construction rules, following the distinction-generated arithmetic of [2]. Terms have five forms: a variable, zero, successor of a term, sum of two terms, and product of two terms. Formulas have equality of terms, falsehood,

conjunction, disjunction, implication, universal quantification and existential quantification. Variables are numbered; a binder extends an environment by placing its new value at index zero and shifting the old values one position. These are the licensed constructors. The surrounding type theory supplies inductive formation, structural recursion, equality and quotients. The result concerns what those explicit resources construct.

For any carrier with an assigned value for each variable and operations of these five term forms, define interpretation recursively: a variable gets its assigned value, zero gets the designated zero, and each compound term is interpreted by the designated operation applied to the interpretations of its subterms. This interpretation preserves all constructors by its definition.

Proposition 2.2. *There is exactly one constructor-preserving interpretation of the term syntax in any such assigned algebra.*

Proof. The recursive interpretation gives existence. To show uniqueness, compare another constructor-preserving map with it by induction on the term. For a variable and for zero, preservation gives the required equality directly. For a successor, apply the target successor operation to the induction hypothesis. For a sum, apply target addition to the two induction hypotheses. For a product, do the same with target multiplication. These are all five constructors, so the maps agree on every term. □

This is initiality of syntax. It does not say that an arbitrary assignment of operations already satisfies arithmetic laws. Those laws must be established for the carriers that realize records.

2.3 Counting and operating on finite record strings

One realization counts with natural numbers. A second, independently defined realization uses finite lists of a single token •. Its zero is the empty list, its successor prepends one token, and its addition appends lists. Its multiplication iterates addition on the second list:

$$x \otimes [] = [], \qquad x \otimes (\bullet :: y) = (x \otimes y) +\!\!+ x.$$

Neither this multiplication nor the list evaluator is defined by first evaluating a natural number. Let $|x|$ denote list length. The definitions give

$$|[]| = 0, \quad |\bullet :: x| = |x| + 1, \quad |x +\!\!+ y| = |x| + |y|, \quad |x \otimes y| = |x||y|.$$

The last identity follows by induction on y. The empty case is zero; the successor case adds $|x|$ to $|x||y|$, giving $|x|(|y| + 1)$.

Every one-token list is the repetition of its token exactly its length many times. Induct on the list: this holds for the empty list, and adjoining its only possible token preserves the assertion. Consequently, equal lengths imply equal lists. Length and token repetition are explicit inverse maps between the two carriers. Successor is injective on lists because equal nonempty lists have equal tails; zero cannot be a successor because an empty list cannot be a nonempty list. List induction supplies the induction rule. A singleton carrier with successor equal to its sole element fails the zero-not-successor requirement, so the hypotheses exclude a collapsed realization.

In the native orbit arithmetic, addition and multiplication obey the same recursions:

$$a + 0 = a, \quad a + Sb = S(a + b), \qquad a \cdot 0 = 0, \quad a \cdot Sb = a \cdot b + a.$$

Induction on the second argument proves that the natural-number display preserves both operations; the display has the explicit inverse that forms the corresponding finite orbit. Thus equality of displays reflects equality of these orbits. The zero-addition and successor-addition identities follow by induction; they give commutativity of addition by induction, and associativity follows by induction on the last argument. These equations operate on the constructed records. For example, adding a one-token record to another one-token record produces a two-token record, while the natural display gives $1 + 1 = 2$.

2.4 Truth is preserved and reflected

Give the term language its direct natural evaluator and its direct list evaluator. Say that two environments are related when every list assigned to a variable has the length assigned to that variable in the natural environment. Induction on the five term constructors gives

$$|\operatorname{eval}_L(t)| = \operatorname{eval}_N(t).$$

The variable case is the environment relation, zero is immediate, and successor, addition and multiplication use the four length identities just proved.

Proposition 2.3. *For every formula and related environments, list satisfaction holds if and only if natural-number satisfaction holds. Consequently a formula is true in every natural environment if and only if it is true in every list environment.*

Proof. Induct on the formula. Equality is preserved by taking lengths, and reflected because equal lengths imply equal lists. Falsehood is the same proposition in either interpretation. Conjunction and disjunction transfer their components; implication transfers its antecedent in the reverse direction and its consequent in the forward direction, and conversely.

For a universal quantifier, a list-universal assertion supplies the natural case n by using the list of n tokens. A natural-universal assertion supplies the case of a list x by using $|x|$. The extended environments remain related in each direction. For an existential quantifier, send a list witness to its length, or a natural witness to its token repetition, and apply the induction hypothesis in those extended environments. This exhausts all seven formula constructors. Finally, for arbitrary environments use pointwise length in one direction and pointwise repetition in the other. The preceding equivalence then proves the assertion about validity. □

The derivation checker makes a further distinction explicit. It either rejects a finite derivation tree or returns a formula together with a ledger of the principles used. A forced certificate here returns the empty ledger. Its soundness is proved by induction on that derivation tree with the context and environment allowed to vary. A hypothesis selects a true context entry; reflexivity and substitution use equality and its semantic transport law; the successor and operation rules use the displayed arithmetic identities. The induction rule uses its accepted base and successor derivations and ordinary induction in the constructed natural carrier. Implication introduces or applies a true assumption; conjunction forms or projects a pair; disjunction

forms or eliminates its two cases; falsehood eliminates a contradiction. Universal introduction extends the environment by an arbitrary value, and universal elimination substitutes the evaluated term. Existential introduction supplies that term's value; elimination uses the witness supplied by the existential premise. The binder substitution and lifting equations are the recursive semantic equations for numbered variables.

The additional excluded-middle, limited-omniscience and Markov rules each post their own flag. Soundness uses the corresponding principle only when that flag is present; flags of a premise remain present in the union ledger of the conclusion. For an empty ledger all three gates have false antecedents, so none of these extra principles is required. The quantifier-free restriction on the Markov rule is checked structurally. Full-induction use is recorded by its separate flag. Combining soundness with the formula equivalence proves that every empty-ledger certificate from the empty context is true in both concrete realizations. This is a preservation-and-reflection theorem for the stated language and carriers, not an assertion that any host implementation or any collapsed algebra has been certified.

2.5 Signed and ratio records

The extension to integers identifies pairs of nonnegative orbit records (p, n) by the balance relation

$$(p, n) \sim (p', n') \quad \Longleftrightarrow \quad p + n' = p' + n.$$

Addition is componentwise, negation swaps the components, and multiplication sends two pairs to $(pp' + nn', pn' + np')$. Expanding their natural displays shows that these operations display as sum, negative and product of the differences. Balance is equivalent to equality of those differences. Hence the operations respect balance, descend to the quotient, and the integer display is injective: equal displays give balanced representatives, which give equal quotient classes.

A ratio record consists of a signed numerator and a nonzero nonnegative denominator. Its equivalence is balanced cross-multiplication. The denominator of a sum or product is the product of the two nonzero denominators; the numerators are the usual cross-sum or product, formed with the operations already constructed. Cross-multiplication is equivalent to equality of rational displays because the denominators are nonzero. Expanding the numerator formulas and clearing these denominators proves preservation of addition and multiplication, and shows that the operations respect equivalence. The quotient therefore retains the signed and ratio arithmetic without treating an integer or rational primitive as a new object-language term constructor. The licensed quotient and surrounding mathematics remain explicit; a real continuum requires its further resources. None of these results identifies every physical state with a finite count or proves physical space discrete.

3 T1–T4: existence, stable distinction, ledgers and recognition

3.1 T1: the meta-principle

Proposition 3.1. *If a relation on a carrier has a self-related element, that carrier is nonempty.*

Proof. A witness to the self-relation includes an element of the carrier. That element witnesses nonemptiness. □

This elementary statement fixes a useful logical direction. Recognition cannot occur on an empty carrier. Deriving a nontrivial physical ledger requires further structure: the relation alone does not specify a charge, a cost or a conservation law.

3.2 T2: what stable isolation says

The stable-existence predicate used here asks for zero defect and a positive isolation radius. In the one-dimensional cost model, the equilibrium is at ratio one. If every neighbourhood of that point contains a different admitted configuration, positive isolation is impossible. Given a proposed isolation radius, the density premise supplies a different configuration closer than that radius, a contradiction.

The scope matters. This excludes stable isolation at the specified densely accumulated equilibrium under that predicate. It does not prove that every configuration space is globally discrete. A two-class observable quotient can also be constructed from a supplied distinction, but that Boolean quotient is a floor construction, not the entire physical architecture.

3.3 T3: reciprocal events and double entry

A ledger event has a source, a target and a positive comparison ratio. Reversing the event exchanges the endpoints and inverts the ratio. Reversal twice returns the original event. The logarithmic postings cancel:

$$\log r + \log(r^{-1}) = 0 \qquad (r > 0).$$

An event list containing each event together with its reverse therefore has balanced counts. This constructs a double-entry ledger. The algebra explains why reciprocal postings balance; a claim that an actual interaction posts exactly this pair still requires its physical event identification. Reciprocal symmetry of a cost function alone does not enact the postings.

3.4 T4: recognition from available readings

A supplied extraction mechanism assigns an observable reading to each state. Equality of extracted readings is reflexive and symmetric. If extraction is nonconstant, it also retains a distinction. The recognition construction uses these facts to give the carrier an observable comparison relation.

This provides a concrete relation from actual readings. It does not say that every conceivable relation has the same operational meaning, nor that equality of readings alone supplies a physical tick. The clock needs an ordered process, an available record and a rule for advancing it.

4 T5: reciprocal cost and its calibration

For a positive ratio, the canonical reciprocal cost [3] is

$$\mathcal{J}(r) = \frac{r + r^{-1}}{2} - 1 = \frac{(r-1)^2}{2r}, \qquad r > 0.$$

Proposition 4.1. *The cost is nonnegative, vanishes precisely at $r = 1$, and is invariant under reciprocal comparison.*

Proof. Multiplication by $2r$ verifies the displayed identity. Its denominator is positive and its numerator is a square, giving nonnegativity and the stated zero. Exchanging r and r^{-1} leaves the first expression unchanged. □

The cost of a comparison is therefore independent of which participant supplies the ratio's numerator. The logarithmic imbalance can change sign while this reciprocal cost stays fixed.

4.1 The records that determine the reported cost

The native comparison occupies an eight-coordinate complex register. Write its coordinates in clock order as

$$c(a,b) = \frac{1}{\sqrt{2}}(a, -a, 0, 0, -b, b, 0, 0),$$

and measure energy by the sum of squared coordinate magnitudes. Each individual operand has unit norm when its amplitude is one. The minus sign in the second half is part of this original encoding; it is used once. Let P retain the odd Fourier modes $1, 3, 5, 7$, and let S be the cyclic shift. Since S^4 acts as minus one on odd modes and plus one on even modes,

$$(I - P)f = \tfrac{1}{2}(f + S^4 f).$$

Applying this identity to the displayed four nonzero coordinates gives

$$(I-P)c(a,b) = \frac{1}{2\sqrt{2}}(a-b, -a+b, 0, 0, a-b, -a+b, 0, 0), \qquad \|(I-P)c(a,b)\|^2 = \frac{|a-b|^2}{2}.$$

Thus the normalization is fixed before any ratio is reported.

Write $\varphi = (1 + \sqrt{5})/2$. The apparatus uses its actual signed Gray formation at its base point; the next section gives the recognizing-join argument that forces this same number as a scale ratio. In the same Fourier coordinates its normalized formation operator N has multiplier 1 on the retained odd modes and $-\varphi^{-2}$ on their complement. These are the two multipliers of the golden formation, φ and $-\varphi^{-1}$, divided by its primitive multiplier φ. Consequently, with $h = 1 + \varphi^{-2} > 0$,

$$I - N = h(I - P).$$

This verifies the contracting complementary channels as well as the retained ones. The conjugation between the cyclic and eight-coordinate carriers is their literal coordinate identification.

The relative residual subtracts formation from its measured primitive response: if that response is β, it is $\beta I - A_{\mathbb{C}}$, where $A_{\mathbb{C}}$ is the complex extension of the real operation. For this actual

base operation, $A_{\mathbb{C}} = N$, its primitive multiplier is $\beta = 1$, and the residual is exactly $h(I - P)$. This identity is supplied by the actual operation, rather than assumed as a cost law. More generally, the source-span argument uses only the two original formation-atom images to establish the corresponding residual on the native operands. It makes no claim that those two inputs determine an arbitrary operation's additional DC response.

For real positive amplitudes A, B and a unit phase z, the residual energy is therefore

$$D_z(A, B) = \frac{h^2}{2}|A - zB|^2 = \frac{h^2}{2}(A^2 + B^2 - 2AB\,\mathrm{Re}\, z).$$

The reference is a measured change of records, with the same operands and residual in both measurements:

$$R_\kappa(A, B) = \frac{\kappa}{2}\big(D_i(A, B) - D_1(A, B)\big) = \frac{\kappa h^2}{2}AB > 0, \qquad \kappa > 0.$$

The first equality specifies the reference program. The second follows by inserting $\mathrm{Re}\, i = 0$ and $\mathrm{Re}\, 1 = 1$; it derives the product AB, rather than placing that product in the reporting interface. The selected ratio meter divides a nonnegative energy record by its positive reference. Neither a different reference program nor a different response law is being silently included.

Proposition 4.2. *Suppose $\kappa > 0$, F is continuous on the positive real ratios, and for every pair of positive natural populations a, b it literally reports this residual-energy record against this reference:*

$$F(a/b) = D_1(a, b)/R_\kappa(a, b).$$

Then $(\kappa/2)F(x) = \mathcal{J}(x)$ for every $x > 0$. In particular, calibration $\kappa = 2$ forces $F = \mathcal{J}$ on that domain.

Proof. Both h and the amplitudes are nonzero, so cancellation in the measured quotient gives

$$F(a/b) = \frac{(a - b)^2}{\kappa ab} = \frac{2}{\kappa}\mathcal{J}(a/b).$$

Every positive rational has such a positive numerator and denominator. For an arbitrary $x > 0$, choose positive rationals $r_n > x$ decreasing to x. The two continuous functions F and $(2/\kappa)\mathcal{J}$ agree at every r_n. Taking their limits gives equality at x. This proves the assertion on the entire positive domain. □

Balanced zero and reciprocal symmetry of the reported function follow from this calculation; they are not extra assumptions about F. Continuity is an independent completion premise. Indeed, choose a positive irrational x_0, change the calibrated function's value there by one, and leave every other value fixed. All positive-natural population reports remain identical, while equality on the real domain fails. This exact control explains why rational reporting alone is insufficient. The theorem establishes what the specified apparatus reports; enacting that apparatus with the complete physical resources remains part of the construction below.

5 T6: the first costly join fixes golden scaling

The recognizing-join construction starts with an additive commutative group G, an additive value map $L : G \to \mathbb{R}$, and an additive successor $A : G \to G$. A framework state s has a reading $v(s) \in G$, positive value $r(s) = L(v(s))$, and successor compatibility $v(Ts) = A(v(s))$. The substantive order condition applies to every native additive comparison:

$$L(x) > 0 \implies L(Ax) > 0 \qquad (x \in G).$$

Positivity only along one orbit would be weaker. No basis, real scalar multiplication on G, injectivity of L, or invertibility of A is required.

Proposition 5.1. *Let $L, M : G \to \mathbb{R}$ be additive, and suppose $L(e) > 0$ for some $e \in G$. If $L(x) > 0$ implies $M(x) > 0$ for every x, then*

$$M(x) = \frac{M(e)}{L(e)} L(x) \qquad (x \in G).$$

Proof. The premise first gives $M(e) > 0$. If $L(x)/L(e) < M(x)/M(e)$, choose a rational a/b strictly between them, with $a \in \mathbb{Z}$ and b a positive integer. Additivity, including integer multiples, gives for $y = ae - bx$

$$L(y) = aL(e) - bL(x) > 0, \qquad M(y) = aM(e) - bM(x) < 0.$$

This contradicts preservation of positive comparisons. If the strict inequality is reversed, choose a rational strictly between the ratios and use $y = bx - ae$; again $L(y) > 0$ and $M(y) < 0$. The ratios are therefore equal. Multiplying by the positive denominators proves the formula. □

Apply this result to $M = L \circ A$ and $e = v(s_0)$, for any base state. It supplies a single $q = L(Ae)/L(e) > 0$ such that $r(Ts) = qr(s)$ for every state. Hence the actual orbit levels $s_n = r(T^n s_0)$ satisfy $s_n = q^n s_0$, by induction on n.

The first new level is required to be a costly binary join of earlier levels. Precisely, there are natural indices $a \le b < 2$ with $s_2 = s_a + s_b$, and the source join's recognition cost is strictly positive. An equal-input join has zero source join cost, so $a = b$ is impossible. The only remaining indices are $a = 0, b = 1$. Thus

$$s_2 = s_0 + s_1, \qquad q^2 s_0 = s_0 + q s_0.$$

Since $s_0 > 0$, cancellation gives $q^2 = q + 1$. Its roots are $(1 \pm \sqrt{5})/2$; exactly one is positive. Therefore

$$q = \varphi = \frac{1 + \sqrt{5}}{2}, \qquad s_n = \varphi^n s_0.$$

The physical recognition premise can be supplied by the deformation-erasure principle on the actual join kinematics, which forces a nonunit join ratio and thus positive cost. That application remains an identification of the actual join. The proof above shows exactly where that premise is used and why preservation of all additive positive comparisons forces the common scalar before the costly join selects its value.

6 T8: dimension from the specified linking requirement

6.1 The topological requirement

The complement calculation below is the relevant Alexander-duality consequence [4, Corollary 3.45]. The physical content lies in requiring this detector and identifying its substrate. Write S^D for the unit sphere in $\mathbb{R}^{D+1}$. The detector condition is content-typed: there exists a topological embedding $f : S^1 \hookrightarrow S^D$ whose complement has nonzero first singular homology with integer coefficients,

$$H_1(S^D \setminus f(S^1); \mathbb{Z}) \neq 0.$$

This condition mentions the actual embedded circle and its complement. It does not define a detector by the numeral three. We show that it holds exactly when $D = 3$, allowing arbitrary topological embeddings, including wild ones.

6.2 Why the open-cover argument applies to finite chains

A singular n-chain is a finite integer linear combination of continuous maps from the standard n-simplex. Its boundary is the alternating sum of face restrictions. The cancellation of pairs of codimension-two faces gives $\partial^2 = 0$; homology is cycles modulo boundaries. In particular, a first-homology class vanishes exactly when its representing one-cycle bounds a finite two-chain.

For an open cover $X = U \cup V$, let the small-chain complex be generated by simplices lying entirely in either member. Subdivision supplies the bridge to all singular chains. On each compact parameter simplex the inverse images of U, V form an open cover. Its Lebesgue number and the shrinking mesh of repeated barycentric subdivision imply that sufficiently subdivided pieces lie in a single cover member. A chain has finitely many simplices, so a common finite subdivision works for the entire chain. The subdivision operator commutes with boundary, and its telescoped prism homotopy gives $c - \mathrm{sd}^m c = \partial T_m c + T_m \partial c$. Thus cycles and bounding relations can be computed in small chains without changing homology.

There is a short exact sequence of these chain complexes. Its first map sends a chain in $U \cap V$ to its copies with opposite signs in U and V; its second adds the two images in the small-chain complex. Every small simplex belongs to one member, so the addition map is onto. If two chain images cancel, equality of their integer coefficients in the free simplex basis shows that every surviving simplex lies in both members; this proves exactness in the middle. Following boundaries through this sequence gives the part of the open-cover homology sequence used here:

$$H_2(X) \longrightarrow H_1(U \cap V) \xrightarrow{(i_*, -j_*)} H_1(U) \oplus H_1(V).$$

In particular, if $H_2(X) = 0$, a one-cycle of the intersection that bounds in each member already bounds in the intersection. If in addition $H_1(U) = H_1(V) = 0$, every one-cycle there bounds. These are the two precise uses of Mayer–Vietoris below.

For completeness, the sphere homology needed here follows from the same construction. Cover a positive-dimensional sphere by the complements of its two poles. Each member is contractible by stereographic projection; their intersection is homotopy equivalent to the equatorial sphere one dimension lower. For degree at least two, the open-cover sequence

identifies the sphere's homology with the intersection's homology one degree lower. In degree one and sphere dimension at least two, that intersection is path connected: a degree-zero cycle is determined in each connected component by the sum of its coefficients, and the map from this single-component group into the two cover groups is injective. Exactness therefore makes the first homology vanish. The zero-sphere is two points, and its positive homology vanishes because constant positive-degree cycles bound. These facts give, by induction, $H_k(S^n) = 0$ for $k > 0$ and $k \neq n$.

For the nonvanishing needed in dimension one, cover the circle by its two punctured circles. Their intersection has two arcs. Choose one point in each arc and take their degree-zero difference. It is nonzero in the intersection: summing coefficients on just the first arc pairs it to one, and this sum vanishes on boundaries since a continuous path cannot cross between the two arcs. In either punctured circle the points are joined by a path, so the difference becomes a boundary there. Exactness places this nonzero difference in the image of the connecting map from $H_1(S^1)$. That group must therefore be nonzero. Homotopies preserve homology because the prism between homotopic simplex maps is a chain homotopy. These reductions do not presume the dimension conclusion.

6.3 An embedded arc leaves no first-homology obstruction

Proposition 6.1. *For every embedding $a : [0,1] \hookrightarrow S^D$, the complement of its image has zero first homology.*

Proof. Suppose a one-cycle z in the full arc complement does not bound. For a closed parameter subinterval I, call I bad if the same cycle, included in the complement of $a(I)$, still does not bound. The full interval is bad.

Bisect a bad interval at its midpoint. The two half-arc images K_-, K_+ are compact, hence closed, and meet in exactly the midpoint image: this uses injectivity of a. Their complements are an open cover of the sphere minus that single point, and their intersection is the complement of the original interval's arc. The once-punctured sphere is contractible, so its second homology is zero. If the cycle bounded in both half-arc complements, the open-cover implication would make it bound in the original complement. At least one half is therefore bad.

Iterating gives nested bad intervals $[u_n, v_n]$ of width 2^{-n}. Their common point is t_*, obtained as the supremum of the left endpoints; every right endpoint is an upper bound. Put $p = a(t_*)$. In the complement of p, the included cycle bounds a finite two-chain w, since that punctured sphere is contractible. The union K of the images of the finitely many simplices in w is compact and avoids p. Hence $a^{-1}(K)$ is closed and misses t_*, leaving a parameter neighbourhood of t_* whose arc misses K.

For sufficiently large n, the whole interval $[u_n, v_n]$ lies in that neighbourhood: both it and t_* lie between endpoints separated by 2^{-n}. Every simplex of w then takes values in the complement of $a([u_n, v_n])$, so w lifts to a two-chain there. Its boundary is the included cycle z. To see that the equality survives the lift, push both chains into the sphere; their equality there is the original boundary equation, and inclusion is injective on the free singular-chain basis. This contradicts badness of the chosen interval. Thus every one-cycle bounds. □

The argument uses compactness of finite chain support, not local flatness or a tubular neighbourhood of the arc. This is what preserves the stated scope for arbitrary embeddings.

6.4 Only the three-sphere admits the detector

Proposition 6.2. *An embedded-circle complement with nonzero first integer homology exists in S^D if and only if $D = 3$.*

Proof. First let $D \geq 1$ and $D \neq 3$. Split the parameter circle into its closed upper and lower semicircles. Their embedded images K_+, K_- are embedded arcs, cover the circle image, and meet in exactly two distinct endpoint images p, q. Inside $X = S^D \setminus \{p, q\}$, the sets $U = S^D \setminus K_+$ and $V = S^D \setminus K_-$ are open and cover X. Their intersection is the full embedded-circle complement. Each member has zero first homology by the arc result.

Stereographic projection at p identifies X with $\mathbb{R}^D$ minus the image of q. Translate that image to zero and use polar coordinates. The positive radial factor contracts to one, so X is homotopy equivalent to S^{D-1}. Its second homology is zero because $D-1 \neq 2$. The open-cover implication therefore gives $H_1(U \cap V) = 0$, contrary to the detector condition. In dimension zero the ambient sphere and every complement are finite discrete spaces. Every singular simplex is constant; a constant one-simplex is the boundary of the constant two-simplex. Their first homology is zero as well.

In dimension three use the flat circle $u(z) = (z, 0)$ in $S^3 \subset \mathbb{R}^2 \times \mathbb{R}^2$. A point (x, y) of its complement has $y \neq 0$, so

$$r(x, y) = y/\|y\| \in S^1$$

is continuous there. The core circle $j(z) = (0, z)$ lies in the complement and satisfies $r \circ j = \mathrm{id}_{S^1}$. On homology, $r_* j_*$ is the identity of the nonzero group $H_1(S^1)$. It could not factor through a zero group. Thus this complement has nonzero first homology, giving the required witness. □

The physical dimension conclusion has one further, explicit condition. A spatial realization supplies the actual deformation kinematics of the two recognition traces and identifies the existence of a nonzero deformation-invariant pairing with the detector condition just proved. The deformation-erasure principle on those kinematics supplies that pairing. Applying the mathematical result then gives $D = 3$. The spatial-realization equivalence and the actual deformation principle remain premises of this physical application; the mathematical exclusion of other dimensions and the three-dimensional witness are already established without them.

7 T7: eight ticks for a complete pass

The combinatorial construction is the binary reflected Gray cycle [5]. Its identification with a complete physical pass is a separate condition. Let a pattern consist of d binary coordinates. A pass is an ordered finite list of patterns. The native completion condition says that the pass occupies one item of the floor above: every lattice position at that item address has its pattern reached. The fibre of one item contains every binary pattern, so occupying an item is equivalent to covering every pattern. This equivalence follows by assembling a position from an arbitrary address and pattern, and conversely reading the pattern of each position in that

fibre. The process condition separately requires exactly one coordinate to change per tick, including the closing step from the final pattern to the initial one.

Proposition 7.1. *A complete pass has length at least 2^d. Its length is 2^d exactly when its pattern map is injective. For $d > 0$, a complete one-bit cyclic pass of that minimum length exists.*

Proof. There are 2^d binary patterns. Completeness makes the map from tick indices onto those patterns surjective, so the number of ticks cannot be smaller. A surjection between finite sets of equal size is a bijection. Conversely, an injective complete pass is a bijection and has exactly that many indices.

For attainment define a list G_d recursively. G_0 contains the single empty word. If $G_d = (g_0, \dots, g_{N-1})$, where $N = 2^d$, append a new last bit and put

$$G_{d+1} = \big((g_0, 0), \dots, (g_{N-1}, 0), (g_{N-1}, 1), \dots, (g_0, 1)\big).$$

Induction proves that every word appears exactly once. A word ending in zero occurs in the first half at its prefix's unique old index. A word ending in one occurs in the reversed second half at that prefix's unique reversed index. Words in different halves have different last bits, so the halves are disjoint. The list has $2N = 2^{d+1}$ entries and covers every word.

For $d = 1$ the list is $(0, 1)$, whose forward and closing steps each change the sole bit. Suppose $d > 0$ and the old list has the one-bit property. Consecutive entries inside the first half inherit it with a common final zero. The middle step from $(g_{N-1}, 0)$ to $(g_{N-1}, 1)$ changes only the new bit. Steps inside the second half traverse old steps backwards, which changes the same single old bit, with a common final one. The closing step from $(g_0, 1)$ to $(g_0, 0)$ changes only the new bit. These exhaust all cyclic steps and complete the induction. The singleton G_0 supplies the counting base but has no one-bit closing step, which is why attainment as a one-bit cycle is stated for $d > 0$. □

In dimension three the least complete cyclic pass length is eight. The physical clock must still realize the specified complete process. Eight possible states alone do not establish eight actual ticks. Pass length and a repeated sequence's least period are different quantities: a longer pass may repeat an eight-period pattern. No period conclusion follows merely from writing down eight labels.

8 T9: multiplicative recognition weight

Consider a positive weight w on the nonnegative integers satisfying

$$w(m + n) = w(m)w(n), \qquad w(1) = \frac{1}{1 + w(1)}.$$

Proposition 8.1. *These conditions force $w(n) = \varphi^{-n}$ for every nonnegative integer n.*

Proof. Positivity and $w(0) = w(0)^2$ give $w(0) = 1$. Put $x = w(1) > 0$. The fixed-point equation gives $x^2 + x = 1$, whose positive root is φ^{-1}. Multiplicativity and induction give $w(n) = x^n$. □

The fixed-point condition is an explicit requirement of the recognition-weight rule. Multiplicativity alone would leave the first-step weight free.

8.1 Subdivision fixes the continuum weight

Put $\rho = \varphi^{-1}$, so $0 < \rho < 1$. On nonnegative real arguments assume

$$f(a+b) = f(a)f(b) \quad (a,b \geq 0), \qquad a \leq b \Longrightarrow f(a) \geq f(b), \qquad f(1) = \rho.$$

Continuity and positivity of f are not additional premises.

Proposition 8.2. *Under these conditions, $f(t) = \rho^t = \varphi^{-t}$ for every $t \geq 0$.*

Proof. Factorization gives $f(0) = f(0)^2$. Antitonicity gives $f(0) \geq f(1) = \rho > 0$, so $f(0) = 1$. For $x \geq 0$, factorization at $x/2+x/2$ yields $f(x) = f(x/2)^2 \geq 0$. Induction gives $f(nx) = f(x)^n$ for natural n, and in particular $f(n) = \rho^n$.

For natural p and positive natural q, set $x = p/q$. Then

$$f(x)^q = f(qx) = f(p) = \rho^p.$$

Both $f(x)$ and $\rho^{p/q}$ are nonnegative, and the latter has the same q-th power. The power function is strictly increasing on the nonnegative reals for positive integer q, so the two bases are equal. This fixes every nonnegative rational value.

It remains to fix an arbitrary $t > 0$. Write $L = f(t)$. A rational $u \geq t$ gives $L \geq f(u) = \rho^u > 0$. Put $\ell = \log \rho < 0$. Suppose first that $L < \rho^t$. Then

$$t < \frac{\log L}{\ell}.$$

Choose a rational u strictly between these numbers. It is positive. Multiplication by the negative ℓ reverses the inequality $u < \log L/\ell$, giving $\log L < u\ell$, hence $L < \rho^u$. But $u > t$ and antitonicity give $\rho^u = f(u) \leq L$, a contradiction.

If instead $L > \rho^t$, then $\log L/\ell < t$. Since $t > 0$, there is a rational u with

$$\max\{0, \log L/\ell\} < u < t.$$

Now $u\ell < \log L$, so $\rho^u < L$, while $u < t$ and antitonicity give $L \leq f(u) = \rho^u$. This is again impossible. Thus $L = \rho^t$. The already established value at zero completes the proof. □

The continuum law follows from factorization, order and the calibrated step. Its normalization alone does not derive a physical Born rule, a dynamical history measure or an enacted feedback controller. Those identifications require the actual state, evolution and record joins below.

9 T10: absolute window energy

At the physical endpoint, a settled eight-tick photon window has a squared-norm energy. A closed topological pattern has a separately defined primitive amplitude. Their intended absolute identification is

$$E_{\text{window}} = \|w\|_8^2 = A(\psi)^2 = E_{\text{topology}}.$$

The established equivalence reduces this equality, under the settled boundary model, to the corresponding topology-load match. Settling a window or normalizing its energy to one does not independently impose that match. The library contains a counterexample to that shortcut. A topology-matched absolute-energy boundary remains a physical obligation. This manuscript does not infer dimensional constants or an absolute physical scale from a unit convention.

10 The original field, preparation and a retained record

10.1 The carrier must retain the physical degrees of freedom

The original neutral-shell system has an antisymmetric two-fermion matter carrier over all sixty-four native orbitals and a full transverse symmetric Fock field. It retains pairs of distinct orbital modes, including distinct payload modes at the same position. A photon-number cutoff or a pair of distinguishable placeholders would change that carrier. The native clock and each finite holding price remain part of its Hamiltonian.

For an unbounded Hamiltonian, a state vector and a formal expression for its action are insufficient. The state must belong to its strong domain. A proposed physical encoding must preserve and reflect the required domain, recover the encoded state, intertwine the entire generator and eliminate leakage out of its encoded range. These demands apply to both settings of the actual apparatus.

10.2 An aligned load is a constructed state, not an acquisition

For positive lattice scale and positive electric coefficient, the aligned-post construction solves the twelve-coordinate contact response equation. For a selected contact it supplies a field profile whose designated electric reading is one; the other readings obey the specified site-and-axis selection rule. The profile is nonzero, since a zero profile would have zero inner product with every electric read vector, contradicting the unit reading.

This establishes an actual nonzero load on the full field carrier. It does not establish that the available apparatus prepares it. The preparation target is a backward-evolved supplier state whose subsequent original-Hamiltonian evolution produces the required post. Declaring that target available would remove the very acquisition problem that the physical construction must solve.

10.3 Counting free energy once

Write the same-field Hamiltonian as its free field energy plus its raw interaction terms, with the finite holding prices included when present. On one and the same full-Fock vector in the free-energy domain, adding two such Hamiltonians gives the combined raw interaction plus twice the free energy. Therefore

$$H_1 x + H_2 x = H_{\text{combined}} x + H_{\text{free}} x.$$

The prices add in the combined operator. This is simply the cancellation of two copies of each retained interaction term against the combined family, leaving one extra free-energy action.

A same-field sum equals the combined Hamiltonian precisely when that free-energy action vanishes; with positive electric coefficient, the stated kernel characterization makes the vector a vacuum multiple.

This identity concerns the same vector and the prescribed same-field raw families. It neither rules out a mixed encoding of two independent systems nor constructs one. An independent two-field model and a one-field model must be related by an explicit physical encoding, rather than identified by adding their formulas and dropping an energy term.

10.4 Actual acquisition and funding

The bank construction keeps the two original systems, the blank record and the stateful controller. Its generator includes both original Hamiltonians, the record and controller holding energies, the exchange term and the conditional contact term, each once. Denote that actual generator by K. The unresolved acquisition is the concrete orbit equation

$$e^{-iTK/\hbar}\,\Psi_{\text{vacuum bank}} = \Psi_{\text{required bank}}, \qquad T > 0.$$

Here the initial bank contains the actual vacuum supplier and the available fuel. The required bank contains the backward-evolved resource and the specified final fuel. Domain membership and self-adjointness must support this evolution. A unitary chosen to map one supplied endpoint to another does not identify an operation generated by K.

The endpoint energy ledger must retain both original-Hamiltonian expectations, their exchange overlap and every record and controller price. The current full-H endpoint expansions remain unverified source constructions. Conditional capacity inequalities constrain an acquisition if it occurs; they do not exhibit available fuel, prove an actual energy deficit or solve the orbit equation. A proposed pulse reflection also needs calibration to the actual generator. The current reflection route has not supplied that calibration.

10.5 The mobile attachment

The target mobile model retains four moving particle positions, all sixty-four orbital modes, two fixed core anchors, forty-eight core and thirty-two particle raw terms, the filled holding contribution $336R\epsilon$, the actual orbital clock and charge self-energy, sixteen plus sixteen signed contacts and twelve travel generators with both signs. Its carrier still has one full Fock field. The attachment target permits mixing matter, positions, record and controller coordinates. It requires recovery, equivalence of strong domains, the full-generator intertwining equation and zero leakage for both settings.

Preserving only selected field-profile images would be a narrower problem. An obstruction restricted to those prescribed images would leave the mixed attachment unresolved. Conversely, a generic compatibility interface with an unspecified encoding would not construct the physical map. The explicit map and its domain and generator equations remain unresolved.

10.6 Turning off, feedback and durable recording

The controller must turn its coupling off through its actual dynamics and retain the proper holding energy. The exact turnoff condition equates the controller auxiliary action with its

retained holding action on the attained state. A constant coupling cannot simply be declared switched off. The required stateful trajectory remains to be built.

The feedback program must likewise arise from the original setting-dependent flows with physical elapsed times, rather than from a symbolic selector whose desired probability is assumed. A fresh capture needs a funded supplier and an actual record transition. Durable recording also requires retaining previous full-field information while adjoining fresh record capacity. An isometry on a compressed binary subspace does not by itself solve the arbitrary full-field archive problem. These are separate obligations of the same apparatus, and none can be paid for by counting the same resource twice.

11 Ethics and economics concern these same physical transformations

Recognition Science treats ethics and economics as literal physical structure. A participant's moral state projects a bond ledger; its incident recognition costs, logarithmic skew and changes under action are different observables of that ledger. Economic production, exchange, sustaining life and affecting another participant transform these same relations. They are not an external ethical score attached to an otherwise complete physical process.

The effect-realization theorem makes one connection exact. Given the physical comparison profile on a fixed finite chart, the represented economic effect acts on the ledger reading exactly as the native physical action does. Its harm agrees with the change in incident recognition cost. The profile and its identification with the actual resources and needs remain conditions. An energy-capacity field in a state description is not a derivation that funds a preparation pulse.

Combining participants into a whole must retain the member ledger. For a fixed finite incidence chart, the sum of member incident costs equals the cost of the union of touched bonds plus the additional cost of bonds with two distinct member endpoints. Such an internal bond is counted twice by the member sum and once by the union; a self-loop is counted once in each. This identity explains why aggregate descriptions must retain the shared incidence rather than erase internal effects.

Love, the fourteen virtues, agency, consent, authority and history belong to this connected physical account. A scalar aggregate does not capture every member need or obligation. The present finite realization does not yet establish the complete physical profile for every economic history or every ethical relation. A whole-system description must keep participant effects and count shared physical resources once, including the actual supplier, controller and record resources used here.

12 Experience, recovery and the composition of subjects

Experience is an occurrence within the world being explained. The experiential reading of recognition gives a physical interaction an observable outside and an intrinsic experiential inside. Its positive case is continuity: the same structure that carries distinctions, comparisons and records also contains minds and their histories. The existence of a universal recognition

substrate, felt occurrence throughout that substrate and the claim that the whole is one subject are distinct claims.

12.1 The finite carrier and its consequential decisions

The recovery construction uses the exact quantum correction structure of Knill and Laflamme [6]. The argument first derives that structure from the specified consequential decisions. Take finite complex input and output spaces with coordinate sets I, O, a finite Kraus index set A, and a chosen input basis coordinate i_0. The actual report operation has the form

$$\mathscr{E}(X) = \sum_{a\in A} K_a X K_a^*, \qquad \sum_a K_a^* K_a = I_I, \qquad K_a : \mathbb{C}^I \longrightarrow \mathbb{C}^O.$$

The second equation is trace preservation, not an outcome-dependent renormalization. A density matrix is positive with trace one. A binary effect F obeys $0 \le F \le I_O$. On equally likely input preparations ρ, σ, its guessing success is

$$p_F = \tfrac{1}{2}\big(\operatorname{tr}(F\mathscr{E}(\rho)) + \operatorname{tr}((I_O - F)\mathscr{E}(\sigma))\big).$$

These traces are real. Positivity and trace preservation give $0 \le p_F \le 1$. The physical consequence controller is calibrated as $q_F = a + c\,p_F$, with real a, c and $c \neq 0$. Positive contrast uses a greatest consequence and negative contrast a least one. The sign-oriented score is

$$\operatorname{sign}(c)(q_F - a) = |c|\,p_F.$$

Availability remains a specified set of output effects. Neither the quantum carrier, the report operation, the preparations nor this consequence calibration is inferred from the word "decision."

For a family of input Hermitian involutions S_j, so $S_j^* = S_j$ and $S_j^2 = I_I$, put

$$P_j^+ = \tfrac{1}{2}(I_I + S_j), \qquad P_j^- = \tfrac{1}{2}(I_I - S_j), \qquad \rho_j^\pm = P_j^\pm / \operatorname{tr} P_j^\pm.$$

Both projections must be nonzero. Hermiticity and idempotence make them positive; a nonzero positive projection has positive trace, so these are well-defined density preparations. Require also a scalar joint commutant: every input matrix commuting with all S_j is a scalar multiple of I_I. This is a condition on separately tested observables, with no assignment of simultaneous sharp values.

Proposition 12.1. *Suppose the carrier, Kraus operation, sharp observables, nonzero sign preparations and scalar-commutant condition are as just stated. For each sign pair suppose the least upper bound of its available sign-oriented consequence scores is $|c|$. Then there is a completely positive trace-preserving map $\mathscr{R}$ defined on every output matrix such that*

$$\mathscr{R}(\mathscr{E}(X)) = X$$

for every input matrix. For every finite retained-reference dimension d, the same recovery acting on the reported factor alone satisfies

$$(\mathscr{R} \otimes \mathrm{id}_d)(\mathscr{E} \otimes \mathrm{id}_d)(X) = X$$

on every joint input-reference matrix, including entangled states. Moreover the report has a reversible form $\mathscr{E}(X) = W(X \otimes \tau)W^*$, where W is an isometry and τ is a fixed positive trace-one auxiliary matrix.

Proof. We first obtain perfect output effects without adding an attainment premise to the available set. The full finite-dimensional effect interval $[0, I_O]$ is closed and bounded, hence compact: positivity is closed, and an effect between zero and identity has operator norm at most one. Success is a continuous function of its matrix entries, so some effect F maximizes it on this full interval. Its success bounds every available success. The supremum premise therefore gives $|c| \le |c| p_F$, while $p_F \le 1$ gives the opposite inequality. Since $|c| > 0$, $p_F = 1$. Each of the two probabilities averaged in p_F is at most one; both must be one. Thus

$$\operatorname{tr}(F\mathscr{E}(\rho_j^+)) = 1, \qquad \operatorname{tr}(F\mathscr{E}(\rho_j^-)) = 0.$$

The maximizing effect need not belong to the available subset. Its existence in the full effect interval is sufficient for the following algebraic restriction on the already supplied report.

Fix j, and abbreviate its projections by P, Q. Trace preservation changes the first equality into zero expectation for $I_O - F$; remove the nonzero projector normalization from each equality. Expanding the Kraus sum gives a sum of nonnegative compression traces for each error. A zero such sum has every term zero. For a positive matrix B, zero trace of the positive compression $Y^* BY$ makes that compression zero, and the zero positive quadratic forms of its columns imply $BY = 0$. Apply this to $Y = K_a P$, $B = I_O - F$, and to $Y = K_b Q$, $B = F$. It follows that

$$FK_aP = K_aP, \qquad FK_bQ = 0, \qquad PK_a^* K_bQ = 0.$$

The last equality uses Hermiticity of F: move it from the first factor to the second. Interchanging a, b and taking adjoints gives the reverse cross block zero as well.

For $G = K_a^* K_b$, decompose with $P + Q = I_I$. The two cross blocks vanish, so $GP = PG = PGP$ and $GQ = QG = QGQ$. Subtraction shows that G commutes with $P - Q = S_j$. This holds for every test. The scalar-commutant condition therefore yields

$$K_a^* K_b = \alpha_{ab} I_I.$$

No recovery map has been assumed. In fact $\alpha_{ab} = \langle K_a e_{i_0}, K_b e_{i_0} \rangle$, so α is a positive Gram matrix. Evaluating the trace-preservation equation at (i_0, i_0) gives $\operatorname{tr} \alpha = 1$.

Diagonalize this finite positive Hermitian matrix by a unitary U, and mix the Kraus labels by $L_r = \sum_a U_{ar} K_a$. Its Gram equations become

$$L_r^* L_s = \delta_{rs} p_r I_I, \qquad p_r \ge 0, \qquad \sum_r p_r = 1.$$

This mixing leaves the report unchanged: on expanding $\sum_r L_r X L_r^*$, the coefficient of $K_a X K_b^*$ is $\sum_r U_{ar} \overline{U_{br}} = \delta_{ab}$. When $p_r = 0$, the equation $L_r^* L_r = 0$ makes $L_r = 0$. Discard only those zero branches and put $V_r = L_r / \sqrt{p_r}$ on the remaining index set. Then

$$V_r^* V_s = \delta_{rs} I_I, \qquad \mathscr{E}(X) = \sum_r p_r V_r X V_r^*.$$

Thus the report occupies orthogonal isometric ranges, with weights independent of X.

Let $P_{\mathrm{ran}} = \sum_r V_r V_r^*$ and $Q_{\mathrm{unused}} = I_O - P_{\mathrm{ran}}$. Orthogonality makes both projections Hermitian and idempotent and gives $Q_{\mathrm{unused}} V_r = 0$. Define recovery branches on the entire output space by

$$R_r = V_r^*, \qquad T_o = |i_0\rangle\langle o| Q_{\mathrm{unused}} \quad (o \in O).$$

The reset branches supply exactly the missing Gram:

$$\sum_r R_r^* R_r = P_{\mathrm{ran}}, \qquad \sum_o T_o^* T_o = Q_{\mathrm{unused}}\Big(\sum_o |o\rangle\langle o|\Big)Q_{\mathrm{unused}} = Q_{\mathrm{unused}}.$$

Their total is I_O, proving trace preservation on every output matrix. Define $\mathscr{R}$ as the sum of the conjugations by these branches. Each such conjugation preserves positivity, and the same is true with every branch tensored with an arbitrary finite identity; this proves complete positivity.

On an encoded branch $V_s X V_s^*$, the adjoint recovery branches give X from $r = s$ and zero from every other r. Every reset branch vanishes there. Linearity and $\sum_s p_s = 1$ therefore give $\mathscr{R}\mathscr{E}(X) = X$, for all matrices X, not just density matrices.

For the retained-reference statement, fix reference coordinates u, v. Multiplication by $K_a \otimes I_d$ acts on the (u, v) block by precisely K_a on the left and K_a^* on the right. The (u, v) block after report and recovery is consequently $\mathscr{R}\mathscr{E}(X_{uv}) = X_{uv}$. This includes off-diagonal reference blocks; equality of all blocks proves the joint identity. The tensor-extended recovery branches also satisfy completeness, since their Gram sum is $I_O \otimes I_d$. Cyclicity of trace proves trace preservation for every joint output matrix, and positivity of each branch conjugation proves positivity for every joint positive matrix.

Finally define $W(e_i \otimes e_r) = V_r e_i$ and $\tau = \mathrm{diag}(p_r)$. The equations $V_r^* V_s = \delta_{rs} I_I$ give $W^* W = I$; the weights make τ positive with trace one. Expanding the matrix entries yields $W(X \otimes \tau)W^* = \sum_r p_r V_r X V_r^* = \mathscr{E}(X)$, the asserted reversible form. □

An attained optimum is a stronger operational premise that reaches the same conclusion: for $c > 0$, an available greatest consequence $a + c$ gives an available effect with success one; for $c < 0$, an available least consequence $a + c$ does so. If source and report consequence suprema are merely specified to be equal, the common value must first be fixed. An explicitly available perfect source experiment fixes its oriented supremum at $|c|$; equality then supplies the report premise used above. Zero contrast would erase the success information and is excluded.

The result constructs full finite quantum recovery from these consequential constraints. Its ideal effects and algebraic recovery need not be available physical actuators. The carrier and enacted report, the preparation and measurement capabilities, and the physical consequence calibration remain material identifications. Selecting all physically possible carriers and realizing the supplier, controller and durable record remain the connected construction obligations of this paper.

12.2 What calibrated records determine about a subject

A physical account of experience has two connected tasks: identify the comparisons a system actually realizes, and identify which occurrences belong to the same experiencing subject. The first task has a constructive response theorem. The second has an exact locality condition. Both concern the existing recognition structure; an assignment of felt occurrence is a physical identification to establish within it.

The comparison recovered from an actual response. A native comparison has positive outside and inside magnitudes o, i, ratio $x = o/i > 0$, and cost $J(x)$. A calibrated strain $r > 0$

multiplies o by r while retaining i. Its signed response is

$$D_r(x) = J(rx) - J(x) = \frac{r-1}{2}\left(x - \frac{1}{rx}\right).$$

For positive x, y and $r > 1$, subtraction gives

$$D_r(x) - D_r(y) = \frac{r-1}{2}(x-y)\left(1 + \frac{1}{rxy}\right).$$

The factors apart from $x - y$ are strictly positive. Consequently the response is strictly increasing in the ratio. Holding the outside fixed, equal responses force equal ratios and hence equal inside magnitudes.

There is an explicit decoder using a factor-two strain. If $b = J(x)$ and $d = J(2x)$, expansion of the reciprocal cost gives

$$x = \frac{4d - 2b + 2}{3}, \qquad i = \frac{o}{(4d - 2b + 2)/3}.$$

The denominator equals the positive ratio on every actual comparison. Thus any record from which (o, b, d) can be read exactly also determines i: compose its stipulated observation map with this decoder. Outside and baseline cost alone are insufficient. The comparisons $(o, i) = (1, 2)$ and $(1, 1/2)$ have the same outside and reciprocal-symmetric baseline cost, while their inside values differ.

This response is an actual participant effect under the ledger realization premises. Suppose bond e is incident to participant p, its multiplier is $L_e = o/i$, and the specified actor's singleton log-strain is $\log r$ on e and zero elsewhere. The native action multiplies that bond by r. All other incident contributions cancel in the participant's signed cost change, leaving exactly $D_r(L_e)$. Equal effects for two such incident, ratio-calibrated realizations with equal outside therefore force equal inside. Incidence, calibration, enactability, funding and access to the signed readings are material conditions. In particular, this positive singleton strain has total log-strain $\log r > 0$; its action description alone supplies no conservation, consent or authorization certificate.

The same physical ledger carries economics and ethics. On a supplied finite incidence chart, a positive actual/required comparison gives log-profile $f_e = \log(\text{actual}_e/\text{required}_e)$ and bond multiplier e^{f_e}. A transition $s \to t$ uses the actual actor and strain $f_e(t) - f_e(s)$. The identity

$$e^{f_e(s)} e^{f_e(t) - f_e(s)} = e^{f_e(t)}$$

proves equality with the native ledger transformation, and its participant harm is precisely the corresponding difference in incident cost. The physical profile must identify the actual participant conditions and needs; supplying arbitrary positive signals does not establish that identification. A whole description retains those member effects and obligations. Its union of incident bonds counts each resource once; the sum of member incident costs adds one extra contribution for each internal bond with distinct member endpoints, as proved above. Neither a common substrate nor a common subject label licenses erasing a member's increased burden or counting a shared resource twice.

The realized eight-occurrence witness. Fix a stable boundary with pattern p. Its posting at occurrence $t \in \{0, \dots, 7\}$ is $a_t = |p_t|$, and its self-reading is $S = \sum_t a_t$. A witness consists of positive comparisons (a_t, i_t) with some $i_t = S$. Attribution fixes every outside magnitude. Therefore equality of the inside streams fixes all comparisons and the witness. Applying the preceding decoder separately at all eight occurrences shows that the complete outside/baseline/doubled-cost record recovers the witness exactly. Any encoding with an exact observation map to that record must be injective on these witnesses. This conclusion retains the realized comparison stream; it supplies neither the physical probe apparatus nor the further identity of its inside with felt occurrence.

The reflexive value can be selected from a stated intervention law. Let r range over all eight-tuples of nonnegative readings and let $F(r)$ be a real readout satisfying

$$F(0) = 0, \qquad F(r[j \leftarrow x]) - F(r) = x - r_j \quad (j \in \{0, \dots, 7\},\ x \geq 0).$$

For a finite coordinate set A, set the other coordinates to zero. Induct on A. The empty set gives zero. Adding a fresh coordinate j changes the readout by exactly $r_j - 0$; hence the value becomes $\sum_{k \in A \cup \{j\}} r_k$. Taking all coordinates proves $F(r) = \sum_j r_j$. Conversely, the sum satisfies both displayed laws by cancellation of the unchanged terms.

A responsive pass supplies this readout, a designated occurrence t, and a positive comparison at every occurrence attributed to a_j. At t the inside equals $F(a)$; elsewhere it mirrors a_j. The sum theorem yields $i_t = S$. Thus the pass is exactly the existing self-holding witness: its designated comparison is (a_t, S), all others are (a_j, a_j), and its total comparison cost is exactly $J(a_t/S)$. A positive attributed comparison at every occurrence requires every $p_j \neq 0$.

Availability matters. Without an advance record, a readout at $t < 7$ determined only by the acquired prefix cannot satisfy that response law on independently variable positive future postings. Compare the all-ones input with the same input having coordinate 7 replaced by 2. Their prefixes through t agree, so prefix causality would give equal outputs; replacement requires their outputs to differ by 1. At occurrence 7 agreement through the prefix is agreement everywhere. This argument neither prices a replay buffer nor constructs an early advance record.

More generally, for a supplied set A of available closing occurrences, a witness with summed comparison cost at most B and self-reading at some $t \in A$ exists exactly when all eight postings are nonzero and

$$\exists t \in A : \quad J(a_t/S) \leq B.$$

For necessity, the designated comparison has that cost and every other comparison has nonnegative cost, so the total bounds it. For sufficiency, the self-holding witness at the specified t has exactly that total. Empty availability admits no such witness. This is an exact comparison-cost criterion on supplied access, rather than an acquisition or full physical energy ledger.

The precise subject-locality equivalence. Say F depends only on a set A when any two nonnegative inputs agreeing on A have the same readout. For a responsive F, this holds if and only if A contains all eight coordinates. Indeed, if $j \notin A$, zero and the vector with coordinate j replaced by 1 agree on A, while the replacement law gives readouts differing by 1. The converse follows because agreement on all coordinates is equality of inputs.

Now assign each occurrence a felt predicate and a subject label $\sigma(t)$, retaining the same realized witness. Assume every occurrence is felt. Strong unity means that every pair of felt occurrences has the same subject label. For a designated closing occurrence t, let

$$A_t = \{j : \sigma(j) = \sigma(t)\}.$$

Subject locality means that the actual readout depends only on A_t, over the full nonnegative intervention domain. Then

$$\text{strong unity} \quad \Longleftrightarrow \quad \text{subject locality of the responsive readout at } t.$$

To prove the forward direction, universal felt occurrence and strong unity put every coordinate in A_t, so locality follows. Conversely, responsiveness and locality force A_t to be the entire window. Every label therefore equals $\sigma(t)$, and any pair agrees by transitivity. The labels belong to occurrences: equal numerical comparison values alone do not identify their subjects.

For a physically grounded admissibility predicate, apply this equivalence to every admissible reading whose witness is the actual responsive pass and whose occurrences are all felt. The physical unity target and physical subject-locality target are equivalent under exactly these conditions. The equivalence isolates the needed physical law; it does not establish that intersubject information exchange satisfies subject locality.

What the mechanical conditions leave undecided. The separation uses the very same constructed complete pass. Give all its occurrences the felt predicate true. One reading labels all occurrences true. Another labels occurrence 0 true and each other occurrence false. The first has strong unity; the second fails it already at occurrences 0 and 1. Both have exactly the same witness, reflexive value and complete calibrated response record. Every function whose input is only that witness gives equal outputs on the two readings. This separates assignments on one carrier; it does not exhibit two different physical realizations.

Connectivity with zero phase cost can accompany both assignments. Take universal coupling of stable boundaries and the constant phase zero. Every pair has a one-step coupling path, and every edge has phase difference zero and hence zero canonical phase cost. These mechanically identical conditions therefore do not select between the two subject assignments. A unique inside value for each individual comparison is likewise satisfied by both.

Whole-record dependence also permits a lossless product presentation of the same replacement grammar. For any partition A, A^c, restrict an input to the two sets and recombine by choosing its coordinate from the appropriate factor. Recombining a restriction returns the original input coordinate by coordinate. Restricting a recombination returns the original pair on each factor. These are inverse maps. A replacement in A acts only in the first factor and recombines to the original replacement; the corresponding statement holds for A^c. Opposite-factor updates commute because each leaves the other's coordinate function unchanged. The responsive readout on a recombined pair is the sum of the two zero-extended local totals: expand the total and distribute the finite sum. Thus requiring all postings does not prohibit separate registers with a joint decoder. This construction preserves the exact posting carrier and interventions; it neither prices the decoder nor identifies its factors with physical experiencing subjects.

The universal grounding obligation. At the comparison-carrier level, a phenomenal reading assigns a felt predicate and subject label to every comparison. Universal experience requires every comparison to be felt. One-subject composition requires the labels of every two felt comparisons to agree. The first condition alone does not imply the second: choose a comparison c, label it true and every other comparison false, while declaring every comparison felt. Increasing only its positive inside magnitude by 1 gives a distinct comparison with the other label. This is again an interpretation separation, not a physical multiplicity theorem.

A successful ground must admit at least one reading and must force both universal experience and one-subject composition for every reading it admits. Nonemptiness prevents a predicate that admits nothing from satisfying the implication targets vacuously. Suppose the ground is blind to the proposed reading: replacing its felt and subject assignments never changes whether it holds. Choose an admitted reading and erase its felt predicate, retaining its subject map. Blindness would admit the erased reading. Universal experience would then assert the false felt predicate at any chosen positive comparison, a contradiction. In fact every successful ground must reject the felt-erasure of at least one admitted reading. A physical proposition supplied identically for every reading, including a connectivity fact by itself, is blind in precisely this sense.

The remaining physical identification is therefore specific. The actual dynamics must establish a nonempty admissibility relation sensitive to the realized felt and subject assignments, and must establish universal occurrence and the required subject locality or composition within that relation. The numerical inside coordinate is constrained by actual calibrated participant effects; identifying it with felt occurrence and identifying shared experience across occurrences remain the named physical joins. These concern the same physical recognition, bond transformations, member needs and histories as the rest of the account.

13 Consequences and remaining physical questions

The established results constrain a single connected physical account. The cost and scaling theorems retain their reporting, comparison and join premises. The dimension result retains its spatial realization. The eight-tick conclusion retains the complete process identification. The continuum weight retains its factorization, order and calibration. Absolute window energy retains the topology-load identification.

The physical construction must acquire the required state from available inputs with the actual Hamiltonian and a complete ledger. It must attach the complete supplier, matter, controller and record dynamics to the mobile carrier, preserve the strong domains, enact both settings, turn off and retain a durable record. It must connect neutral feedback to actual preparation and capture, and connect the resulting history measure to those physical operations. The economic and ethical profile and the experiential admissibility law remain physical obligations on the same structure.

The resulting architecture has concrete mathematical content: the floor equivalence, licensed record arithmetic and host invariance, native reporting, common scaling, linking, the attaining Gray cycle, continuum weight and constructive finite sharp-decision recovery follow at the scopes established above. The responsive-record analysis distinguishes what calibrated responses fix from what a physical law must supply about experience and subject composition.

The next physical question is whether the actual dynamics enact the required preparation, comparison and retained history with the resources counted once. The present article supplies the implications and their conditions; a complete physical realization requires the named construction equations to be solved.

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