The Moral Code of Reality

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Physical Structure, Agency, and the Conditions of Living Together

Jonathan Washburn

Love and the work of living together

Someone needs help. Someone else can give it. Between those two facts lies much of moral life: what help changes, what it costs, who can provide it, what has been promised, and whether both people can go on afterward.

Love belongs at the center of this account. It directs attention to a damaged relation and the possibility of repair. In Recognition Science that possibility has an exact physical form. A relation compares quantities, its imbalance has a cost, and an action changes it. For a pair of positive comparison ratios whose product is conserved, equalizing the ratios gives the unique endpoint of least total recognition cost. Love names this conserving repair. The result concerns the same bonds and transformations that physics describes.

The theorem makes one part of love precise. Human love also involves attention, affection, fidelity and the ability to keep caring through time. Explaining those capacities requires the people, their histories and their means of action to remain in view. A lower total burden can leave the person helping unable to continue. A perfectly balanced record can conceal an unkept promise. A welcome effect can occur without the recipient's consent. Each fact changes what a loving action must attend to.

Consider Alice, who has agreed to bring Bob a meal. The food must exist, Alice must be able to do the work, and Bob must receive it in time. If she spends her last capacity helping today, tomorrow's care may become impossible. If she brings the meal after the checkpoint her promise concerned, she can repair Bob's present condition without changing what happened earlier. The physical account has to retain both the repair and the missed obligation. These are the concrete demands from which the argument begins.

The virtues describe different parts of this work. This book develops fourteen named virtues, with Love among them. Their number is the size of the catalogue being examined; the physical results establish the operations and their conditions, without deriving fourteen as the number of every possible virtue.

Love repairs a relation through conserving reduction of recognition cost.

Justice carries out the correction owed on the retained terms and facts.

Forgiveness releases a claim through the authority of the one entitled to release it, while retaining what happened.

Compassion responds to another's need through care that the participants can actually sustain.

Sacrifice bears a personal burden for another's benefit; the burden, benefit and willingness each need their own account.

Gratitude recognizes a benefit and its author in the response that follows.

Wisdom chooses from real alternatives using the consequences the decision must preserve.

Prudence attends to costs and exposures that an attractive total can hide.

Temperance keeps a commitment within the capacity to complete it and continue afterward.

Courage concerns acting well when the actor is exposed to a real burden.

Patience lets needed information arrive while attending to the cost of waiting.

Humility corrects an assessment in response to evidence.

Hope concerns a future reachable through actual, funded continuations.

Creativity searches for useful alternatives when the present choices are inadequate.

These descriptions say what each virtue has to accomplish. The chapters distinguish what an operation already establishes from what its fuller realization still needs. Love has a conserving uniqueness theorem. Justice has an exact execution on shared bonds once the correction is specified. Care has exact conditions for finite continuation in its stated environment. Several other rules retain chosen response rates or depend on an uncompleted identification. A standing disposition also requires a person whose records and capacities reliably select the fitting operation. A named operation alone does not establish that disposition.

The virtues therefore belong together without becoming interchangeable. Justice needs truth about what occurred. Forgiveness needs authority over the claim. Sacrifice needs an account of who bears the cost. Temperance protects the capacity on which continued Compassion depends. Hope and Creativity concern the future actions that remain available. Love's repair is central because relations can be changed for the better; the other virtues keep that repair connected to the people who must enact it and live with its consequences.

The argument starts with the meal, establishes the physical comparison and its geometry, then develops repair, care, undertaking and consent. The later chapters follow the remaining virtues into choices, information, economics, institutions and minds. Formal propositions state their conditions beside their conclusions. The notation guide is a reference to consult as those objects arise; the meal in Chapter 1 is the starting point for reading straight through.

A physical moral structure can leave more than one action available, and sometimes more than one action meets the same requirement. Its necessity lies in the relations every adequate judgment must preserve: people and their needs, actual effects, scarce resources, authored commitments and the history that connects them. Understanding those relations gives us something definite to repair, sustain and answer for.

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Notation and terms

Core terms

Term	Symbol	Plain gloss
recognition		telling occurrences apart, comparing them and keeping what a later comparison needs
participant	i	a person, or anything else that can stand at an end of a bond, such as Alice or Bob
bond	b	a directed relation from one participant to another, or to themselves, comparing two positive quantities of the same kind. Between two people, examples set a bond's multiplier until the pair identification (Section 3.2) is derived. A person's own bond records their response to exposure, the load of getting through an interval (Chapter 6)
multiplier	$x_b > 0$	the quotient of the two quantities a bond compares, first over second; the bond is balanced at $x_b = 1$
recognition cost	$J(x)$	the cost of imbalance, a pure number: zero at balance, the same for x and $1/x$. For two signals of amplitudes u and w, with $x = u/w$, it is their energy $u^2 + w^2$ above $2uw$, in units of $2uw$ (Chapter 3)
ledger	L	the active bonds with their multipliers; the shared record of relations
whole-ledger cost	$C(L)$	the cost of every active bond, each counted once
incident cost	$C_i(L)$	the cost of the bonds touching participant i; the burden i carries
skew	$\sigma_i(L)$	$\sum \log x_b$ over bonds leaving i minus the same sum over bonds entering i; a participant's signed net imbalance

strain	v_b	how much an action changes a bond's log ratio: $x_b \mapsto x_b e^{v_b}$
harm	$C_i(L') - C_i(L)$	the change in i's incident cost when an action takes L to L'; an increase is harm, a decrease is relief
reserve		what a participant can still spend on repair, work and keeping going, counted in units of recognition cost; the whole-unit reserves of Chapters 13 and 15 and Appendix B are separate accounts
maintained		of a care checkpoint: the person's own bond is balanced and their reserve strictly positive (Chapter 6)
carrier		the physical system a construction runs on, with the numbered entries (signal components or registers) that describe its states
undertaking		a proposal and its acceptance with the terms retained; an undertaking formed by a proposal and its recipient's acceptance
installed		said of the setting the accepted write reaches once it has completed: the agreement's requirement is then written into the register taken to stand for the means of action (Chapter 7)
native		belonging to the physical comparison or ledger itself: the number a meter reports is native, a curve fitted to such numbers is not, and two of the three cost theorems require the curve to agree with the meter (Chapter 2)
golden ratio	$\phi = \frac{1+\sqrt{5}}{2}$	the positive root of $\rho^2 = \rho + 1$. The cost J does not depend on it. Under the three conditions of Theorem 2.5, each update multiplies a record's size by ϕ; some virtue constructions use its powers as chosen parameters (Chapters 9–11)

Quantities kept apart

Ratios, flows and changes. A bond's signed log ratio, or logarithmic flow, is $\log x_b$, and an action with strain v_b changes it by

$$x_b' = x_b e^{v_b}, \qquad \log x_b' = \log x_b + v_b.$$

Skew adds signed flows, so opposite departures cancel in it: a participant with one bond leaving at multiplier 2 and one arriving at 2 has skew $\log 2 - \log 2 = 0$, while the absolute bond sum $\sum_b |\log x_b| = 2 \log 2$ keeps both departures.

Cost and incidence. The recognition cost is

$$J(x) = \frac{x + x^{-1}}{2} - 1.$$

The incident cost $C_i(L)$ counts every bond touching participant i once, an internal bond from i to i included. A bond between two distinct participants appears in both of their incident costs and is still one physical bond, so the incident costs of all participants add up to $C(L)$ plus the cost of every bond between two distinct participants.

Reserves and energy. A reserve is a stock a participant spends; a budget renews it each interval. Neither is incident cost or the physical energy of a comparison (Chapter 3). In Chapter 6 a meal's work and intake are chosen for the care environment. Accounts in different constructions are separate quantities unless a conversion between them is derived.

States and histories. An ordered history keeps which states and events occurred, and when, so equal endpoints can follow different histories (Chapter 6). Signed harms add along a history to the difference between its endpoints: a harm of $1/4$ followed by a relief of $1/4$ sums to zero, and the burden carried in between stays part of what happened.

Authorship and effect. The actor of a physical action, the authors of a record, its beneficiary and its payer can be different participants. Authored consent records an act and its content; a nonincrease of incident cost records an effect. Neither record supplies the other.

Local symbols. Sections define their own uses of p, q, η and λ; the arguments and accompanying formulas identify each. Capital letters such as these change meaning from chapter to chapter, and each use is defined where it appears:

H the half-turn, which exchanges opposite entries of the comparison's carrier (Chapters 2 and 12, Appendix A); a participant's harm H_i, subscripted by the participant (Chapters 3 and 14); a Hamiltonian or a holding observable in a physical construction (Chapter 7, Appendix B).

R the comparison's reference (Chapter 2); a reserve, R_n after n intervals (Chapter 5); the renewal map $R((m, e), q)$ (Chapter 10); Rényi's half-order measure $R = -2 \log \beta$ (Chapters 12 and 17, Appendix A); the restoration predicate $R(a)$ (Chapter 13 and Appendix B).

W the register's exchange matrix W_κ (Chapter 7); the work store's occupation observable (Chapters 9 and 14); the catalogue's Wisdom score $W(s)$ (Chapter 10); the observation list $W(t)$ (Chapter 11); a policy's finite-horizon guarantee W_X^π (Chapter 15); a controlled exchange (Appendix B).

P the projector $(I - H)/2$ (Chapters 2 and 12, Appendix A); a requirement $P(i)$ on a participant (Chapter 4); a condition on actions (Chapter 7); the prudent score P_λ (Chapter 10); a space of physical states (Chapter 16); a polynomial (Appendix A); a product of operators or a projector (Appendix B).

D a residual energy D_z (Chapter 2); a domain of participants (Chapter 4); a holding observable (Chapter 7); the fall $D = \eta(\nu - \mu)$ in the work store's mean occupation (Chapter 9); the imbalance penalty $D(s)$ (Chapter 10); energy records and their pooled sum (Chapter 12); the cost of bonds between members of a group, D_M (Chapter 15); an energy form D_γ (Appendix A); a free evolution $D(t)$ (Appendix B).

Part I

Physical morality

Morality concerns physical relations between people who act and undergo. The cost of those relations is forced, their geometry is exact, and every person keeps their place in any description of the whole.

Chapter 1

What an action leaves behind

Alice has agreed to feed Bob. Relying on her, Bob spent his one interval of labor working at a service instead of getting food, so in the coming interval he cannot feed himself. Alice has food within reach and the strength to carry it to him. When the interval arrives she can bring the meal, or she can stay where she is. Both actions are physically possible. A moral law has to explain why one of them is owed, and it has to do so in a world where the other can still happen.

Put numbers on the situation. Each person carries a bond of their own, from themselves to themselves (Definition 3.1), whose ratio records their response to the interval. In the care model we use, each interval doubles that ratio, from balance at 1 to 2, and getting through the interval means repairing it, bringing it back to 1. The repair is paid from the person's *reserve*, the capacity they can spend on work and on keeping going; reserves are pure numbers, in units of the cost J of the next section. The care model adopts one rule for the price: a repair debits the cost of the bond it rebalances from its owner's reserve, here $J(2) = 1/4$. Alice begins with a reserve of 2, Bob with $1/4$. Delivering the meal costs Alice a further $1/4$ of work and adds $1/2$ to Bob's reserve. The opening reserves, the doubling, the work and the food's value are chosen for the example (Chapter 6). Table 1.1 shows where each choice leaves them.

Both histories end with every bond balanced. If we judged actions by the imbalance they leave in the world, we would score feeding and staying identically, because both leave none. Yet staying exhausts Bob's reserve. To *continue*, a person must keep a strictly positive reserve; zero falls short even when, as here, the present repair has been paid. The case is built so that the bond total is blind for one interval, and the blindness does not last: one interval later, with no further meal, Bob cannot pay his repair, and his own bond stays out of balance at a cost of $1/4$ that feeding would have left at zero (Chapter 7), the same quarter the repair would

	Alice's reserve	Bob's reserve	Cost of every bond
Opening	2	1/4	0
Alice feeds Bob	3/2	1/2	0
Alice stays	7/4	0	0

Table 1.1. The two histories of the first care interval. Each person pays 1/4 of reserve to repair their own bond. The meal changes no bond; it costs Alice 1/4 of work, and the food adds 1/2 to Bob's reserve.

have debited. The difference that matters is there from the start. It sits where a single total does not look: in Bob's remaining capacity, in the undertaking Alice accepted, and in the record of who agreed to what.

1.1 The physical object of morality

Morality's objects are people, the relations through which they affect one another, their capacities to act and to undergo effects, the resources that sustain them, and the histories in which their actions have consequences. Recognition Science, the physics in which we work, starts from what every act of recognition does: it tells occurrences apart, compares them using information about both, and keeps what a later comparison needs. From requirements on how a comparison is physically reported it derives the cost of imbalance as a uniqueness theorem (Chapter 2), and follows these objects through conservation and transformation.

The basic object is a *bond*: a directed relation from one participant to another, or from a participant to themselves, carrying a positive number x, the ratio of the two quantities the relation compares (Chapter 3 defines it). At $x = 1$ the bond is balanced; $x > 1$ means the first quantity exceeds the second, and seen from the other end the same bond has ratio $1/x$. Its departure from balance has a cost,

$$J(x) = \frac{x + x^{-1}}{2} - 1,$$

which is zero at balance, the same for x and $1/x$, and grows without bound in either direction; in the coordinate $t = \log x$ it is $\cosh t - 1$. An action keeps every bond and multiplies its ratio by a positive factor, as any change that keeps both compared quantities positive must; ending a relation is a different transformation (Chapter 4). A participant's burden

is the cost of the bonds that touch them, and the change an action makes to that burden is its *harm*: an increase harms, a decrease relieves. Harm is a technical term, narrower than the everyday word: it is the damage the ledger itself reports. In the first interval staying away does Bob no harm in this sense, since every bond ends balanced, and it exhausts his reserve, a damage tracked by a quantity of its own.

Our claim is that morality is literal physical structure, its quantities physical ones and not analogies: physics, ethics and economics concern the same *ledger*, the set of active bonds with their multipliers, with the same participants and the same transformations. Inside their conditions the theorems that follow cannot fail. The identifications that place Alice's and Bob's relations inside those conditions can, and Chapter 17 lists each one, with the steps still open. The physical identification must be specific. Authored account transitions and economic effects already have exact representations on the same native bond ledger. Calibrated interventions can identify its active relations and their multipliers (Chapter 14). Applying those results to a particular person's physiology, need or usable reserve still requires identifying the quantities that person's bond compares (Section 3.2). The care environment states those quantities and response laws as premises, then computes their consequences.

A description of an action can keep track of who was affected, what was spent and who acted, or lose it. A total of several people's burdens does not say who bears them. An endpoint does not say whether anyone consented, whether an earlier requirement was met, or whether the same person persisted through the change. If two histories differ in what they mean for Bob, a description that merges them cannot decide what Bob was owed.

This gives a working test. State the judgment a description is meant to support and list what the description records. Then look for two situations it records identically in which the correct judgments differ. If such a pair exists, no procedure using only that description can be right about both, and more calculation on the same input cannot help; the description has to keep more. In the cases that follow, the judgments differ because a fact they answer to differs: whether Bob can pay his next repair, whether food arrived, who accepted.

1.2 A whole contains its members

Care consumes resources at both ends of the relation. An account that tracks only Bob's intake can describe a successful sequence of meals long after Alice's strength has run out; one that tracks only Alice's reserve can show her flourishing while Bob goes hungry. The joint question keeps both: does one actual sequence of actions meet both people's requirements with the resources that exist? Food Bob eats is not available for anything else, so two separate accounts cannot each spend the same reserve and add their successes.

The same constraint holds for a household, an institution and a society. Grouping members changes the description; it removes no one's needs and discharges no one's undertaking, so an aggregate account is adequate only if it keeps what determines what happens to each member.

Necessity here constrains action without dictating a single choice: in Chapter 6 two different four-interval schedules, each using two meals, keep Alice and Bob maintained and meet the same exact condition. Necessity fixes the condition, not the schedule.

1.3 What an undertaking changes

A proposal and its acceptance are events with authors and content, and they create a new dependence: a particular person has undertaken a particular action for a particular recipient.

Chapter 7 follows the agreement into a physical record. Picture a register taken to stand for Alice's means of action: her next action is read from it. Before the agreement it holds 'idle'; Bob's acceptance, once detected, writes 'feed', and when that write has completed the setting is *installed*. Call a set of requirements on her next action a *reading*. Every setting the accepted write can reach is the same one, so the record grounds exactly one reading, the one the installed setting implements. That reading keeps both people going and contains the requirement to perform the accepted care. In the technical sense Chapter 7 makes exact, that requirement *governs* Alice's next action: it belongs to that one grounded reading, an available action meets every requirement of the reading, and another available action violates it. Feeding Bob meets them all; staying away fails the requirement and remains executable. Governs does not mean controls. The installed register settles which action it selects, not what

Alice does: she can still act through another controller, another physical state that selects her action, and staying away is that. The proof explains a requirement that can be broken, which is the kind of requirement a moral law is.

Uniqueness is immediate once the write is fixed. What the construction adds is timing: the write is a physical evolution that completes in finite time, with exact energy accounts, before Alice acts. The governing reading is settled in the physical system before the outcome, so it cannot be chosen after we see what she did.

The step from what governs Alice's action to what she owes Bob is a further identification, which Chapter 7 adopts as a premise; Bob's reliance is the reason for it. He gave up feeding himself because he had accepted Alice's proposal, and the record asks her to restore exactly the position that reliance cost him.

The harder cases are those where claims compete. In Chapter 13 Alice has a second undertaking, to Carol, and one unit of food for both; each claim fixes exactly what would repair it, and no available operation meets both. The account of what governs must carry that shortage, who is responsible for it and what remains possible.

1.4 A history continues after a repair

Suppose Alice misses an interval and brings the meal later. The late meal can restore Bob's present capacity and make further care possible; the missed interval remains part of what happened. A successful repair changes the condition an action left. It does not turn the history into one in which the breach never occurred.

Release works differently. Bob can author the act that clears Alice's current requirement (Chapter 8); Alice cannot produce that act by changing her own account. An effective release changes what is required now, and the record of who released whom, and of any omission before, stays in the history.

Correction differs from both. It leaves what happened and what was owed as they were, and changes whether the books say so truly. An accurate correction can reveal a shortfall the old books concealed. The displayed imbalance goes up, and the correction is still true. A moral comparison has to tell creating an obligation apart from accurately recording one that already existed (Chapter 8).

1.5 The meaning of necessity

A necessary result shows that nothing else satisfies its conditions. The four central results are necessary in different ways, each at its own scope.

- The comparison cost is a uniqueness theorem. Take a linear comparison of two signals with amplitudes u and w, set up as in Chapter 2, its runs measured as energy and reported against a reference built from its own runs. If the report on the ratio $x = u/w$ is zero at balance, the same for x and $1/x$ and never smaller for a larger imbalance, and the reference is positive with both signals present and zero with one absent, the report can only be J, or the fixed multiple of J that the reference's unit sets (Theorem 2.3). The reason is quadratic: energy makes every run quadratic in the amplitudes, and the requirements leave $(u - w)^2$ over $2uw$, which is $J(u/w)$. Two further routes, one with no apparatus at all, reach the same function from their own requirements with the unit fixed.

- Conserving repair is a strict-convexity minimum. A conserving repair moves log ratio between two bonds and keeps the sum of their log ratios, the logarithm of their product. Because the cost is strictly convex in that coordinate, the total has exactly one minimum among such endpoints, where the two multipliers are equal; every other conserving endpoint costs more (Chapter 5).

- The care criterion is an exact existence condition. A finite care schedule keeping both participants funded at every checkpoint exists exactly when the opening reserves, the horizon and the number of meals Alice is authorized to deliver satisfy two derived inequalities, and when they hold the proof names a schedule that works (Chapter 6).

- The governing reading is unique for the installed record: the accepted write leaves one setting, so the record grounds exactly one reading of Alice's next action, and the accepted care it requires governs (Chapter 7).

Necessity also leaves room for wrongdoing. The care constructions contain omissions Alice can perform, and the authored histories contain disagreement, correction, release and repair. The necessity concerns the form of the governing structure. It does not make every agent obey it.

The same exactness makes the account correctable. A simplification fails if it erases a participant, a uniqueness claim if another function satisfies the same conditions, and a physical identification if the measured quantities do not behave like the objects in the derivation. Each failure points to one claim and leaves the independent results standing (Chapter 17 sets out the tests).

An action leaves a transformed world, affected people and a history. The physical account of morality begins by keeping them together.

Chapter 2

Distinction, recognition and necessary structure

Each bond between Alice and Bob carries a positive number x, the ratio of the two quantities it compares (Chapter 3 defines it). In Chapter 1 each bond also carried a cost,

$$J(x) = \frac{x + x^{-1}}{2} - 1 = \frac{(x-1)^2}{2x}, \qquad x > 0.$$

Balance, $x = 1$, costs nothing; ratios 2 and $1/2$ each cost $1/4$. Every later number about burden, harm and repair is computed with this function. Why this one, and not another that also vanishes at balance?

The answer is a theorem about how a physical comparison is reported. In the comparison studied here, the two quantities are the amplitudes of two signals; the bonds between Alice and Bob wait on the pair identification (Section 3.2). Let F be any real-valued report that agrees with the apparatus at whole-number amplitudes, where counting can set and check the values exactly. If F is continuous, it equals J, or a fixed multiple of J if the reference is rescaled. Nothing about the shape of F is assumed: its zero at balance, its symmetry between x and $1/x$ and its upward curvature come from the apparatus.

The apparatus is built with those properties in view, so a second theorem leaves its design open and places four requirements on how any linear comparison of two signals on eight entries is reported. They force the comparison to subtract one operand from the other and fix its reference, the unit in which it reports, and the cost is again J, or the fixed multiple of it that the reference's unit sets. A third route uses no apparatus: a law for combining comparisons, with the cost's unit fixed near balance, reaches the same function.

2.1 What every act of recognition already does

Recognition is telling occurrences apart, comparing them, and keeping what a later comparison needs. Three facts about it hold in any actual act of recognition.

First, there are distinguishable occurrences. Following this sentence already involves distinctions: this mark rather than another, a response rather than its absence, the present occurrence rather than the one being recalled. The claim is existential: some occurrences can be told apart, not necessarily every pair of states.

Second, a comparison requires information representing both of its terms. If Alice is to tell whether Bob has eaten since yesterday, something representing yesterday's state must remain available until her comparison uses it. Without information about one term, the comparison cannot tell apart different possibilities for that term, since the same input would have to serve for all of them. Naming the earlier occurrence does not help: the word "yesterday" tells Alice which state she means and nothing about what it was. The record need only last until it is used and keep what the distinction needs; afterward it may be altered, consumed or lost.

Third, a later examination is also an occurrence. A record whose history ends at a particular stage cannot already contain, as a completed event, an examination at a later stage. Alice's examination of yesterday's record today is a new occurrence, one the record did not contain. The record may contain a prediction or description of that examination; that description is a different occurrence.

These facts motivate the setting below; they are not premises of its theorems, each of which rests on the hypotheses in its statement. The second motivates the requirement that decides the form of the cost (Section 2.5), the third the operation the apparatus runs (Section 2.3), and the first that operation's cost (Section 2.10).

A numerical report needs four things: a physical system that holds both operands, an operation that acts on them, a measured quantity, and a reference to compare it with. The physical system a construction runs on, together with the entries that describe its states, is its *carrier*. The *apparatus* below defines all four exactly, as a model of one physical comparison.

2.2 The comparison carried by eight entries

Here the carrier holds a signal: eight complex numbers, its entries, arranged in a cycle. There are eight because the apparatus gives one entry to each joint state of three binary coordinates, one for each dimension of the three-dimensional realization of this apparatus that Recognition Science's foundation constructs. The theorems of the first two routes to the cost are proved on this carrier, yet the cost carries no trace of the count: the calculation uses one feature of the cycle, the half-turn, which moves every entry four places and so exchanges each entry with the one opposite it.

Each operand is written as a balanced pair, a value and its negative on adjacent entries. The second operand's pair sits half a cycle from the first, with its sign reversed, so that the half-turn sets one operand against the other. For positive amplitudes u, w and a unit phase z, the input is

$$f(u, zw) = \frac{1}{\sqrt{2}}(u, -u, 0, 0, -zw, zw, 0, 0). \tag{2.1}$$

The phase rotates the second operand relative to the first. The factor $1/\sqrt{2}$ normalizes the input:

$$\|f(u, zw)\|^2 = u^2 + w^2.$$

The layout is the apparatus's own; Theorem 2.3 leaves the placement of the operands free and reaches the same cost.

Write the half-turn as H. Then $H^2 = I$, and the two operators

$$P = \frac{I - H}{2}, \qquad Q = \frac{I + H}{2}$$

separate a signal into complementary parts:

$$P + Q = I, \qquad P^2 = P, \qquad Q^2 = Q, \qquad PQ = 0.$$

The part Pf changes sign under the half-turn; the part Qf does not. The half-turn also preserves the ordinary squared norm, so the two parts are orthogonal. Because the second operand is written with reversed sign, Pf carries the operands' sum $u+zw$ and Qf their mismatch $u-zw$ (Table 2.1): they are the *sum part* and the *mismatch part*. Applying Q brings the two operands onto the same entries:

$$Qf(u, zw) = \frac{1}{2\sqrt{2}}(u - zw, -(u - zw), 0, 0, u - zw, -(u - zw), 0, 0).$$

Four entries have squared magnitude $|u - zw|^2/8$. Consequently,

$$\|Qf\,(u, zw)\|^2 = \frac{|u - zw|^2}{2}. \tag{2.2}$$

Entry	0	1	2	3	4	5	6	7
$\sqrt{2}f$	u	$-u$	0	0	$-zw$	zw	0	0
$\sqrt{2}Hf$	$-zw$	zw	0	0	u	$-u$	0	0
$2\sqrt{2}Pf$	$u + zw$	$-(u + zw)$	0	0	$-(u + zw)$	$u + zw$	0	0
$2\sqrt{2}Qf$	$u - zw$	$zw - u$	0	0	$u - zw$	$zw - u$	0	0

Table 2.1. The comparison on the eight entries. The half-turn H moves every entry four places; $Q = (I + H)/2$ averages each entry with the one opposite it, and $P = (I - H)/2$ takes half their difference. The sum part Pf carries $u + zw$. In the mismatch part Qf the two operands meet only through their mismatch $u - zw$.

When the inputs agree in phase and amplitude, the mismatch part vanishes, while the input still has squared norm $u^2 + w^2 > 0$: the comparison registers nothing though its inputs carry energy.

2.3 The comparison measures what its operation changes

An operation acts on the input, and the apparatus measures what it changes. The derivation of the cost uses one property of that operation: it leaves the sum part unchanged and multiplies the mismatch part by a fixed factor other than one. Every such operation gives the same report. What it changes is a fixed nonzero multiple of the mismatch part; the reference is built from the same runs (Section 2.5), so the multiple enters both and cancels. Leaving the sum part alone lets equal operands report zero, since at balance the sum part still carries $2u$.

The apparatus's actual operation is the *join*, the step that forms each new stage of a record from the two before it. The apparatus runs it because comparing is itself a new occurrence (the third fact): the operands meet, and what remains is the record's next stage. Section 2.10 computes the join on the eight entries: divided by the golden ratio $\phi = (1 + \sqrt{5})/2$, it

keeps the sum part and multiplies the mismatch part by $-\phi^{-2}$. What it changes, its *residual*, is $Tf = (1+\phi^{-2})\,Qf$, which for the comparison input carries $u - zw$. In squared-norm energy units, the measured residual energy is

$$\begin{aligned} D_z(u,w) &= \|Tf(u,zw)\|^2 \\ &= \tfrac{1}{2}(1+\phi^{-2})^2\,|u-zw|^2 \\ &= K(u^2+w^2-2uw\,\mathrm{Re}\,z), \qquad K = \tfrac{1}{2}(1+\phi^{-2})^2 > 0. \end{aligned} \tag{2.3}$$

The factor K carries ϕ and, like any common change of energy units, cancels in every report, so the golden ratio does no work in the value of J.

2.4 What an energy meter must report

Let $m(d,r)$ be a meter's report for nonnegative energy d against a positive reference r. It reports nothing negative, pools separate records by adding their reports, and reports its own reference as one:

$$\begin{aligned} m(d,r) &\geq 0, \\ m(d+d',r) &= m(d,r) + m(d',r) \qquad\qquad (d,d' \geq 0), \\ m(r,r) &= 1. \end{aligned}$$

The addition pools independent energy records at the same reference. It does not assert that coherent signal amplitudes have additive energy; their interference is already included in D_z.

Proposition 2.1 (The meter's ratio law). *Under these requirements,*

$$m(d,r) = \frac{d}{r} \qquad (d \geq 0,\ r > 0).$$

Proof. Fix $r > 0$. Additivity gives $m(0,r) = 0$. If $0 \leq d \leq d'$, then $d' = d + (d'-d)$, so nonnegativity and additivity give $m(d,r) \leq m(d',r)$.

Partition the reference into n equal nonnegative records. Their reports add to one, hence $m(r/n,r) = 1/n$. Pooling k such records gives

$$m\left(\frac{k}{n}r, r\right) = \frac{k}{n}.$$

Thus the report is fixed at every nonnegative rational multiple of the reference. Rational multiples below and above d, together with monotonicity, squeeze $m(d,r)$ between rational numbers below and above d/r. Their bounds can be made arbitrarily close, so $m(d,r) = d/r$. □

2.5 Constructing the reference with the same inputs

The reference is built by the same apparatus from the same two operands, so it carries exactly the scale of the residual. Run the comparison twice, first with the second operand in phase, $z = 1$, then turned by a quarter phase, $z = \mathrm{i}$. Define the reference by the measured increase

$$R(u, w) = D_{\mathrm{i}}(u, w) - D_1(u, w). \tag{2.4}$$

Equation (2.3) gives

$$D_1 = K(u - w)^2, \qquad D_{\mathrm{i}} = K(u^2 + w^2),$$

so the measured reference is

$$R(u, w) = 2Kuw > 0. \tag{2.5}$$

A real alternative uses the opposed phase:

$$D_{-1} - D_1 = 4Kuw = 2R(u, w).$$

Half the in-phase to opposed-phase swing therefore supplies the same reference.

A single run would not serve, and the reason decides the shape of the cost. Used as the reference, the quarter-phase run would give the rival $(u - w)^2/(u^2 + w^2)$, which vanishes at balance, is symmetric and grows with the imbalance. It measures the mismatch against the energy of the whole input, which always exceeds it: $(u-w)^2 = u^2+w^2-2uw < u^2+w^2$. So the rival stays below one, while $J(x) > x/2 - 1$ grows without bound. With the second operand absent, when there is nothing to compare, the quarter-phase run still registers $D_{\mathrm{i}}(u, 0) = Ku^2$, the energy of the first operand alone; the opposed-phase run alone fails in the same way. In the difference that energy cancels, since with $w = 0$ both runs give Ku^2: $R(u, 0) = 0$.

This requirement decides between J and its bounded rival; of the requirements Section 2.8 places on a report, it is the one the rival fails. A reference is the unit in which a comparison is reported, so it must need both operands present. The second fact about recognition, applied to the calibration itself, motivates this requirement: recognition compares two things, a comparison with one operand absent compares nothing, and its unit should then be zero. Zero, and not undefined: the reference

is built from measured energies, which the runs leave whether or not both operands are present, and the requirement fixes its value when one is absent. The report is never asked for there, since the ratio of the amplitudes is then not a positive number. Of the energy a run leaves, $K(u^2 + w^2 - 2uw\,\mathrm{Re}\,z)$, only the cross term uw needs both operands present. Section 2.8 shows that a reference with $R(1,0) = 0$ keeps only that term, and against it the cost grows without bound.

The in-phase report follows:

$$\begin{aligned} m(D_1, R) &= \frac{K(u-w)^2}{2Kuw} \\ &= \frac{(u-w)^2}{2uw} \\ &= \frac{1}{2}\left(\frac{u}{w} + \frac{w}{u}\right) - 1. \end{aligned} \tag{2.6}$$

Both the join's scale and the common energy unit have canceled. Multiplying both input amplitudes by the same positive factor multiplies the numerator and reference by its square, leaving the report unchanged, so the report depends on the operands only through their ratio.

With $u = 2$ and $w = 1$, the in-phase run leaves $D_1 = K$, the quarter-phase run leaves $D_\mathrm{i} = 5K$, and the reference is $R = 4K$. The meter reports $K/4K = 1/4$, which is $J(2)$. Amplitudes 4 and 2 multiply all three energies by four and give the same report.

A different finite reference unit is possible. For $\gamma > 0$, use

$$R_\gamma = \frac{\gamma}{2} R = \gamma K u w.$$

The resulting report is

$$\frac{D_1}{R_\gamma} = \frac{2}{\gamma}\left[\frac{1}{2}\left(\frac{u}{w} + \frac{w}{u}\right) - 1\right].$$

The original reference corresponds to $\gamma = 2$. Keeping this unit visible distinguishes a change in calibration from a different physical law.

2.6 For this apparatus, the reported cost is forced

Let F be any real-valued report on positive ratios, and require it to report the actual apparatus at positive whole-number amplitudes:

$$F(a/b) = m(D_1(a,b), R_\gamma(a,b)) \qquad (a, b \in \mathbb{N},\ a, b > 0). \tag{2.7}$$

At whole numbers these reports can be checked literally, by counting; an arbitrary real amplitude can be set and checked only to finite precision. We call a quantity *native* when it belongs to the physical comparison or ledger itself, as opposed to a description of it: the meter's report D_1/R_γ is native, while F is a function written down to describe it, as an account describes a ledger. Requirement (2.7) is *native reporting*: wherever the apparatus speaks, F says what it says. Because the apparatus's report depends only on a/b, the requirement is consistent, and it fixes F at every positive rational ratio. One further requirement carries those literal reports to the continuum: F is continuous on the positive reals.

Theorem 2.2 (Continuous native reporting). *Use the eight-entry inputs* (2.1)*, the join's residual T, squared-norm energy, the quarter-phase reference* (2.4)*, and a meter satisfying the pooling, nonnegativity and self-reference requirements. Let $\gamma > 0$.*

If F satisfies (2.7) *and is continuous on* $(0, \infty)$*, then*

$$\frac{\gamma}{2} F(x) = J(x) = \frac{x + x^{-1}}{2} - 1 \qquad \text{for every } x > 0.$$

In the original unit, $\gamma = 2$, the reported cost itself is J.

Proof. By Proposition 2.1, the report in (2.7) is D_1/R_γ. The pulse calculation, the join's residual and the measured reference then give, for positive integers a, b,

$$F(a/b) = \frac{(a-b)^2}{\gamma ab} = \frac{2}{\gamma} J(a/b).$$

Define $G(x) = (2/\gamma)J(x)$ for $x > 0$. This function is continuous there: addition, multiplication and inversion away from zero preserve continuity.

Fix $x > 0$. Positive rational numbers can be chosen to approach x. Write them as a_n/b_n, with positive integers a_n, b_n. Native reporting gives $F(a_n/b_n) = G(a_n/b_n)$ at every term. Continuity gives limits $F(x)$ and $G(x)$ for these equal sequences. Uniqueness of the limit gives $F(x) = G(x)$. □

Continuity does real work. Raise the continuous report G by one at a single positive irrational point. Every report at a rational ratio is unchanged, so the altered function meets every whole-number comparison, yet it differs from G at that point, where it is discontinuous.

Given the apparatus, the theorem leaves no cost to choose. A continuous report that differs from J at some positive ratio must, in the

apparatus's own unit, disagree with the apparatus at some ratio of whole numbers.

2.7 Symmetry and order follow from the apparatus

Exchanging the operands changes $x = u/w$ to $x^{-1} = w/u$ and leaves both the in-phase residual $K(u - w)^2$ and the reference $2Kuw$ unchanged, so the cost is the same in either order:

$$J(x^{-1}) = \frac{x^{-1} + x}{2} - 1 = J(x).$$

In the original unit,

$$J(x) = \frac{(x-1)^2}{2x} \geq 0, \qquad J(x) = 0 \iff x = 1.$$

The denominator is positive, and a square vanishes exactly when its argument does. In logarithmic coordinates, write $x = e^t$. Then

$$J(e^t) = \frac{e^t + e^{-t}}{2} - 1 = \cosh t - 1.$$

The cost is even in t. Because $\sinh t > 0$ for $t > 0$, the cost increases strictly with the magnitude of the logarithmic imbalance.

The curvature is explicit:

$$\frac{d^2}{dt^2} J(e^t) = \cosh t > 0.$$

In ratio coordinates,

$$J'(x) = \frac{1 - x^{-2}}{2}, \qquad J''(x) = x^{-3} > 0.$$

The cost is strictly convex on the positive ratios and in the logarithmic coordinate. Figure 3.2 in Chapter 3 plots the curve.

2.8 How the comparison is reported fixes its subtraction and reference

The apparatus subtracts the second operand and reports against the quarter-phase increase. Neither is a free choice. The properties that came

out of the apparatus, a zero at balance, the same value for x and $1/x$, and growth with the magnitude of the imbalance, can be required of any report instead, with the operation, the placement of the operands and the reference left open.

Write the input as $u\xi + zw\eta$ for any two fixed signals ξ, η on the eight entries, let T be any linear operation on such signals, and measure, in squared-norm units, the residual energy

$$D_z(u,w) = \|T(u\xi + zw\eta)\|^2.$$

Let a reference be any real combination of runs at unit phases,

$$R(u,w) = \sum_j c_j D_{z_j}(u,w), \qquad |z_j| = 1,$$

and let the report agree with a meter at whole-number amplitudes:

$$F(a/b) = m(D_1(a,b), R(a,b)) \qquad (a,b \in \mathbb{N},\ a,b > 0).$$

The apparatus is the case $\xi = f(1,0)$, $\eta = f(0,1)$, with T the join's residual.

The reason is quadratic. A linear operation measured in squared-norm energy makes every run a quadratic form in the amplitudes. A zero at balance leaves only multiples of $|u - zw|^2$, and a reference that vanishes when the second operand is absent leaves only the cross term uw. Their quotient is a multiple of $(u-w)^2/(2uw) = J(u/w)$.

Theorem 2.3 (Requirements on the report fix the comparison). *Let the meter satisfy the pooling, nonnegativity and self-reference requirements. Suppose that the reference is positive when both operands are present and zero when the second is absent, so $R(a,b) > 0$ for positive whole a, b and $R(1,0) = 0$; that equal operands report zero, $F(1) = 0$; that the order of the operands does not matter, $F(x) = F(1/x)$ for $x > 0$; and that a larger imbalance never reports smaller, so $F(e^t)$ is nondecreasing for $t \geq 0$. Then $T\eta = -T\xi$, $K = \|T\xi\|^2 > 0$ and $\sum_j c_j = 0$. Moreover*

$$D_z(u,w) = K|u - zw|^2,$$
$$R(u,w) = \gamma K uw \quad \text{with} \quad \gamma = -2\sum_j c_j \operatorname{Re} z_j > 0,$$

and $(\gamma/2)F(x) = J(x)$ for every $x > 0$.

Proof. At $a = b = 1$ the meter reports the quotient $D_1(1,1)/R(1,1)$ (Proposition 2.1), and this equals $F(1) = 0$. So $\|T(\xi + \eta)\|^2 = 0$ and $T(\xi + \eta) = 0$. Hence $T\eta = -T\xi$, and by linearity $T(u\xi + zw\eta) = (u - zw)\,T\xi$, which gives $D_z(u,w) = K|u - zw|^2$. For $|z| = 1$ and real u, w, $|u - zw|^2 = u^2 + w^2 - 2uw\,\mathrm{Re}\,z$. Summing the runs,

$$R(u,w) = K\Big[\Big(\sum_j c_j\Big)(u^2 + w^2) - 2uw \sum_j c_j \,\mathrm{Re}\, z_j\Big].$$

If K were zero, every run and so every reference would vanish, against $R(1,1) > 0$; so $K > 0$. At $(u,w) = (1,0)$ the reference is $K\sum_j c_j = 0$, so $\sum_j c_j = 0$ and $R(u,w) = \gamma K uw$. Positivity at $a = b = 1$ gives $\gamma > 0$.

For positive whole a, b the meter again reports the quotient, so

$$F(a/b) = \frac{K(a-b)^2}{\gamma K ab} = \frac{2}{\gamma} \cdot \frac{(a-b)^2}{2ab} = \frac{2}{\gamma} J(a/b).$$

Put $G = (2/\gamma)J$, which is continuous on $(0, \infty)$. Fix $x > 1$. For rational y, y' with $1 \le y < x < y'$, the order requirement gives $G(y) = F(y) \le F(x) \le F(y') = G(y')$. Letting y and y' approach x gives $F(x) = G(x)$. At $x = 1$ both are zero, and for $0 < x < 1$ symmetry gives $F(x) = F(1/x) = G(1/x) = G(x)$. □

A single run has one nonzero weight, so its weights cannot sum to zero as the theorem requires. The quarter-phase increase $D_{\mathrm{i}} - D_1$ has weight $+1$ at $z = \mathrm{i}$ and -1 at $z = 1$, so its weights sum to zero and $\gamma = 2$: the report is J itself.

The conclusion $T\eta = -T\xi$ is the sense in which the reversed sign in (2.1) is forced: in any linear comparison on these entries in which equal operands report zero against a positive reference, the operation takes the second operand's signal to the negative of what it makes of the first's.

2.9 Combining comparisons gives the same cost

The third route uses no apparatus. It asks how the costs of chained comparisons combine and what a small imbalance costs. Chain a comparison with ratio x to one with ratio y: set in the same direction they give the ratio xy, and set against each other they give x/y. Require of a cost F on positive ratios that the two chained costs together be fixed by the costs of the parts, in this form:

$$F(xy) + F(x/y) = 2F(x)F(y) + 2F(x) + 2F(y) \qquad (x, y > 0). \tag{2.8}$$

The recognition cost obeys this law: for $F = J$, $x = e^t$ and $y = e^\tau$, both sides equal $2\cosh t \cosh \tau - 2$. Require also a unit near balance: a small logarithmic imbalance t costs $t^2/2$ to leading order, as $\cosh t - 1$ does.

The form of this law is the route's premise, as the apparatus was the first route's. For $F + 1$ in logarithmic coordinates it is d'Alembert's equation, the addition law of cosh, and the theorem below shows that the unit near balance leaves it one solution. Its product term $2F(x)F(y)$ makes the costs of the parts interact. Without it the law becomes $F(xy) + F(x/y) = 2F(x) + 2F(y)$, which $\frac{1}{2}(\log x)^2$ satisfies with the same unit, since $(t+\tau)^2/2 + (t-\tau)^2/2 = t^2 + \tau^2$; the product term is what selects J.

Theorem 2.4 (Composition and unit calibration force the cost). *Let $F : (0, \infty) \to \mathbb{R}$ satisfy the composition law* (2.8) *for all $x, y > 0$, and*

$$\lim_{t \to 0,\ t \neq 0} \frac{2F(e^t)}{t^2} = 1.$$

Then $F(x) = J(x)$ for every $x > 0$.

Proof. Put $h(t) = F(e^t) + 1$. The law becomes $h(t+\tau) + h(t-\tau) = 2h(t)h(\tau)$ for all real t, τ, and the calibration says $2(h(t) - 1)/t^2 \to 1$. Taking $\tau = 0$ gives $h(t) = h(t)h(0)$ for every t. If $h(0) \neq 1$, then h vanishes identically and $2(h(t) - 1)/t^2 = -2/t^2$ has no finite limit. So $h(0) = 1$.

By the calibration there is $\delta > 0$ with $h(t) > 1$ for $0 < |t| < \delta$. For such t, let $\theta(t) > 0$ be the number with $\cosh \theta(t) = h(t)$. Then $h(t) \to 1$, so $\theta(t) \to 0$. Since $(\cosh \theta - 1)/(\theta^2/2) \to 1$ as $\theta \to 0$, the calibration gives $\theta(t)/|t| \to 1$.

Fix such a t. The law at (nt, t) gives $h((n+1)t) = 2h(t)h(nt) - h((n-1)t)$ for $n \geq 1$. The numbers $\cosh(n\theta(t))$ obey the same recurrence, since $\cosh((n+1)\theta) + \cosh((n-1)\theta) = 2\cosh\theta\cosh(n\theta)$, and they start from the same values 1 and $h(t)$. By induction, $h(nt) = \cosh(n\,\theta(t))$ for every $n \geq 0$.

Now fix $t_0 \neq 0$ and take $t = t_0/n$ with n large enough that $|t| < \delta$. Then $h(t_0) = \cosh(n\,\theta(t_0/n))$, and $n\,\theta(t_0/n) = |t_0| \cdot \theta(t_0/n)/|t_0/n| \to |t_0|$. The left side does not depend on n, so $h(t_0) = \cosh|t_0| = \cosh t_0$. Together with $h(0) = 1$, this gives $h = \cosh$ on the whole real line. Hence $F(x) = h(\log x) - 1 = \cosh(\log x) - 1 = J(x)$. □

Continuity and the zero at balance were not assumed; both follow. Reciprocity sits close to the law's form: once $F(1) = 0$, setting $x = 1$ in

the law gives $F(y) + F(1/y) = 2F(y)$, so $F(1/y) = F(y)$. This route is the composition law and quadratic calibration of [1].

2.10 Golden scaling is forced

The join builds a record in stages, each a *level* with a positive *size*. The theorem below shows that, under three conditions, successive sizes must scale by the golden ratio. Among the moral results golden scaling returns once, and conditionally: Temperance's cap on a single commitment is $1/\phi$ of the amount committed if that amount, the cap and the remainder are successive levels of this kind (Chapter 10).

The simplest join acts on a record of two whole counts, the latest level's and the previous level's, and forms the next level by adding them: $(p, q) \mapsto (p + q, p)$. From $(0, 1)$ it runs through $(1, 0)$, $(1, 1)$, $(2, 1)$, $(3, 2)$, and so on. With $\phi = (1 + \sqrt{5})/2$, the positive root of $\phi^2 = \phi + 1$, the join multiplies the coordinate $\phi p + q$ of a pair by ϕ and the coordinate $p - \phi q$ by $-1/\phi$:

$$\phi(p + q) + p = \phi\,(\phi p + q), \qquad (p + q) - \phi p = -\phi^{-1}(p - \phi q),$$

since $\phi^2 = \phi + 1$ and $\phi - 1 = 1/\phi$.

This is the computation Section 2.3 used. The apparatus writes a pair on the eight entries in the layout (2.1) with $z = 1$ and real values u, w, here of either sign (for the pair $(0, 1)$, $u = 1 - \phi$), given by $u + w = 2(\phi p + q)$ and $u - w = 2(p - \phi q)$. The growing coordinate goes to the sum part, so the join divided by ϕ keeps that part and multiplies the mismatch part by $-\phi^{-2}$; written the other way round, it would change the sum part, and equal operands would not report zero. The apparatus's operation does the same on every signal; it is defined by

$$\mathcal{A}f = Pf - \phi^{-2}\,Qf,$$

and what it changes is the residual

$$Tf = f - \mathcal{A}f = (1 + \phi^{-2})\,Qf.$$

The nth level is the state reached from a starting state after n updates; take each state to be a record and its size to be what the record counts. The first condition says that sizes add: adding two records adds what they count, the arithmetic on which counting rests, and the update of a

sum of records is the sum of their updates. Signed differences of records, the comparisons between them, are included. The second says that the update may rescale but never reverses or erases which of two records was larger. The third says that the second level joins two earlier ones and that this join is a recognition whose two parts are told apart, so that it posts a nonzero cost. For each of the three index pairs the condition allows, ℓ_{j-i}/ℓ_0 is the ratio ℓ_j/ℓ_i of the two parts joined, and J vanishes only at balance (Section 2.7), so the cost clause says that the parts differ. It excludes $\ell_2 = 2\ell_0$ and $\ell_2 = 2\ell_1$, a level joined with a copy of itself, which distinguishes nothing and would give the factors $\sqrt{2}$ and 2. The first two conditions make every update multiply sizes by one factor ρ; the third supplies the recursion $\ell_2 = \ell_0 + \ell_1$, so $\rho^2 = \rho + 1$, whose one positive root is ϕ.

Theorem 2.5 (Golden scaling). *Let S be a countable set of states with an update $U : S \to S$ and a positive size $\ell : S \to (0, \infty)$. Fix a state s_0 and write $\ell_n = \ell(U^n s_0)$. Suppose:*

(1) Additive sizes. *There are an abelian group Γ, an additive map $\alpha : \Gamma \to \Gamma$, a map $\iota : S \to \Gamma$ and an additive map $\nu : \Gamma \to \mathbb{R}$ with $\nu(\iota(s)) = \ell(s)$ and $\iota(Us) = \alpha(\iota(s))$ for every state s.*

(2) Comparisons survive the update. *For every $g \in \Gamma$, $\nu(g) > 0$ implies $\nu(\alpha g) > 0$.*

(3) The first new level is a costly join. *There are whole numbers $i \le j < 2$ with $\ell_2 = \ell_i + \ell_j$ and $J(\ell_{j-i}/\ell_0) > 0$.*

Then $\ell(Us) = \phi\, \ell(s)$ for every state s. In particular, $\ell_n = \phi^n \ell_0$.

Proof. The update multiplies every size by one factor. Put $g_0 = \iota(s_0)$. Then $\nu(g_0) = \ell_0 > 0$, and by (2), $\nu(\alpha g_0) > 0$. Take any $g \in \Gamma$ and suppose $\nu(g)/\nu(g_0) < \nu(\alpha g)/\nu(\alpha g_0)$. Choose a rational number k/n strictly between them, with $n > 0$. The element $g_1 = k g_0 - n g$ of Γ has

$$\nu(g_1) = n\,\nu(g_0)\Big(\frac{k}{n} - \frac{\nu(g)}{\nu(g_0)}\Big) > 0,$$

so $\nu(\alpha g_1) > 0$ by (2). But α and ν are additive, so

$$\nu(\alpha g_1) = n\,\nu(\alpha g_0)\Big(\frac{k}{n} - \frac{\nu(\alpha g)}{\nu(\alpha g_0)}\Big) < 0,$$

a contradiction. The reverse strict inequality fails in the same way, with $g_1 = ng - kg_0$. Hence $\nu(\alpha g) = \rho\,\nu(g)$ for every g, where $\rho = \nu(\alpha g_0)/\nu(g_0) > 0$. Applied to $g = \iota(s)$,

$$\ell(Us) = \nu(\iota(Us)) = \nu(\alpha\,\iota(s)) = \rho\,\nu(\iota(s)) = \rho\,\ell(s),$$

and so $\ell_n = \rho^n \ell_0$.

The join has two different parts. If $i = j$, then $\ell_{j-i}/\ell_0 = 1$ and $J(1) = 0$, against (3). So $i < j < 2$, which leaves $i = 0$, $j = 1$ and $\ell_2 = \ell_0 + \ell_1$.

The factor is ϕ. Substituting $\ell_n = \rho^n \ell_0$ gives $\rho^2 \ell_0 = \ell_0 + \rho \ell_0$, and $\ell_0 > 0$, so $\rho^2 = \rho + 1$. The roots are $(1 \pm \sqrt{5})/2$, and only $(1 + \sqrt{5})/2$ is positive. Hence $\rho = \phi$. □

The pairs above show the theorem at work. A state is a pair (p, q) of whole numbers, not both zero, the update is the join, and the size is $\lambda p + q$ for a fixed $\lambda > 0$. Signed pairs of integers form Γ; the join is additive on them, and so is the size. From $(0, 1)$ the sizes are $1, \lambda, \lambda+1, 2\lambda+1, 3\lambda+2, \ldots$. The second level is the join of the first two, $\ell_2 = \ell_1 + \ell_0$, and that join costs $J(\lambda) > 0$ whenever $\lambda \neq 1$. With $\lambda = \phi$, the join multiplies every size by ϕ, as the coordinate $\phi p + q$ showed, so a positive comparison stays positive. The sizes are then exactly $1, \phi, \phi^2, \phi^3, \phi^4, \ldots$, although the total counts $p + q$ are the whole numbers $1, 1, 2, 3, 5, \ldots$.

For any other $\lambda \neq 1$, hypotheses (1) and (3) still hold, so (2) must fail: otherwise the theorem would give $\ell_1 = \phi \ell_0$, that is $\lambda = \phi$. With $\lambda = 3/2$, the signed pair $(-6, 10)$, a comparison of two records, has value $-9+10 = 1 > 0$, but its update $(4, -6)$ has value $6-6 = 0$: the update has erased a comparison. Condition (2) asks more than keeping the records: with $\lambda = 3/2$ every record survives the update, yet a comparison that was positive becomes zero.

Golden scaling is forced under these three conditions: once sizes add, comparisons survive each update and the first new level is a costly join, every level is ϕ times the one before.

2.11 What necessity has established

Each route to the cost is a uniqueness theorem from its own hypotheses. With the reference unit $\gamma = 2$ in the first two routes and the unit fixed near balance in the third, J meets them and no other function does; another reference unit forces the fixed multiple $(2/\gamma)J$. The routes overlap,

since the second asks of a report the properties the first derived, but each removes a different freedom: the report of a fixed apparatus, the apparatus's subtraction and reference, and the apparatus itself.

These theorems identify a relative recognition cost; they do not identify every expenditure of physical energy with the dimensionless number $J(x)$. Applying them to a relation takes one further derivation; for a bond between two people it is the pair identification (Section 3.2), and it is still to be done. Once it is supplied, the cost of a relation is a calculation from its ratio, and Chapter 3 uses it to compute burden, direction and harm on a ledger of bonds.

Chapter 3

The geometry of comparison

Alice and Bob are connected by three relations. One runs from Alice to Bob, one from Bob to Alice, and one begins and ends at Bob, each with a positive number saying how far it stands from balance: 2, 1/2 and 2 (Figure 3.1). The numbers are set for illustration, and this is not the ledger of Chapter 1's care interval. How much burden does each of them carry? In which direction does their imbalance point? If Alice acts on one relation, what does her action add to each person's burden, or remove from it?

Recognition Science answers with a physical recognition ledger, a structured account of actual relations and their transformations; the word implies no financial transaction and no bookkeeper. Its geometry keeps the direction of a relation apart from the burden it carries, and both apart from the resources its participants hold.

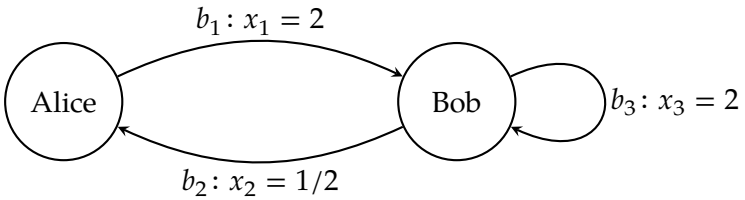


Figure 3.1. Alice and Bob's ledger. Each arrow is a bond from its source to its destination, labeled with its multiplier; b_3 is internal to Bob. Each bond costs $J = 1/4$.

3.1 A comparison has a direction and a magnitude

A *bond* is a directed relation that brings two positive quantities of the same kind into comparison. Its *multiplier* x_b is their quotient, first over second: $x_b = 2$ says the first is twice the second. The bond is balanced, $x_b = 1$, exactly when the two agree, and a change of unit leaves x_b unchanged.

The direction says which quantity stands on top; read from the other end, the same comparison has multiplier $1/x_b$ and is still one bond. So b_1 and b_2 are two relations, each comparing its own pair, that happen to stand at reciprocal ratios.

Chapter 2 derived the dimensionless recognition cost

$$J(x) = \frac{x + x^{-1}}{2} - 1, \qquad x > 0. \tag{3.1}$$

It equals $(x-1)^2/(2x)$, so it vanishes only at balance, and $J(1/x) = J(x)$ (Section 2.7). Every bond of Figure 3.1 costs $J(2) = (2-1)^2/4 = 1/4 = J(1/2)$.

The natural signed coordinate is the *log ratio*

$$t = \log x,$$

also called the bond's logarithmic flow. Multiplying ratios adds flows, and reciprocal ratios have flows t and $-t$. In this coordinate

$$J(e^t) = \frac{e^t + e^{-t}}{2} - 1 = \cosh t - 1, \tag{3.2}$$

which is even in t: the cost cannot tell a comparison from its reverse, and the direction survives in the sign of $\log x_b$, which the skew below keeps.

The cost is also strictly convex in t: with the average of two log ratios held fixed, moving them apart raises their combined cost, and bringing them together lowers it. Log ratios $\pm \log 2$ cost $1/2$ together; at their common average $t = 0$ they cost nothing (Figure 3.2). Chapter 5 uses this to find the unique least-cost endpoint when the product of two multipliers is fixed.

3.2 What a bond between two people compares

Each kind of relation supplies its own pair. In the comparison of Chapter 2 it is two signal amplitudes, and that chapter derives the cost of their quotient. In an agreement it is the amount the source's record claims and the amount the destination's record acknowledges (Chapter 8); in provision, an actual signal and the signal required (Chapter 14). The figure leaves its pairs unspecified: b_1 could be Alice's provision for Bob, and b_2 an agreement comparing what Bob's record claims from Alice with what hers acknowledges: two relations between the same people,

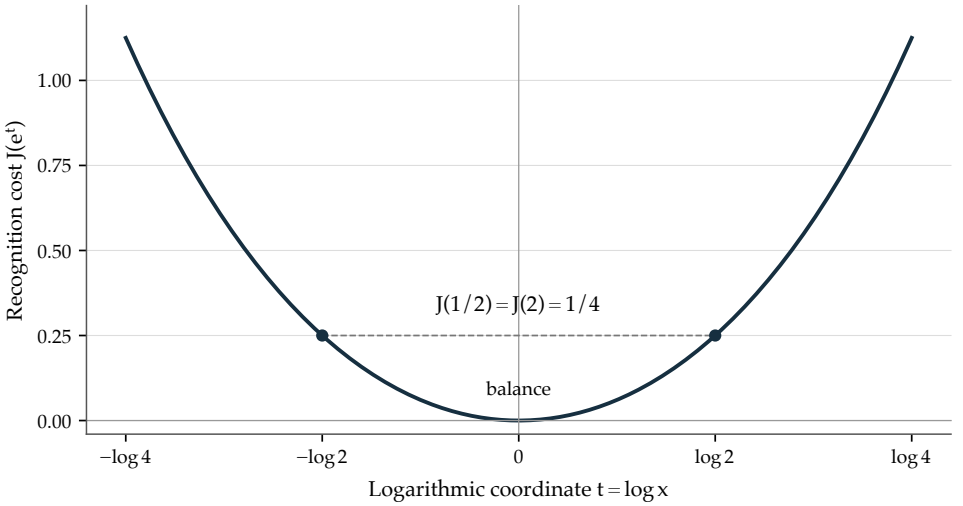


Figure 3.2. Recognition cost in logarithmic coordinates. Reciprocal multipliers lie at opposite coordinates and have equal cost. Balance is the unique minimum. The marked ratios 1/2 and 2 each carry cost 1/4.

with different pairs. A person's own bond, such as b_3, compares their response under *exposure*, the load of getting through an interval: in the care chapters each interval doubles its multiplier and a repair returns it to 1, and which two physical quantities of the person it compares belongs to the derivation of the care environment (Chapter 6).

Take the provision bond from Alice to Bob: it compares an actual signal, what reaches Bob, with a required signal, what he needs. What is still to be derived is which physical quantities of theirs those signals are, and why that bond, from Alice to Bob, carries the comparison. In general this is the *pair identification*: for a bond between two people, which physical quantities form its pair, the actual condition of one compared with what the other's undertaking or need requires, and whose relation carries the comparison. The authored-book and economic constructions identify particular comparisons and their native effects. For a particular human need, the remaining task is to identify its operands and explain why the corresponding response and reserve laws hold.

The algebra below uses only the multiplier and holds whatever the pair. Once a bond's pair is identified as a comparison of the kind Chapter 2 treats, the bond's cost is J by that chapter's theorems, and the incident cost and harm defined below are, by definition, the burden and harm of the people at its ends. Where that application-specific identification

has not been derived, the examples supply the operands and multipliers explicitly. Fixed in advance, the identification can be tested: with the per-interval charge of Table 5.2 and a reserve that tracks usable capacity, keeping a relation at multiplier 4 through an interval should use $J(4)/J(2) = (9/8)/(1/4) = 9/2$ times the capacity that keeping one at 2 uses, in any unit of capacity (Chapter 17).

3.3 The whole counts a bond once, and both ends carry it

A *participant* is one who can stand at an end of a bond, such as Alice or Bob (Chapter 4 shows how participants are told apart).

Definition 3.1 (Ledger). A *ledger* L consists of finitely many active bonds, each a directed relation between participants. Each bond b has an identity, a source participant, a destination participant, and a positive multiplier x_b. Its endpoints specify the ordered relation; its multiplier specifies the relation's present departure from balance. A bond whose source and destination are the same participant is *internal*. The total recognition cost of L is

$$C(L) = \sum_{b \text{ active}} J(x_b).$$

The ledger is balanced when every active multiplier is one.

Alice and Bob's ledger has total cost $C(L) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$. The total is zero exactly when the ledger is balanced, since each term $J(x_b)$ is nonnegative and vanishes only at $x_b = 1$.

A nonzero total does not say who carries it. We assign each participant the whole of every bond incident to them, as source or as destination, and keep the direction of their relations separately. This is a definition. Its reason is that each end is party to the whole relation: when b_1 is out of balance, Alice stands in an unbalanced relation and so does Bob, and neither stands in half of one. A split would need an allocation rule the ledger does not supply.

Definition 3.2 (Incident cost and skew). The *incident cost* of participant i is

$$C_i(L) = \sum_{b \text{ incident to } i} J(x_b),$$

where an internal bond whose source and destination are both i is included once. The participant's net logarithmic *skew* is

$$\sigma_i(L) = \sum_{b:\,\mathrm{source}(b)=i} \log x_b - \sum_{b:\,\mathrm{destination}(b)=i} \log x_b.$$

Alice is incident to b_1 and b_2; Bob is incident to all three, with his internal bond counted once. Writing A for Alice and B for Bob,

$$C_A(L) = \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{1}{2}, \qquad C_B(L) = \tfrac{1}{4} + \tfrac{1}{4} + \tfrac{1}{4} = \tfrac{3}{4}.$$

Bob carries more because one unbalanced relation touches only him. A bond between two participants enters both incident costs while remaining one physical bond. Here $C_A(L) + C_B(L) = 5/4$ exceeds $C(L) = 3/4$ by the cost of the two shared bonds, so adding participant totals without retaining incidence can count a shared relation more than once.

For Alice, b_1 is outgoing and b_2 incoming; for Bob, b_2 and b_3 are outgoing, and b_1 and b_3 incoming:

$$\begin{aligned}\sigma_A(L) &= \log 2 - \log \tfrac{1}{2} = 2\log 2,\\ \sigma_B(L) &= (\log \tfrac{1}{2} + \log 2) - (\log 2 + \log 2) = -2\log 2.\end{aligned}$$

A self-loop contributes equally to the outgoing and incoming sums, so the signed contribution of b_3 cancels while its cost stays in $C_B(L)$. Had b_2 carried the multiplier 2, its contribution would have canceled that of b_1, leaving Alice's skew at zero while both bonds still cost $1/4$. The signed account tells us about orientation and reciprocity; the incident-cost account tells us about carried burden.

The two skews also sum to zero, and so do the skews on every ledger. Each bond adds $\log x_b$ to its source's skew and subtracts the same amount from its destination's, and on an internal bond the two cancel. Summed over every participant at an end of an active bond,

$$\sum_i \sigma_i(L) = 0.$$

An action keeps every endpoint, so it can only move skew between participants. Justice (Chapter 5) acts on skew: on two bonds in opposite directions between the same two people, it moves a participant's skew by a required amount in the one way that keeps their total log ratio.

3.4 Actions transform the relations we measure

Definition 3.3 (Action, strain and harm). An action specifies an acting participant, a finite set of supported bonds, and a real logarithmic *strain* v_b on each supported bond. Outside its support the strain is zero. The action transforms the ledger L into L' with multipliers

$$x'_b = x_b e^{v_b}. \tag{3.3}$$

The active bond identities and their endpoints remain unchanged. The *harm* to participant i is the resulting change in incident cost,

$$H_i = C_i(L') - C_i(L). \tag{3.4}$$

Positive H_i is increased incident recognition cost. Negative H_i is relief.

The acting participant, recorded for later questions of authorship, need not be an endpoint of the bonds the action strains.

Burden and harm are technical terms, and they part from the everyday words in two ways. Burden lies in relations, not in holdings, so a giver's burden can fall. Let the provision bond from Alice to Bob carry actual signal 1 and required signal 2: its multiplier is $1/2$, and it costs each of them $1/4$. When Alice's meal brings the actual signal to 2, the bond balances and her harm is $-1/4$, a relief, although she has given the meal up (Chapter 14). And burden and harm miss damage that does not show in relations. Staying away in Chapter 1 leaves every bond balanced and exhausts Bob's reserve, and severing Bob's relations when one is out of balance lowers his burden, although it can end his life (Chapter 4). The account tracks those damages with quantities of their own: the reserve and whether a participant continues (Chapters 4 and 6), and the undertaking (Chapter 7).

Positivity is preserved, since both factors are positive, and the logarithmic coordinate changes by exactly the strain:

$$\log x'_b = \log x_b + v_b.$$

Successive strains add. A strain followed by its negative returns the original multiplier.

Restoring one multiplier to balance takes exactly one strain; the repairs of Chapters 5, 8 and 14 start from this.

Proposition 3.4 (Restoration fixes the strain). *For $x > 0$,*

$$xe^{v} = 1 \quad \Longleftrightarrow \quad v = -\log x.$$

Proof. The equation is equivalent to $e^{v} = x^{-1}$. Taking logarithms gives $v = -\log x$. Conversely, $xe^{-\log x} = xx^{-1} = 1$. □

Suppose Alice restores b_1, bringing the two quantities it compares into agreement. The proposition fixes her strain at $v_1 = -\log 2$, and b_1 moves from 2 to $2e^{-\log 2} = 1$; every other multiplier and every endpoint stays as it was. In the new ledger L', $C_A(L') = 1/4$ and $C_B(L') = 1/2$, so

$$H_A = \tfrac{1}{4} - \tfrac{1}{2} = -\tfrac{1}{4}, \qquad H_B = \tfrac{1}{2} - \tfrac{3}{4} = -\tfrac{1}{4}.$$

Both are relieved by $1/4$, the opening cost of the restored bond, which touched both of them. Their skews become $\log 2$ and $-\log 2$.

In general, applying these strains to selected active bonds restores them to unity while retaining every unselected multiplier and every endpoint. Each participant's relief is exactly the opening cost on the selected bonds incident to that participant.

Who bears a harm depends on incidence. If an action instead strains Bob's internal bond by $v_3 = \log 2$, that multiplier doubles to 4 and its cost rises from $1/4$ to $J(4) = 9/8$. Bob's harm is $H_B = 7/8$; Alice's is zero, because b_3 does not touch her.

The opening ledger matters. From a completely balanced ledger every participant begins at zero incident cost, so the resulting incident-cost change is nonnegative; from an unbalanced ledger, repair can give a negative change, as Alice's did.

3.5 Dimensionless cost and physical energy

Recognition cost is a pure number, a comparison has an energy, and a participant has a reserve to spend. Later chapters pay repairs from reserves and build records with exact energy accounts, so none of the three may stand in for another without a derived conversion. One comparison shows how cost and energy are related, and that the cost leaves the energy scale open.

Take the comparison pulse of Chapter 2, the input (2.1) with $z = 1$ and real positive amplitudes u and w: $(u, -u)/\sqrt{2}$ on entries 0 and 1, $(-w, w)/\sqrt{2}$ on entries 4 and 5, half a cycle away, and zero elsewhere. The

eight entries are the carrier's count of Section 2.2, and nothing below uses it: the energy identity holds on any cycle of $N \geq 4$ entries, where the same pulse, with its second pair $\lfloor N/2 \rfloor$ places around the cycle, keeps the two pairs apart. We call this pulse the *comparison port*, since it holds the two quantities being compared. Its ratio and amplitude product are

$$x = \frac{u}{w}, \qquad p = uw.$$

Its input energy, the sum of its squared entries, is

$$E = u^2 + w^2 = 2p(J(x) + 1). \tag{3.5}$$

The second equality follows directly:

$$2uw\left(\frac{u/w + w/u}{2}\right) = u^2 + w^2.$$

Recognition cost is the energy above $2p$, measured in units of $2p$.

Opposite half-strains on the operands,

$$u' = ue^{v/2}, \qquad w' = we^{-v/2},$$

preserve their product and change their ratio to xe^v. They therefore realize the multiplier endpoint in Equation (3.3). At fixed product,

$$E' - E = 2p(J(xe^v) - J(x)).$$

Restoring the ratio to one leaves energy $2p$, so the port's energy falls by

$$2pJ(x) = (u - w)^2.$$

With $u = 2$ and $w = 1$, the ratio is 2, the product 2 and the input energy 5. The restoring strain $v = -\log 2$ moves both amplitudes to $\sqrt{2}$, leaving energy 4, a fall of 1. Amplitudes 4 and 2 have the same ratio, product 8 and energy 20, and restoring them lowers the energy by 4. The product is a physical scale factor: restoration lowers the energy of two ports with the same ratio $x \neq 1$ and different products by different amounts.

Since $J(x) = E/(2p) - 1$, an energy limit $E \leq E_{\max}$ bounds the cost only through the product: with $p \geq m > 0$ it gives $J(x) \leq E_{\max}/(2m) - 1$. Without such a floor, small amplitudes can hold a large imbalance at little energy: $u = 1$ and $w = 1/x$ with $x \geq 1$ give $E = 1 + x^{-2} \leq 2$ for

every x, while $J(x)$ grows without bound. So an energy and a cost can be compared only with the scale included (Chapter 17).

In the port a restoration at fixed product lowers the energy the comparison holds. In the care environment of Chapter 6 a repair lowers its owner's reserve by the cost of the bond it rebalances, a debit rule the care model adopts (Table 5.2). The two changes belong to different quantities, the energy a comparison holds and the capacity a participant spends, and they require an explicit physical connection. The following construction supplies one restricted connection; identifying a particular person's reserve and maintenance with it remains application-specific.

3.5.1 An executed repair can debit an accessible reserve

A particular physical construction connects the two accounts without identifying every repair with the same payment law. It keeps a retained comparison signal, a finite bank of prepared donor signals, blank outputs and all previously exported records. A first exchange reconstructs the exposed comparison from the retained signal and one donor. A second exchange retains the balanced comparison and exports its complementary output. The complete store conserves energy; exported energy is no longer accessible under this protocol.

For positive operands A, B, measure accessible energy above the retained signal's baseline in units of the positive calibration $\varepsilon_0 AB$, where ε_0 is the apparatus's unit energy. With n donors remaining after the preparation, the exposed reserve is $(n+1)J(A/B)$. Completing the second exchange leaves reserve $nJ(A/B)$. Its executed debit is therefore exactly $J(A/B)$, while the retained comparison becomes balanced and every old exported record remains present. The measured debit holds for the prepared store. When the post-export reserve is strictly positive, assigning the retained comparison to its owner's internal bond also gives the native repair update, with the same owner, one advanced tick and that reserve decrease. Strict affordability is exactly this positivity condition.

The export restriction matters. Other available controls can retain and recycle the complementary record instead, with a different accessible-reserve account. This construction establishes one physical payment law, with its baseline, calibration, prepared donors and access restriction stated. Identifying a particular organism's maintenance with that protocol requires the corresponding physical argument; neither conserved total energy nor the cost formula alone supplies it.

3.6 Balanced bonds do not settle who can continue

Every question about the numbers on Alice and Bob's ledger is now a calculation. The ledger does not hold everything an action changes: whether a repair is funded and authorized depends on the participants' reserves and on the undertaking at issue. Chapters 6 and 7 follow recognition cost and reserve through one care execution, where a balanced ledger and an exhausted reserve first appear side by side.

Chapter 4

The beings who act and undergo

Bob is asleep when Alice acts on a relation they share. He authors nothing, notices nothing and cannot object, and the change still reaches him. An infant who needs care is in the same position, and so is a person who cannot describe an injury. Moral attention has to reach the side of an action where its effects arrive before it asks whether the one who bears them can reason, bargain or reply.

Authorship and exposure to another's action are different positions in one physical event. Two further questions concern the beings themselves: what the event is like for the one who undergoes it, and whether that one persists through the change. A description adequate for a bond's cost can omit what these questions need.

4.1 A participant has a distinguishable place in the relations

Beside the ledger, each participant keeps a record of their own: participant i has a row of k accounts,

$$r_i = (r_{i1}, \dots, r_{ik}) \in \mathbb{R}^k.$$

Each entry is an amount the participant's record retains; in the account books of Chapter 8 a row holds what the participant claims from another, what they acknowledge owing, and an offset against which their own writes balance. A row is not a bond. Chapter 8 builds an agreement's bond from two rows: its multiplier is the amount the source's row claims over the amount the destination's row acknowledges, and its cost rests on the pair identification (Section 3.2). What carries a record physically is left open here: the results below use only which rows an intervention changes.

One physical premise carries this section: for each participant there is an available intervention that changes that participant's record and no one else's, and the participant's row is the whole of the record such an intervention reaches. Writing down a vector supplies no actuator. The premise takes the participants as given; the proposition below adds that their responses keep them apart. Given such interventions, telling participants apart is bookkeeping.

The interventions are writes a participant makes on their own record. A *conserved own write* by participant i adds a vector w to row i, leaves every other row unchanged, and satisfies

$$\sum_{a=1}^{k} w_a = 0.$$

They are the writes of the later records: in the double-entry books of Chapter 8 and in Chapter 7's record of a proposal and an acceptance, an own write moves amounts between its author's accounts and creates none. With two accounts, Alice's write $w = (1, -1)$ takes her row from $(3, 1)$ to $(4, 0)$; her total stays 4, and Bob's row is untouched. With a single account the only conserved write would be zero, which is why the result below needs two.

A *participant domain* is a set of participants. It *responds* to a write when at least one member's row changes. Domains may overlap.

Proposition 4.1 (Responses recover participant domains). *For this record system with at least two accounts in each row ($k \geq 2$) and the available own writes, two domains respond identically to every write exactly when they have the same members.*

Proof. Equal domains have equal response conditions. Conversely, suppose domains D_1 and D_2 agree on every response. Choose two distinct accounts a and z, and probe participant i with the vector that has entry $+1$ at a, entry -1 at z and zero elsewhere. It is conserved and nonzero, so it changes row i, because adding a nonzero vector cannot leave a vector unchanged; no other row changes. Thus D_1 responds exactly when $i \in D_1$, and D_2 responds exactly when $i \in D_2$. Their agreement gives

$$i \in D_1 \quad \Longleftrightarrow \quad i \in D_2.$$

This holds for every participant, so $D_1 = D_2$. □

What the proposition rules out is a description that merges people. Suppose a description gives Alice and Bob the same code, the label it uses to tell participants apart. Anything that works from the code must then answer every probe of either one alike, yet probing Alice changes her row and not Bob's. No such code preserves the complete response family. Names can be replaced; the distinctions the actions detect must survive.

The proposition tells participants apart. Whether a participant so distinguished is one person belongs to the question Section 4.3 leaves open: which organization of recognition makes one subject.

4.2 Being affected does not require being the author

A relation can connect records belonging to different participants. In the account books of Chapter 8, the bond from Alice to Bob compares an entry of Alice's row with an entry of Bob's, so Alice's own write at her end changes the comparison while Bob writes nothing. Bob is then on the undergoing side of the event.

His exposure needs no capacity to reason and no actuator under his control, only the incident relation through which the effect reaches him. Whether the change harms him, benefits him, was authorized or was owed depends on further facts.

The distinction runs both ways. Receiving a benefit does not show that Bob authored or accepted the act that produced it, and being unable to initiate an act does not show that he cannot bear its consequences.

Undergoing is also different from experiencing. A meter undergoes a physical change; a person undergoes pain. A moral account has to retain both the physical route of an effect and the kind of being whose life it reaches.

4.3 Experience belongs to the reality being explained

The ledger says where an effect lands and how far a relation is from balance. Harm, as Chapter 3 defines it, is a change in that burden and makes no reference to feeling. Whether anyone feels an effect is a further question. Its answer decides which harms are suffered, and whether removing a person's relations ends a subject.

When Bob hears a sound, sees a color or feels a pain, the experience is an occurrence in the world, and an account of reality has to make room for it and its content: the evidence of experience comes from inside the reality under examination.

The positive case for embodied experience starts from episodes we know on other grounds to be experienced, such as a pain, and asks which organized physical activity constitutes them. The evidence is how the experienced content changes when its physical pathways change: block the nerve and the pain goes. A pathway can contribute to an episode without belonging to the subject who has it, as a hearing aid shapes what Bob hears without being part of Bob, so finding a cause and identifying the subject are different tasks.

The causal argument has three premises. Physical effects have sufficient physical causes; experience makes a causal difference; and the same effects are not systematically produced twice by independent sufficient causes. If Bob's pain acted independently of the physical activity, each effect it made, such as his flinch, would already have a sufficient physical cause and would be produced twice, which the third premise excludes. The difference experience makes therefore runs through the physical activity itself. If experience makes that difference, it cannot be a second cause acting on the activity; an account that fits both facts is that it is the activity, organized as it is. Embodied constitution says exactly that, and it gives one account of the experience and its causal work.

Recognition supplies the structure through which events are distinguished, compared, retained and acted upon, Bob's pain among them. The argument covers episodes already known to be experienced, and we extend it by a premise it does not entail. We adopt the dual-aspect premise: every occurrence of recognition has an observable outside and an experiential inside. The reason to adopt it is continuity. Experience is then intrinsic to the activity already doing the physical work, and feeling and physical action belong to one process; without the premise, some boundary within recognition activity would divide occurrences with an inside from those without, and that boundary would need a law of its own. No calculation of cost, harm, repair or care uses the premise; each holds whatever is felt. The premise bears on the two questions feeling decides; Chapter 16 follows it to artificial systems and to the whole.

Under the premise even a meter's comparison has an inside, a strong consequence and the premise's alone. It does not make the meter some-

one: an occurrence can have an inside without being anyone's experience. The open question is the subject. Which organizations of recognition constitute one subject's experience, as Bob's pain belongs to Bob, is not yet derived. Nor is how subjects compose: whether a household, or the whole, is a subject that contains Bob's experience without erasing him. Local organization and retained history are the candidate marks of one subject, since they are how a particular perspective develops. Until these identifications are made, it is open when a severing ends an actual subject; Chapter 16 states what an occurrence, a subject and a whole must satisfy for them to be made.

For a given severing, the question is whether the removed relations carry occurrences someone feels, and the evidence that could settle it has a definite form. A felt occurrence provides a firm local anchor. If a physical description is shown to refer to the very occurrence Bob feels, the fact that he feels it transfers with the identity, by Leibniz's law: whatever is true of the occurrence is true of anything identical to it, however it is described. Extending the conclusion to a distinct occurrence of the same physical type requires a law connecting that type to a subject's experience. Equality of cost, resemblance of a report and identity of an occurrence are different kinds of evidence.

4.4 A lower burden can conceal a lost life

Recognition cost registers imbalance in bonds that exist. What does it report when a transformation removes the bonds themselves?

For participant i, the incident burden of Chapter 3 is

$$C_i(L) = \sum_{b \in I_i(L)} J(x_b),$$

where $I_i(L)$ is the set of bonds incident to i in ledger L. Every term is nonnegative. If all those bonds are balanced, the burden is zero.

A *severing* transformation S_i removes all bonds incident to i and leaves the remaining bonds unchanged. The strain actions of Chapter 3 change multipliers and never remove a bond; severing changes which bonds exist. After severing, $I_i(S_iL)$ is empty. Therefore

$$C_i(S_iL) = 0.$$

Suppose Bob's only bond has multiplier 2, so his burden is $J(2) = 1/4$. Severing it leaves burden 0, and a burden assessment records relief of

1/4. From a balanced bond it would record no change at all. Neither assessment registers that every relation Bob had is gone.

Proposition 4.2 (Burden alone cannot register every ending). *Severing never increases the affected participant's incident burden. It strictly decreases that burden if an incident bond was out of balance. From a balanced state the burden change is zero. More generally, let an assessment F compare only that participant's burden before and after, with $F(c, c) = 0$ for every c; then F assigns zero to the severing of a balanced participant.*

Proof. The final burden is zero and the initial burden is nonnegative. If an incident multiplier differs from one, its cost is strictly positive; the sum is then positive, so the change to zero is a strict decrease. At balance every summand is zero, so the initial and final burdens are both zero. The last assertion follows from $F(0, 0) = 0$. □

If the removed relations sustain a subject, their removal can end that subject while the burden assessment stays constant or improves. Reducing Bob's burden and removing Bob, whose life bears that burden, are physically different transformations. An account meant to register an ending must keep more than the burden before and after: the participant, the reserve and whether it still admits a continuation (Chapter 6), and the relations a repair is meant to preserve.

4.5 The whole retains the lives of its members

Let $P(i)$ express a particular requirement for participant i, such as having positive reserve at a checkpoint. A requirement on a group has to be checked member by member, because a total can pass while a member fails. In the history of Table 1.1 in which Alice stays, the two reserves total $7/4 + 0 = 7/4$, comfortably positive, while Bob holds nothing.

A domain meets the *memberwise requirement* when every member meets it, and such requirements combine across overlapping domains without loss. Take the household $D_1 = \{\text{Alice}, \text{Bob}\}$ and the pair $D_2 = \{\text{Bob}, \text{Carol}\}$, where Carol, a third person, needs the same food (Chapter 13); the two domains share Bob. All three people meet the requirement exactly when the household does and the pair does:

$$(\forall i \in D_1 \cup D_2,\ P(i)) \quad \Longleftrightarrow \quad (\forall i \in D_1,\ P(i)) \text{ and } (\forall i \in D_2,\ P(i)),$$

since each domain lies inside the union and every member of the union lies in at least one domain.

Enlarging the scope therefore keeps every member's requirement, which a total cannot do. Shared resources are still counted once (Chapter 1): Bob's need does not create a second copy of the food, and the household's larger scope does not make his need disappear.

Participants are distinguishable through their responses, and their lives are connected through shared constraints. A moral calculation therefore has to keep each person distinct, keep the relations that sustain each life and check every member of a whole; the cost alone does none of these.

Part II

Action, care and obligation

Love brings a strained relation to its unique conserving minimum, and Justice executes an exact correction. Care has an exact condition for keeping two people alive on finite resources, and an accepted undertaking governs the next action while its violation stays possible.

Chapter 5

Love, justice, and the work of repair

Bob carries a relation out of balance, and Alice can help by taking part of its imbalance onto a relation of her own. What must the repair conserve? Who receives relief, and does someone else bear more? Can both continue? What actions and resources produce the change, and who pays?

Love and Justice answer different parts of these questions. Love finds the conserving endpoint of least recognition cost. Justice executes a specified correction on the actual relations. The endpoint, the correction owed, the action that reaches it and the capacity to continue are distinct parts of the same physical event.

The fourteen virtues introduced at the beginning of the book share this concern with what people can do and undergo. Here the two central geometries make the relation between Love and Sacrifice visible. When Alice and Bob each carry both reciprocal bonds, equalization relieves both. When each carries a separate internal bond, the same conserving endpoint can relieve Bob by placing a burden on Alice. Incidence determines who bears the change.

The exact results follow the physical operations. Their extension to a stable virtue in a person requires retained records that reliably select the appropriate operation when its conditions hold. The accepted care construction of Chapter 7 supplies a concrete case of action selection; the full disposition account remains to be derived. Table 11.2, after the individual virtues have been developed, distinguishes their proof strength and remaining conditions.

5.1 Equalization is the unique conserving minimum

Love's work is to repair a pair of bonds out of balance without changing their total logarithmic flow, and the best such repair makes the two multipliers equal. Take two bonds between Alice and Bob, one in each direction: the bond from Alice to Bob has multiplier 1, the bond from Bob to Alice has multiplier 4, and together they cost $J(1) + J(4) = 0 + 9/8$. Keep their product at 4 and move them toward each other. At $(4/3, 3)$ the total is $1/24 + 2/3 = 17/24$; at $(2, 2)$ it is $1/4 + 1/4 = 1/2$. The equal pair is the lowest point on the curve of conserving endpoints (Figure 5.1).

Why keep the product? Each bond carries a logarithmic flow $\log x$ (Chapter 3), and keeping the product keeps the sum of the two flows, the pair's *total logarithmic flow*. For opening multipliers x, y and endpoints x', y',

$$x'y' = xy \quad \Longleftrightarrow \quad \log x' + \log y' = \log x + \log y.$$

In the example, $\log 2 + \log 2 = \log 1 + \log 4$. The total is not a skew: Alice's skew on the pair is the difference of the two flows, $\log 1 - \log 4 = -\log 4$ before the repair and $\log 2 - \log 2 = 0$ after.

We define Love's repairs by conservation: they move logarithmic flow between the two bonds and keep its total. The theorem below finds the best of them. Equalizing by adding flow instead, raising the first bond to 4, also leaves equal multipliers, but the pair then costs $J(4) + J(4) = 9/4$, twice what it started with. Removing flow until both multipliers are one is full restoration. It leaves less cost, zero instead of $1/2$, but it changes the pair's total flow, so it is not a move within the pair; Section 5.6 debits its cost from a payer by a convention. Which actual repairs between two people bring in no new flow depends on the pair identification (Section 3.2); the theorem below holds for every one that does.

Theorem 5.1 (The conserving minimum). *Among all positive endpoints x', y' with $x'y' = xy$, the unique minimum of $J(x') + J(y')$ occurs at*

$$x' = y' = \sqrt{xy}.$$

Its value is $2J(\sqrt{xy})$.

Proof. Put $g = \sqrt{xy} > 0$. Since $x'y' = g^2$, the arithmetic–geometric mean inequality gives

$$x' + y' \geq 2g.$$

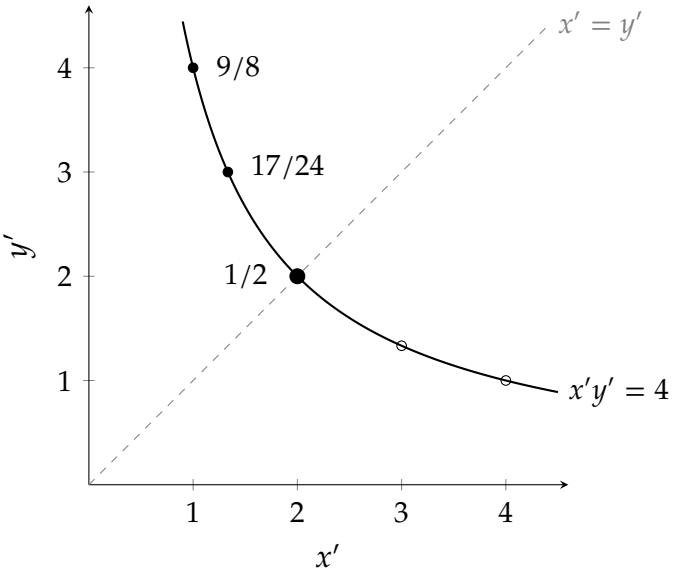


Figure 5.1. The conserving endpoints of the pair $(1, 4)$. Every point on the curve $x'y' = 4$ keeps the product of the two multipliers. Each labeled point shows its total cost $J(x') + J(y')$: $9/8$ at the opening $(1, 4)$, $17/24$ at $(4/3, 3)$, and the least value, $1/2$, at the equal point $(2, 2)$, where the curve meets the diagonal $x' = y'$. The open circles, $(3, 4/3)$ and $(4, 1)$, are mirror images with the same costs.

Applied to the reciprocals, it gives

$$x'^{-1} + y'^{-1} \geq 2g^{-1}.$$

By definition, $J(x') + J(y') = \frac{1}{2}[(x' + y') + (x'^{-1} + y'^{-1})] - 2$ and $2J(g) = \frac{1}{2}[2g + 2g^{-1}] - 2$. Adding the two bounds therefore yields

$$J(x') + J(y') \geq 2J(g).$$

If $x' \neq y'$, the first inequality is strict, so the cost bound is strict. Equality therefore requires $x' = y'$, and positivity together with $x'y' = g^2$ then gives $x' = y' = g$. This endpoint satisfies the constraint and attains the bound. □

Every unequal conserving endpoint does strictly worse, but who gains depends on whose bonds these are. Here the two bonds form a *reciprocal pair*, one in each direction between the same two people, so Alice and Bob are each incident to both and each carries the pair's whole cost: $9/8$ before and $1/2$ after. By the theorem, equalization strictly reduces both incident burdens whenever the opening multipliers differ.

The name Love claims a combination, and its parts have different sources. The theorem supplies two: the repair has the least total recognition cost among endpoints with the pair's original total logarithmic flow, and it is the only conserving endpoint that treats the two bonds alike. Its support supplies the third: it acts on the pair's two bonds alone, so everyone else keeps exactly the same incident cost. Incidence, as Chapter 3 defines it, supplies the fourth: each person carries both bonds, so both gain what the pair gains, here $5/8$ each, and the least total is the least burden for each. A settlement divides a loss between opposed claims; this repair is chosen for the pair as a whole and favors neither bond, treating each one's good as bound up with the other's, as love does. That last clause is the naming step every virtue takes: an operation proved to do the work its name was given. When each person has a bond of their own under a separate limit, the same principle selects the feasible endpoint nearest equal, as does any strictly increasing concave weighing of the two burdens alike (Section 5.4).

5.2 A conserving correction has exactly one execution

Justice's work is to make relations carry exactly the correction that is owed, to the right people and no one else. Two uniqueness results bear on it. The record fixes the content: once the accounts are fixed, the correction that makes a claim truthful to the recorded events is unique (Chapter 8). Conservation fixes the execution: on a reciprocal pair, a correction that keeps the pair's total logarithmic flow has exactly one execution, equal and opposite half-strains. Connecting the two is still to be derived: showing that the correction a truthful record fixes is carried out as the conserving correction on the same pair of people. Where Love selects an endpoint, Justice starts from a required effect and asks which action produces exactly that effect.

Take a reciprocal pair of distinct active bonds, one from participant i to participant j and one from j to i, with $i \neq j$, and strains v_1, v_2 on them. Suppose the bond from i to j carries $1/2$ and the bond from j to i carries 2, so the skew of i on this pair is $\log \frac{1}{2} - \log 2 = -\log 4$. A correction that raises it by $\delta = \log 4$ and conserves the pair's total flow takes strains $\log 2$ and $-\log 2$, and both bonds return to 1. They reach balance only because this pair's product is one. A conserving correction keeps the product, so on Love's pair $(1, 4)$ the same correction gives $(2, 2)$: the skew is cleared and neither bond is at balance. That is Love's endpoint, reached by the same strains, and the theorem below shows why: on a reciprocal pair Love's repair is the conserving correction that clears the pair's skew. The two acts part when the correction owed is not the clearing one, or the bonds are not a reciprocal pair. Correction and restoration are different acts.

Why conserve the total flow? A correction has a direction, raising one person's side of the pair and lowering the other's, and only skew carries direction: a bond's cost is the same at x and $1/x$. Any strains with $v_1 - v_2 = \delta$ raise σ_i by δ and lower σ_j by δ. They differ only in their common part $v_1 + v_2$, which leaves both skews where the correction puts them and only adds flow to the pair or removes it. A nonzero common part is a second act on the whole pair, a partial restoration or a further strain; it can lower the pair's cost, as restoration does, and whether it is owed and who pays are its own questions (Section 5.6). The correction

owed is the change in skew, and keeping the pair's flow is what doing exactly that means. With the common part at zero, one execution remains.

Theorem 5.2 (The half-strains of a conserving correction). *Suppose an action is supported on this reciprocal pair, preserves its total logarithmic flow, and increases σ_i by a specified real amount δ. Its strains are necessarily*

$$v_1 = \frac{\delta}{2}, \qquad v_2 = -\frac{\delta}{2}.$$

They increase σ_i by δ, decrease σ_j by δ, and leave every other participant's skew and incident cost unchanged.

Proof. Conservation gives $v_1 + v_2 = 0$. The first strain changes an outgoing flow of i, while the second changes an incoming flow, so the source increment is $v_1 - v_2 = \delta$. Adding and subtracting the equations gives the displayed strains.

Participant j has the reversed incidence and therefore receives increment $v_2 - v_1 = -\delta$. Every other participant is incident to neither bond. All bonds outside the pair have zero strain. □

Two conserving corrections on the same pair compose by adding their amounts, the opposite correction undoes one, and a zero correction preserves the entire ledger, including unrelated relationships. That is what giving each their due comes to on the ledger: the amount the record requires, executed exactly and borne by no one else. The execution alone does not make a correction just. What makes it just is that the record it answers to is truthful to what happened; which correction is owed and who may authorize it come from that record and its history (Chapters 7 and 8).

5.3 Sharing a burden retains the helper

When Alice takes part of Bob's imbalance onto a bond of her own, her side of the move has to be counted: the repair that relieves Bob raises her burden. The move of Figure 5.1, $(1, 4)$ to $(2, 2)$, keeps the product and lowers the total by the same amount whoever owns the bonds; what changes is who carries them. Now let each bond be internal to one person (Chapter 3): one is Alice's own bond, the other Bob's, each carrying its owner's response to exposure (Section 3.2), with multipliers

$$(1, r^2), \qquad r > 1.$$

Alice begins without cost on her bond; Bob carries the exposed burden. Strains $(\log r, -\log r)$ reach the conserving endpoint (r, r), moving imbalance from Bob's bond onto Alice's and keeping the two bonds' total logarithmic flow. Unlike a reciprocal pair, these bonds share no participant, so nothing in the ledger ties them: keeping their total is a property of the operation that moves the imbalance, which needs a physical contact acting on both bonds at once. And Alice carries only her own bond.

Let $s = J(r) > 0$. The identity

$$J(r^2) = 2J(r)^2 + 4J(r) \tag{5.1}$$

shows what each of them bears.

Theorem 5.3 (Sharing burdens the helper and relieves the recipient). *Alice's incident cost increases by s. Bob's decreases by $s(2s+3)$. The total cost strictly decreases, and every other participant's incident cost is unchanged.*

Proof. Since $(r + r^{-1})/2 = s + 1$ and $(r + r^{-1})^2 = r^2 + r^{-2} + 2$, we have $J(r^2) = \frac{1}{2}(r + r^{-1})^2 - 2 = 2(s+1)^2 - 2 = 2s^2 + 4s$, which is (5.1). Initially Alice's cost is $J(1) = 0$ and Bob's is $J(r^2) = 2s^2 + 4s$. Afterwards each carries $J(r) = s$. Their changes are therefore

$$s, \qquad s - (2s^2 + 4s) = -s(2s+3).$$

The initial total is $2s^2 + 4s$, whereas the final total is $2s$. Their difference is strictly positive because $s > 0$. Every other participant is incident to neither bond. □

Bob's relief is $2s + 3$ times Alice's added burden: always more than three times, and set by the opening state, so no fixed fraction describes it. With $r = 2$ this is the pair $(1, 4)$ again, and $s = 1/4$. Alice goes from zero to $1/4$; Bob goes from $9/8$ to $1/4$, a relief of $7/8 = s(2s+3)$, while the total falls from $9/8$ to $1/2$ (Table 5.1).

	Alice's bond	Bob's bond	Alice's cost	Bob's cost	Total cost
Before	1	4	0	9/8	9/8
After	2	2	1/4	1/4	1/2

Table 5.1. Sharing a burden at $r = 2$. Alice's cost rises from 0 to $1/4$ while Bob's falls from $9/8$ to $1/4$. One member is worse off, and the total is lower.

The endpoint is the equal point of Love's theorem; the burden Alice takes on to reach it is Sacrifice. The converter of Section 9.6 is such a contact: granted the identification of its two comparisons with Alice's and Bob's internal bonds, it carries out this sharing at the single value of r its pulse fixes.

Relief now does not settle whether Alice can keep providing it. For that we adopt a law of the reserve, the *charged law*. Each participant holds a reserve, the capacity they can spend, and a *budget* $\beta > 0$ renews it each interval; each interval the recognition cost of the bonds they keep, their *charge* c, is debited from the reserve, unit for unit. Recognition cost and reserve are different quantities (Chapter 3), and this debit is the accounting premise that connects them: keeping a bond out of balance uses capacity in proportion to its cost, and counting reserve in units of recognition cost makes the proportion one. Here only the continuation conditions and covered endpoints below use it. With constant charge c on an unchanged ledger, opening reserve $R \geq 0$ and no fresh exposure, a participant's reserve after n intervals is

$$R_n = R + n(\beta - c).$$

The charge is one of three rules that debit recognition cost from a reserve. The other two belong to repairs: the *restoration cost* of Section 5.6, debited once from one designated payer when bonds are returned to balance, and the care *maintenance* of Chapters 1 and 6, which is that restoration cost levied on each interval's repair, with the owner as payer. Table 5.2 sets the three side by side. Each is an adopted law of the construction: none is the energy a comparison port releases on restoration, and the general physical derivation of these laws remains application-specific. Section 3.5.1 derives one restoration debit from an executed export, retaining the apparatus and access conditions on which it depends.

Theorem 5.4 (Constant charged continuation). *Under this reserve law, reserve is positive at every nonnegative integer time exactly when*

$$R > 0 \quad \textit{and} \quad c \leq \beta.$$

Proof. Positivity at time zero requires $R > 0$. If $c \leq \beta$, every later reserve is at least R. If $c > \beta$, the negative increment $\beta - c$ eventually exhausts any finite opening reserve: choose an integer $n \geq R/(c - \beta)$. □

Rule	What it debits	When it applies	Where it is used
Charge	The recognition cost c of the bonds a participant keeps, from that participant's reserve, unit for unit; a budget $\beta > 0$ renews the reserve	Every interval the bonds are kept	Sections 5.3 and 5.4; Chapters 7, 9 and 10; the capacity test of Chapter 17
Restoration cost	The opening cost Q of the bonds restored, each counted once, from one designated payer's reserve	Once, when the bonds are returned to balance	Section 5.6
Care maintenance	$J(2m)$, the cost of the owner's internal bond after the interval's exposure doubles its multiplier m, from the owner's reserve; from balance, $J(2) = 1/4$. It is the restoration cost of that bond, with the owner as payer	Each interval, when the reserve and the interval's funding cover it; otherwise nothing is debited and the bond stays exposed	Chapters 1 and 6, and the care calculations built on them

Table 5.2. The three rules that debit recognition cost from a reserve. Only the charge comes with a budget. In the care model, apart from maintenance, a reserve changes only by each interval's net funding, such as a meal's intake or the work of delivering it (Chapter 6).

Under this law a participant continues when their reserve is positive at every time, the opening included; the law's own funded future is then a continuation in the sense of Chapter 6. Once the budget covers the charge, any positive reserve, however small, stays positive, because each interval's renewal pays that interval's charge and the reserve never has to pay one alone. The theorem is about this one funded future. With only the reserve given and the policy and renewal free, some funded future exists exactly when the reserve is positive, whatever the charge under this law (Chapter 7).

At (r, r) Alice and Bob each pay the charge $J(r)$ of their own bond, so both continue exactly when each opens with a positive reserve and each budget is at least $J(r)$. Suppose Alice's budget is $1/8$ at $r = 2$. Her bond charges $c = J(2) = 1/4$ each interval, so $c > \beta$, and her reserve is exhausted by the first integer $n \geq 8R$, even though the shared cost has fallen. With budgets of $1/4$ each and positive opening reserves, both continue.

5.4 Separate capacities determine the feasible optimum

Full equalization can ask more of Alice than she has. With separate budgets, the best repair is the feasible endpoint closest to equal.

Return to Alice's and Bob's own bonds, opening at $(1, r^2)$, now with $r \geq 1$, and compare every positive conserving endpoint

$$(x, r^2/x).$$

Each owner has a separate nonnegative budget, β_0 for Alice and β_1 for Bob: the most that person can pay toward their own bond's charge in an interval. Define

$$U(\beta) = 1 + \beta + \sqrt{\beta(\beta + 2)}, \qquad u_0 = U(\beta_0), \quad u_1 = U(\beta_1).$$

For $z > 0$, solving $J(z) \leq \beta$ gives

$$\frac{1}{U(\beta)} \leq z \leq U(\beta). \tag{5.2}$$

Indeed, multiplying by $2z > 0$ gives $z^2 - 2(\beta + 1)z + 1 \leq 0$. Its two roots are reciprocal, with upper root $U(\beta)$. So u_0 is the largest multiplier Alice's

budget can carry, and u_1 the largest Bob's can. An endpoint is *covered* when each budget covers its owner's charge. In the theorem below, x_* is the equal point r when it is covered and otherwise the covered endpoint nearest r, which lies at a budget limit.

Theorem 5.5 (Capacity-constrained conserving repair). *A positive endpoint satisfying*

$$J(x) \le \beta_0, \qquad J(r^2/x) \le \beta_1$$

exists exactly when $r^2 \le u_0 u_1$. When it exists, its unique least-total-cost endpoint is

$$x_* = \max\{r^2/u_1, \min\{r, u_0\}\}, \qquad (x_*, r^2/x_*).$$

Proof. Any covered endpoint has $x \le u_0$ and $r^2/x \le u_1$, so $r^2 \le u_0 u_1$.

Conversely suppose $r^2 \le u_0 u_1$. Since $r, u_0, u_1 \ge 1$, the displayed x_* satisfies

$$1 \le x_* \le u_0, \qquad 1 \le r^2/x_* \le u_1.$$

For example, $x_* \le u_0$ follows because both r^2/u_1 and $\min\{r, u_0\}$ are at most u_0. The second upper bound follows from $x_* \ge r^2/u_1$; its lower bound follows from $x_* \le r^2$. Equation (5.2) therefore covers both charges.

To prove uniqueness, write $T(x) = J(x) + J(r^2/x)$. Direct simplification gives, for positive x, y,

$$T(x) - T(y) = \frac{(r^2+1)(x-y)(xy-r^2)}{2r^2xy}.$$

Its signs show that T strictly decreases up to r and strictly increases after r.

If $u_0, u_1 \ge r$, the geometric mean r is covered and uniquely minimizes T. If $u_0 < r$, feasibility gives $r^2/u_1 \le u_0$, and every covered x is at most u_0; the unique minimum is u_0. In the remaining constrained case $u_1 < r$, every covered x is at least $r^2/u_1 > r$, so that lower endpoint uniquely minimizes T. These cases give x_*. □

Take $r = 2$ again, with $u_0 = 3/2$ and $u_1 = 3$. These are $U(1/12)$ and $U(2/3)$, so Alice's budget is $1/12$ and Bob's is $2/3$. Complete equalization would charge Alice $J(2) = 1/4$, more than her budget. Feasibility holds, since $r^2 = 4 \le u_0 u_1 = 9/2$, and $x_* = \max\{4/3, \min\{2, 3/2\}\} = 3/2$. The selected endpoint is

$$(3/2, 8/3),$$

with individual charges 1/12 and 25/48 (Figure 5.2).

Under the charged law each budget also renews its owner's reserve, which needs positive budgets, $\beta_0, \beta_1 > 0$. With nonnegative opening reserves and no fresh exposure, this endpoint preserves both continuations whenever any conserving endpoint does: by Theorem 5.4, an endpoint preserves both exactly when both opening reserves are positive and both charges are covered.

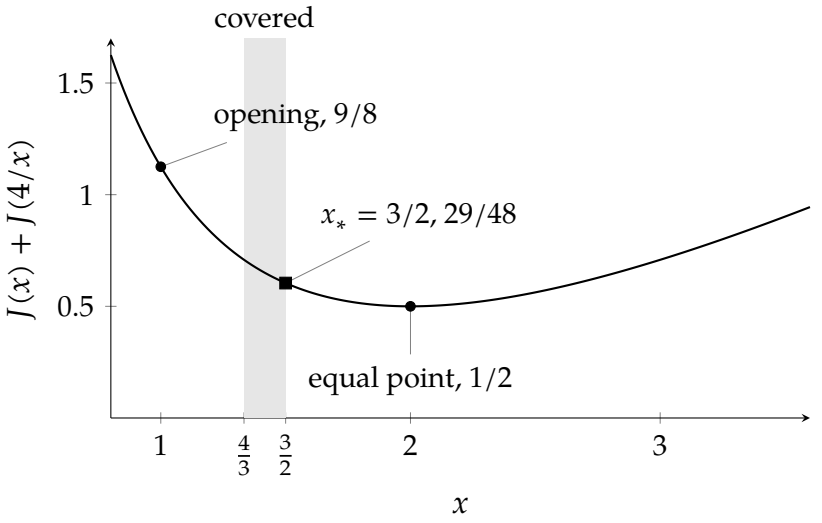


Figure 5.2. Total cost along the conserving endpoints $(x, 4/x)$ of the pair $(1, 4)$. Without budgets, the least cost is at the equal point $x = 2$. Alice's budget 1/12 allows $x \le 3/2$; Bob's budget 2/3 requires $4/x \le 3$, so $x \ge 4/3$. The shaded band holds every covered endpoint, and the best of them is the edge nearest the equal point, $x_* = 3/2$.

The selected endpoint does not depend on how the two burdens are weighed, so long as both are weighed alike, more burden always counts as worse, and each further unit of burden counts at least as much as the one before. Precisely, let f be any function on the nonpositive numbers, the same for both burdens, that is strictly increasing and concave. The same x_* uniquely maximizes

$$f(-J(x)) + f(-J(r^2/x))$$

over the covered endpoints.

To see why, put $t = \log x$ and $m = \log r$. The function $h(t) = f(-J(e^t))$ is strictly concave: the inner function is strictly concave, and increasing

concavity of f preserves that strictness. The pair sum $h(t) + h(2m - t)$ is strictly concave and symmetric about m. It strictly improves toward that center from either side. The separate budget bounds therefore select the same covered endpoint nearest the peak.

5.5 Love among many people retains each person's capacity

The same question extends beyond two people. Suppose each of finitely many participants owns one internal bond. Conserving their total logarithmic flow leaves many possible endpoints. Their capacities need not be equal, and a common endpoint can exceed what one of them can maintain. The repair must therefore retain each person's own bound.

Let participant i have positive opening reserve R_i and positive renewable budget B_i. Adopt the constant charged law with no fresh exposure, as in Theorem 5.4. Write t_i for the logarithm of the endpoint multiplier, and let S be the conserved sum of those logarithms. Since $J(e^{t_i}) = \cosh t_i - 1$, the personal maintenance condition is

$$J(e^{t_i}) \le B_i \quad \Longleftrightarrow \quad -c_i \le t_i \le c_i, \qquad c_i = \operatorname{arcosh}(1 + B_i).$$

The budgets remain separate; no participant spends another's renewable capacity.

Theorem 5.6 (The conserving minimum with separate personal capacities). *A conserving endpoint covered by every personal budget exists exactly when*

$$-\sum_i c_i \le S \le \sum_i c_i.$$

When it exists, choose a common level a such that

$$t_i^* = \max\{-c_i, \min\{a, c_i\}\}, \qquad \sum_i t_i^* = S.$$

This endpoint uniquely minimizes $\sum_i J(e^{t_i})$ among all covered endpoints with sum S. Every participant continues under the stated charged law.

Proof. Adding the individual bounds gives the necessary condition. Conversely, the sum of the clipped coordinates is a continuous function of a. At $a = -\sum_i c_i$ it equals $-\sum_i c_i$; at $a = \sum_i c_i$ it equals $\sum_i c_i$. The intermediate value theorem supplies the required level.

For minimality, set $f(t) = J(e^t) = \cosh t - 1$. Its derivative is $\sinh t$, and strict convexity gives the tangent inequality

$$f(y_i) - f(t_i^*) \geq \sinh(t_i^*)(y_i - t_i^*),$$

strict whenever $y_i \neq t_i^*$. For every allowed y_i, clipping also gives

$$\sinh(t_i^*)(y_i - t_i^*) \geq \sinh(a)(y_i - t_i^*).$$

In the interior the slopes agree. At the lower bound, the allowed displacement is nonnegative and $a \leq -c_i$; at the upper bound it is nonpositive and $a \geq c_i$. Monotonicity of sinh gives the inequality in each case. Summing, the common-slope term vanishes because both endpoints have sum S. Total cost therefore cannot fall, and it rises strictly if any coordinate differs. Finally, each selected charge is covered and every opening reserve is positive, so Theorem 5.4 gives each separate continuation. □

The equalizing level is common wherever personal capacity permits it. A participant at a bound keeps that bound while the others adjust. The whole minimum thus retains the members whose lives it concerns. On the same ledger, changing each initial log multiplier to its selected value is an action with zero total log strain. The endpoint result does not supply the available controls, payment for the actuator, consent or authority to carry it out. Those belong to the action and its history.

5.6 The path and the payer

An endpoint does not record how it was reached or who paid for it, and each needs its own account.

Let a finite action history pass through ledgers $L_0, \dots, L_N$. For participant i, the sum of the successive signed harms is

$$\sum_{k=0}^{N-1} (C_i(L_{k+1}) - C_i(L_k)) = C_i(L_N) - C_i(L_0). \tag{5.3}$$

Every intermediate cost appears once positively and once negatively, so those terms cancel, and the sum keeps no trace of the path. A nonzero strain v on a balanced bond, followed by its inverse, passes through the multiplier $e^v \neq 1$ at positive cost and ends at zero cost again: with $v = \log 2$, the participant carries $J(2) = 1/4$ in between and ends with

net change zero. So the endpoint cannot establish that every stage met the participant's requirements.

Funding supplies another effect. Let S be a finite set of selected active bonds and restore each to unity by its one restoring strain (Proposition 3.4), leaving every other bond unchanged. Define the restoration cost and each participant's relief by

$$Q = \sum_{b \in S} J(x_b), \qquad \rho_i = \sum_{b \in S \cap I_i} J(x_b).$$

The restoration cost counts each selected bond once. If two participants share a bond, its relief appears in both incident accounts while its cost appears once in Q: restoring one bond between Alice and Bob from multiplier 2 relieves each of them by $1/4$, and $Q = 1/4$.

Proposition 5.7 (Restoration and funding retain different effects). *Restoring S lowers shared cost by Q and participant i's incident cost by ρ_i. Under the payment convention that debits Q once from one designated payer's reserve, all initially positive reserves remain positive exactly when the payer's opening reserve is greater than Q.*

Proof. Each selected bond changes from cost $J(x_b)$ to $J(1) = 0$, while every unselected bond retains its cost. Summing over all bonds gives the shared decrease Q; summing over participant i's incident bonds gives ρ_i.

Write the opening reserves as $M_i > 0$, and let p be the payer. The convention specifies $M'_p = M_p - Q$ and $M'_i = M_i$ for every $i \neq p$. Those other reserves remain positive. The payer's remains positive exactly when $M_p > Q$. □

The convention names who is debited; cost descent alone does not choose the payer, and the repairing actor and the payer can be different people. A payer incident to none of the repaired bonds, such as Carol paying for the bond between Alice and Bob, has no change in incident cost, yet her reserve falls by Q, here $1/4$. A description that tracks only incident harm would miss the payment.

For two people, these results calculate a repair's endpoint, the strains of an exact correction, each person's change in burden, whether each continues under the charged law, what was carried along the way and who paid. Chapter 6 follows the reserves through time and derives the exact condition under which care keeps both positive at every checkpoint; Chapter 7 ties the repair to the undertaking that makes it owed.

Chapter 6

The conditions of continued care

Alice is caring for Bob, with the numbers of Chapter 1. She opens with a reserve of 2 and he with 1/4. Getting through an interval costs each of them 1/4. Delivering a meal costs Alice a further 1/4 of work and gives Bob 1/2 of intake. Over two intervals Alice delivers one meal and leaves the other idle; Table 6.1 follows both orders.

	Opening	First checkpoint	Second checkpoint
Meal, then idle	$(2, 1/4)$	$(3/2, 1/2)$	$(5/4, 1/4)$
Idle, then meal	$(2, 1/4)$	$(7/4, 0)$	$(5/4, 1/4)$

Table 6.1. Alice's and Bob's reserves, in that order, under the two orderings of one meal and one idle interval. Both end at $(5/4, 1/4)$. Only the idle-first ordering takes Bob to zero at the first checkpoint.

Both orderings use the same food, work and maintenance and end at the same reserves. In the idle-first ordering, though, Bob pays the first interval's maintenance with exactly the 1/4 he has and reaches the first checkpoint with nothing left. The later meal restores his capacity. It cannot make the first checkpoint one at which he had any.

Chapter 5 asks what two separate budgets can fund at one moment. Care asks it across time: whether Bob stays funded depends on when Alice acts, and her capacity on what she has already spent. The answer is an exact existence condition, one inequality in the opening reserves for each person, and the proof builds the schedule.

6.1 Staying balanced is a repair paid from reserve

The quarter-unit cost of getting through an interval is the price of repairing a bond that exposure has doubled. Each person carries an internal

bond, from themselves to themselves (Definition 3.1), whose multiplier $m > 0$ records their response to exposure, balanced at $m = 1$. The results below use only this multiplier, whatever pair it compares (Section 3.2). In the environment we take here, each interval's exposure doubles the multiplier, to $2m$, so the bond costs $J(2m)$.

Each person also carries a *reserve* e, the capacity they can spend on repair and work, counted in units of recognition cost. Repair returns the exposed bond to multiplier one and debits its cost once from its owner's reserve: the care maintenance of Table 5.2, a restoration cost with the owner as payer, levied each interval because exposure doubles the multiplier again. A repaired bond costs nothing again, so what maintenance spends shows only in the reserve.

Let q be the interval's net funding for the person. When the available amount covers the repair, the response is repaired and its cost is paid:

$$(m, e) \longmapsto (1, e + q - J(2m)) \qquad \text{if } J(2m) \le e + q.$$

Otherwise the exposure remains and nothing is debited:

$$(m, e) \longmapsto (2m, e + q) \qquad \text{if } e + q < J(2m).$$

The end of each interval is a *checkpoint*. A checkpoint is *maintained* for a person when the repair is paid and some capacity remains: the response is balanced and the reserve strictly positive. Otherwise it is *missed*. The funding condition for maintenance is therefore strict:

$$J(2m) < e + q.$$

Equality pays the repair and leaves nothing, as in Bob's first idle-first interval.

For a balanced opening,

$$J(2) = \frac{1}{4},$$

so each maintained interval costs each person a quarter. Once a repair goes unpaid, the next repair's cost follows the response actually carried forward: from $m = 2$ the next exposure costs $J(4) = 9/8$. Every result below uses J only through this quarter, since each exposure in them starts from balance: any cost with the same value at 2 would give the same results, and the cost of Chapter 2 sets that value.

Why count zero as a miss? Any positive reserve can pay to repair some departure from balance, and zero can pay for none (Equation (5.2)). A small positive reserve may still be unable to pay the next repair alone: from $1/8$, as from zero, Bob needs a meal before the next repair at $1/4$. The checkpoint judges what remains now and leaves the next repair to future funding.

To *continue* is to keep a strictly positive reserve, now and at every later stage, so zero is a miss even when the present repair is paid. A *continuation* is a future course of action and funding under which the person continues; a person who has one can continue. With the funding free to be chosen, one exists exactly when the reserve is positive now (Chapter 7). Bob at $1/8$ can continue; at zero he cannot, though a later meal resumes his care without undoing the miss (Section 6.6).

A *food job* is one delivered meal: a quarter of Alice's work, half a unit of Bob's intake. In the repair map it funds Alice $q = -1/4$ and Bob $q = 1/2$; an idle interval funds neither. The map records $e + q$ whatever its sign, so a negative value records work taken on beyond what the reserve could pay: a job begun from below a quarter leaves Alice's reserve negative, at a missed checkpoint that no maintained history reaches. The food comes from stock, not from Alice's bodily reserve; food consumed, work done and bodily maintenance are separate entries in one account.

The doubling and these amounts define the care environment we analyze, and every result below is exact for them. For a particular organism and task they would have to be derived from its physical response, which would also say what a person's internal bond compares, with the reserve tracking a capacity the person can actually use; Chapter 17 states the correspondence a measured reserve must meet.

6.2 Each person must stay funded at every checkpoint

Let Alice's and Bob's opening reserves be real numbers A and B with

$$A \geq 0, \qquad B \geq 0.$$

Both begin balanced. A history of N intervals is a binary sequence

$$(j_1, \dots, j_N) \in \{0,1\}^N,$$

where $j_k = 1$ means that a food job occurs in interval k. Put

$$n(0) = 0, \qquad n(k) = \sum_{r=1}^{k} j_r,$$

so $n(k)$ counts the jobs delivered by checkpoint k.

While both remain maintained, their reserves are

$$A_k = A - \frac{k + n(k)}{4}, \qquad B_k = B - \frac{k}{4} + \frac{n(k)}{2}. \tag{6.1}$$

Alice pays for k intervals of maintenance and $n(k)$ jobs; Bob pays the same maintenance and receives the jobs' intake.

A history is maintained when each of its checkpoints is maintained for both people. Here those checkpoints begin at one; the opening reserve may be zero, provided the first transition funds the first checkpoint. A continuation whose definition includes the opening checkpoint instead requires positive opening reserve. The formulas hold only while the stated maintenance lasts, so the proof is an induction.

Theorem 6.1 (Chronological maintenance). *A binary history is maintained at every checkpoint* $1, \dots, N$ *if and only if*

$$A - \frac{k + n(k)}{4} > 0, \qquad B - \frac{k}{4} + \frac{n(k)}{2} > 0 \tag{6.2}$$

for every $1 \le k \le N$. *When these inequalities hold, both responses are balanced and their reserves are given by* (6.1).

Proof. At the opening, $n(0) = 0$, so (6.1) gives $A_0 = A$ and $B_0 = B$.

Suppose all earlier checkpoints have been maintained. Both people enter interval k balanced. Alice's funding from the food job is $-j_k/4$; Bob's is $j_k/2$. After paying the quarter-unit maintenance cost, their next reserves are therefore

$$A_{k-1} - \frac{1 + j_k}{4}, \qquad B_{k-1} - \frac{1}{4} + \frac{j_k}{2},$$

provided these amounts are positive.

Substituting the reserve formulas at $k-1$, and using $n(k) = n(k-1) + j_k$, gives precisely (6.2) at k. Positive amounts pay the repairs and leave both responses balanced. Conversely, maintenance at checkpoint k requires those same amounts to be positive.

Induction proves both directions and the reserve formulas. □

The condition holds person by person and checkpoint by checkpoint: at the first idle-first checkpoint Alice holds $7/4$, Bob holds 0, and her surplus does not save the history.

One weighted total ignores the schedule:

$$2A_k + B_k = 2A + B - \frac{3k}{4}. \tag{6.3}$$

Substituting (6.1) shows why: under the weights 2 and 1, Alice's quarter-unit of work and Bob's half-unit of intake cancel, and maintenance lowers the total by $3/4$ per interval.

Positive reserves at checkpoint N therefore require

$$3N < 8A + 4B.$$

This bounds the horizon without deciding it, because a schedule must still satisfy both inequalities at every checkpoint.

6.3 Feeding only when Bob needs it decides every horizon

Cap Alice's food jobs at a nonnegative integer G. In the theorem G is a bare cap; in the cases below it is the number of meals Alice is authorized to deliver, and work beyond it is unauthorized however much food is in stock (Chapter 7). Bob's inequality fixes how many meals he needs by each checkpoint, and each meal costs Alice a quarter, so her best chance is to give him exactly the least he needs. If that exhausts her, every schedule does.

Bob's inequality can be written

$$n(k) > \frac{k - 4B}{2}.$$

Because jobs are counted by integers, the least count satisfying it is

$$\ell_B(0) = 0, \qquad \ell_B(k) = \max\left\{0, \left\lfloor \frac{k - 4B}{2} \right\rfloor + 1\right\} \quad (k \geq 1). \tag{6.4}$$

The floor plus one is needed because the inequality is strict. With $B = 1/4$ and $k = 3$ the right side is 1; one meal by then would leave Bob at exactly zero, so the least count is 2.

Lemma 6.2 (The least count can be scheduled). *For $B \geq 0$, the sequence $\ell_B(k)$ is nondecreasing and increases by at most one per interval. Hence*

$$j_k^* = \ell_B(k) - \ell_B(k-1)$$

belongs to $\{0, 1\}$.

Proof. For $k \geq 1$, the argument of the floor in (6.4) increases by $1/2$ from k to $k+1$. Its floor therefore stays fixed or increases by one. Adding one and replacing negative values by zero preserves these two possibilities.

The first interval needs a separate check because $\ell_B(0)$ was defined directly. Since $B \geq 0$,

$$\frac{1-4B}{2} \leq \frac{1}{2}.$$

Its floor is at most zero, so $0 \leq \ell_B(1) \leq 1$. This proves the claim at every interval. □

Theorem 6.3 (Finite-horizon care). *Let $A, B \geq 0$, let $N \geq 1$, and let $G \geq 0$ be an integer. There is a binary history of length N, containing at most G food jobs, that maintains both participants if and only if*

$$\ell_B(N) \leq G, \qquad N + \ell_B(N) < 4A. \tag{6.5}$$

When these conditions hold, the history

$$j_k^* = \ell_B(k) - \ell_B(k-1) \qquad (1 \leq k \leq N)$$

is maintained.

Proof. Suppose a maintained history exists. At every checkpoint, Bob's strict inequality requires $n(k) \geq \ell_B(k)$. At the final checkpoint,

$$\ell_B(N) \leq n(N) \leq G.$$

Alice's positive reserve gives

$$N + \ell_B(N) \leq N + n(N) < 4A.$$

Thus both conditions in (6.5) are necessary.

Conversely, suppose those conditions hold. Lemma 6.2 makes j^* a binary history. Its cumulative job count telescopes:

$$n^*(k) = \sum_{r=1}^{k} (\ell_B(r) - \ell_B(r-1)) = \ell_B(k).$$

By definition, this count satisfies Bob's strict reserve inequality at every positive checkpoint.

For Alice, monotonicity gives

$$k + \ell_B(k) \le N + \ell_B(N) < 4A \qquad (1 \le k \le N).$$

Alice therefore also retains positive reserve at every checkpoint. Theorem 6.1 establishes maintenance throughout. The total number of jobs is $\ell_B(N) \le G$. □

The criterion has one condition per person: the authorized meals cover the least Bob needs, and Alice can pay for N intervals of her own maintenance plus those meals. The schedule j^* delivers a meal exactly when Bob's least required count rises.

Every maintained history has $n(k) \ge \ell_B(k)$, and Alice's reserve falls with $n(k)$ while Bob's rises. Among maintained histories, j^* therefore gives Alice the greatest reserve and Bob the least at every checkpoint. It marks the boundary of feasible care, which does not make it the preferred allocation; what Alice owes belongs to the undertaking she and Bob made (Chapter 7).

If a history maintains both people for N intervals within the cap, so do its first $N-1$: they meet the same checkpoints with no more jobs. The feasible horizons are therefore $0, 1, \dots, N_{\max}$, the empty history counting as zero. Equation (6.3) makes the range finite, and its positive horizons are exactly those passing (6.5). A job cap can end the range by limiting Bob's intake, and Alice's reserve can end it even when more jobs are authorized.

6.4 Within two jobs, two schedules reach the fourth checkpoint; none reaches the fifth

Return to $A = 2$ and $B = 1/4$ with two jobs authorized. The stock holds three food units, one more than the jobs, so food never runs short; the authorization and the reserves bind.

The chronological inequalities become

$$k + n(k) < 8, \qquad k < 1 + 2n(k). \tag{6.6}$$

Bob's least counts through the first five checkpoints are

$$(\ell_B(1), \dots, \ell_B(5)) = (1, 1, 2, 2, 3).$$

At $N = 4$ with $G = 2$ the criterion holds, since $\ell_B(4) = 2 \le 2$ and $4+2 < 8$, and the least schedule is $j^* = (1, 0, 1, 0)$.

Proposition 6.4 (The two maintained four-interval histories). *With at most two jobs, a four-interval history maintains both participants exactly when it is*

$$(1, 1, 0, 0) \qquad \textit{or} \qquad (1, 0, 1, 0).$$

Proof. Alice's inequality is automatic over four intervals with at most two jobs: $k + n(k) \le 4 + 2 < 8$. Bob needs one job by the first checkpoint and two by the third. Hence $j_1 = 1$ and $n(3) \ge 2$. The two-job cap then gives $n(3) = n(4) = 2$. Thus exactly one of intervals two and three contains the remaining job, and interval four is idle.

Conversely, each displayed history has Bob's required count at every checkpoint and satisfies Alice's bound. □

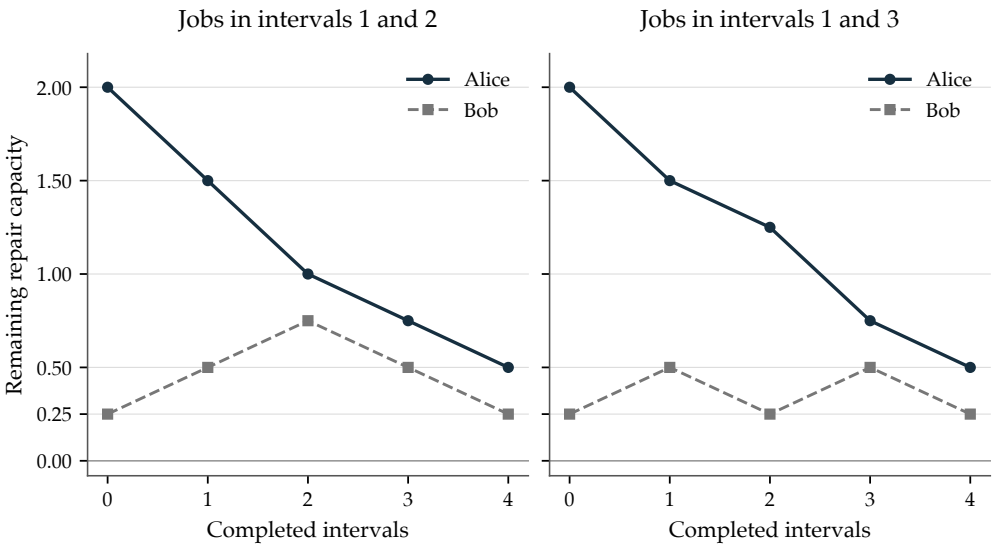


Figure 6.1. The two maintained four-interval histories from opening reserves $A = 2$ and $B = 1/4$, using at most two food jobs. Alice's reserve is the solid line and Bob's the dashed line. Both stay positive at every marked checkpoint and end at $(1/2, 1/4)$ by different paths. Lines only connect the discrete checkpoints.

Both histories feed Bob first, because his opening quarter cannot carry him to a positive first checkpoint; they differ only in where the second meal falls (Figure 6.1).

No schedule reaches a fifth checkpoint, even with the job cap removed. At $k = 5$, Bob requires $5 < 1+2n(5)$, so $n(5) \geq 3$. Alice requires $5+n(5) < 8$, so $n(5) \leq 2$. No integer count meets both requirements, although the weighted bound allows a fifth interval: $3 \cdot 5 = 15 < 17 = 8A + 4B$. With these opening reserves, Alice cannot fund the work that Bob's continued maintenance requires.

6.5 When the schedule can actually be performed

A binary list says when meals occur; performing it also takes food, labor and authority. It suffices that every binary history within the horizon and the cap G is available: its food units accessible, Alice able to work at the chosen times, and the work and the consumption authorized. Then the least-job history is maintained and can be performed. If access, timing or authority narrows the menu, the inequalities remain necessary, but their sufficiency must be checked against it. Work Alice could do later does not feed Bob at the checkpoint where he needs it, and a calculation of what Alice can afford gives no one authority over her work.

6.6 A missed checkpoint stays missed

Care can resume after Bob's miss in the idle-first ordering. The tail $(1, 1, 0)$ can be performed after the omission, within the two-job authorization: Alice delivers two meals and then waits, and the reserves run from $(7/4, 0)$ through $(5/4, 1/4)$ and $(3/4, 1/2)$ to $(1/2, 1/4)$, meeting all three remaining checkpoints. The final state, including stock, food consumed and work used, agrees with that of the maintained histories of Proposition 6.4.

The first checkpoint remains missed, and neither a later meal nor a later release changes that. Its verdict rests on two things: the state at the checkpoint, which the actions up to it fixed, and the terms in force there. A later release by Bob changes the terms from then on and leaves those in force over the executed past as they were (Chapter 8).

Repair and retained breach are therefore compatible. After a missed checkpoint, no future tail makes the whole history compliant from its beginning. So we cannot find what Alice owes by looking at the compliant futures: there are none, and a requirement quantified over an empty set would hold whatever it said, even a contradiction. Remaining care is evaluated instead over the futures that can still be performed from the

actual state, against the checkpoints still ahead; here that set contains the tail $(1, 1, 0)$.

Continued care is now decidable. From two opening reserves, a horizon and a meal cap, Theorem 6.3 says whether both people can be kept funded, and j^* shows how. Chapter 7 derives what the undertaking requires of Alice's first action, and Chapter 8 who can release her from it.

Chapter 7

An undertaking between two people

Return to the first interval of Alice's care for Bob. Alice has offered to feed him, Bob has accepted, and their agreement is active. When the interval arrives she can bring the meal or stay where she is. Both actions are physically possible. What makes the meal owed, in a world where staying away can still happen?

Food, work and maintenance fix the two reserves. Bob's reliance on the promise fixes a reference: where he would have stood had he never relied on Alice. Before Alice acts, the accepted terms are written into a record whose verdict selects her first action and grounds exactly one reading of what she must do, the reading that keeps both people going. Under it the meal performs the undertaking and staying away, still executable, violates it. The reserves, the reference and the record are exact. Three identifications, adopted as premises, tie them to the people: that the record's selection is what Alice owes, with Bob's reliance as the reason (Section 7.7), and that its detector reads Bob's act of accepting and its register is Alice's means of action (Section 7.8).

7.1 Food, maintenance and remaining capacity

The care law of Chapter 6 applies, from the opening of Table 6.1: both internal multipliers balanced, Alice's reserve 2 and Bob's 1/4. Each interval's exposure pushes each balanced multiplier to 2, and the repair back to 1, each person's maintenance (Table 5.2), costs its owner

$$J(2) = \frac{1}{4}.$$

A food job consumes stock, costs Alice 1/4 of reserve in work and gives Bob intake worth 1/2.

Alice has one unit of labor in this interval and each food job uses one, so her first action is either staying or a single meal. These two classes are the *physically legal* first actions, omission of the care Bob accepted included, and every comparison below is proved over them. An action is physically legal when it passes the operation check (the technology admits the action now and its material balances), the resource checks (food in stock; each person's labor and reserve can absorb the work asked) and the history checks, which ask only whether the action agrees with retained records: work covered by an accepted work grant for that worker, window and total; only food units the record holds; each actual delivery credited once, toward an active undertaking with quantity still due. By design, then, authorization enters legality only as a matter of record. Staying passes every check; two meals in one interval do not.

Feeding leaves the (multiplier, reserve) pairs

$$\left(1, \frac{3}{2}\right) \quad \text{and} \quad \left(1, \frac{1}{2}\right)$$

for Alice and Bob. Omission leaves

$$\left(1, \frac{7}{4}\right) \quad \text{and} \quad (1, 0).$$

Each person can afford the maintenance repair in both histories, so every multiplier returns to 1 and the bond costs tie at zero, the least possible value. Both actions leave Alice positive reserve. What separates them is Bob's: positive after feeding, exhausted after omission.

7.2 Bob's reliance fixes where he would have stood

Bob's reserve of $1/4$ has a history. In the interval before the opening, Alice has reserve $9/4$, Bob has $3/4$, and both multipliers are balanced. Bob has one interval of labor and two uses for it. Feeding himself consumes an accessible food unit, costs a quarter-unit of work and gives him half a unit of intake. Working at a service costs the same and gives him no intake: its product is a unit of service, recorded apart from food, and whom it serves plays no part in what follows. Each person also pays for the quarter-unit maintenance repair.

Bob works when Alice's proposal and his matching acceptance have put the terms in place, and feeds himself otherwise: he works because

he relies on Alice to feed him. After the accepted work the reserves are $9/4 - 1/4 = 2$ and $3/4 - 1/4 - 1/4 = 1/4$, the opening already calculated; only these reserves, the responses and the stock carry over into it.

We call the proposal and acceptance that put the terms in place the undertaking's *formation*. Now prevent the agreement from forming: either Alice's offer or Bob's acceptance is prevented before Bob acts, with his rule for choosing unchanged. In both cases Bob feeds himself, leaving reserves 2 and $3/4$ after maintenance. In their next interval Alice waits, and another quarter-unit of maintenance leaves $7/4$ and $1/2$, with both multipliers 1. Both formation interventions recover the same response for Bob: $(1, 1/2)$.

They agree because Bob's rule looks only at whether the terms are in place. The *intervention criterion* takes as Bob's reference a response that every formation intervention recovers; their agreement makes that reference exist and frees it from any choice of whose act was removed. Feeding gives Bob $(1, 1/2)$; omission gives $(1, 0)$. Call an action a *closure of the authored reliance*, or say it *closes* the reliance, when Bob's response after it equals the reference. In plain words, it puts Bob back in the position the agreement led him to give up, on which Section 7.7 rests the obligation. Closure asks for equality because the reference records what reliance displaced. The example is built so that the meal returns exactly the half-unit of intake Bob gave up by working, so on this domain "at least" would select the same action.

7.3 Both reserve conditions come down to a positive reserve

Two of the five conditions that pick out the meal in Proposition 7.2 concern what a reserve still allows: which future repairs it can pay for, and whether its owner can go on. Both come down to whether the reserve is positive.

A reserve e can pay a finite sequence of positive multipliers $x_1, \dots, x_n$ exactly when the total cost $J(x_1) + \cdots + J(x_n)$ does not exceed e: paying the first repair leaves $e - J(x_1)$, and induction over the rest gives the total-cost condition and the final reserve.

The payable sequences determine the reserve, because every non-negative amount is the cost of a single repair: for $e \geq 0$, the multiplier $U(e) = 1 + e + \sqrt{e(e+2)}$ at the upper end of the interval (5.2) of Chapter 5

costs exactly $J(U(e)) = e$. Bob's opening reserve, for instance, is exactly one maintenance repair: $U(1/4) = 1 + \frac{1}{4} + \sqrt{\frac{1}{4} \cdot \frac{9}{4}} = 2$, as it must be, since $J(2) = 1/4$. If every sequence payable from e is payable from e', the single request $U(e)$ gives $e \leq e'$; conversely, $e \leq e'$ makes every total affordable from e affordable from e'. Two reserves that can pay the same repairs are therefore equal: a reserve is the largest total cost it can pay.

Two people share a *positive repair margin* when some positive reserve exists whose payable sequences each of them can pay; by the comparison just made, that holds exactly when both reserves are positive. Feeding gives Alice and Bob such a margin, since any finite sequence payable from $1/2$ is payable from either remaining reserve. Omission leaves Bob none.

The two actions still do not rank simply. A repair at multiplier 5 costs $J(5) = 8/5$, between Alice's $3/2$ after feeding and $7/4$ after omission, so only omission lets her fund it. A repair at multiplier 2 costs $1/4$, within Bob's $1/2$ after feeding and beyond his zero after omission. Preserving both people is a specific physical condition; feeding does not enlarge every possible opportunity.

Whether a person can go on is a question about futures. A *continuing extension* of a reserve is a future policy and renewable budget, starting from that reserve, under which the reserve, debited each interval by the charge of Table 5.2, stays positive at every *stage*: the opening itself, stage zero, and each interval after it.

Proposition 7.1 (The continuation boundary). *A reserve admits a continuing extension if and only if it is strictly positive.*

Proof. Any continuing extension must be viable at its initial stage, where its reserve is the opening reserve. That reserve must therefore be positive.

Conversely, from any positive opening reserve, choose a balanced future trajectory and a positive renewable budget. The balanced trajectory has zero charge. The reserve is the positive opening amount plus accumulated nonnegative funding, and it stays positive at every stage. □

The test is existential: it asks whether some policy and renewable budget would keep the reserve positive, so it forecasts nothing about whether such funding will come. Its content is the strictness of continuing (Chapter 6): from zero no future helps, because the opening is itself a stage, while from any positive reserve some future works, even from

$1/8$, which cannot pay $J(2) = 1/4$ alone. This is the strict positivity in the checkpoint condition of Chapter 6; any positive reserve pays for some departure from balance, since $U(e) > 1$ when $e > 0$.

7.4 Closure, continuation and the accepted terms select the same action

The terms Bob accepted name him as recipient, with a care window covering checkpoints 1 to 4 (start 1, finish 5). At an active checkpoint they require his response to be maintained: balanced, with positive reserve. An action *performs* requirements when it is physically legal and meets them all; the meal performs the accepted care content at the first checkpoint, and staying does not.

Proposition 7.2 (A unique reference and a joint continuation condition). *The formation interventions determine exactly one reference response. On every physically legal first action of the fixed care opening, the following conditions are equivalent:*

1. *the action closes the authored reliance;*
2. *Bob admits a continuing extension;*
3. *every represented participant admits such an extension;*
4. *Alice and Bob retain a common positive repair margin;*
5. *the action performs the active accepted care content.*

Proof. Both formation interventions yield the same response. Any reference satisfying the intervention criterion must equal the response obtained by preventing Alice's offer, which proves uniqueness.

The legal first actions fall into the two response classes already calculated: no food job, or one food job. In both, Alice keeps positive reserve and both responses are balanced. Bob has zero reserve in the first class and $1/2$ in the second. The continuation boundary therefore makes Bob's continuation and joint continuation equivalent.

The payable-sequence characterization gives the same split for a common positive repair margin. Finally, the accepted terms require Bob's maintained first checkpoint. Evaluating those terms, and testing equality with the recovered reference, both select the food-job class. All five conditions therefore agree on the complete legal first-action domain. □

With two response classes the conditions are checked class by class, so their agreement is no convergence of independent principles: each condition, computed from the physics, comes down on this domain to whether Bob keeps positive reserve, and puts the meal on one side and omission on the other. A condition can fail that test, and one does. A reading built on Alice's continuation alone admits omission, because $7/4$ is positive, while remaining logically consistent and closed under consequences. A record that governs Alice's action must retain Bob and the relation the two of them actually authored.

7.5 One more interval turns Bob's zero into bond cost

Run the same exposure and repair law for one further interval, with no new food or other provision. Feeding had left Alice and Bob at

$$(1, 3/2), \qquad (1, 1/2).$$

Both can pay the next quarter-unit repair. Their new responses are

$$(1, 5/4), \qquad (1, 1/4).$$

Omission had left them at $(1, 7/4)$ and $(1, 0)$. Alice can pay; Bob cannot. The resulting responses are

$$(1, 3/2), \qquad (2, 0).$$

Bob's unrepaired multiplier now contributes $J(2) = 1/4$ to the actual bond ledger, and the feeding continuation contributes zero. No comparison cost has been added: these are the bonds after the next actual interval, with each repair debited from its owner's reserve.

Apply the identical interval to either formation-prevention history. The recovered responses become

$$(1, 3/2), \qquad (1, 1/4).$$

Both recovered bond multipliers are 1: had the undertaking never formed, both bonds would end this interval balanced, and the two interventions supply the same reference on each active bond. Comparing a ledger with this reference divides each multiplier by the recovered one on the same

bond, which measures how far the ledger stands from that never-formed position. Because the reference is balanced, the comparison changes nothing here: the compared ledger is the actual continuing ledger.

Proposition 7.3 (Closure on the continuing ledger). *On the physically legal first-action domain of this opening, closure of the authored reliance is equivalent to zero cost on the next actual ledger. The cost is exactly zero for the food-job class and one quarter for the omission class. The comparison against either recovered formation reference has this same cost.*

Proof. The work bound gives the two legal response classes: no food job, or one food job. The displayed debits compute the next responses in each class. In the food-job class both multipliers are one, so both bond costs vanish. In the omission class the multipliers are one and two, giving total cost zero plus one quarter.

The formation interventions recover unit multipliers on the same two bonds, so the compared ledger and the actual continuing ledger have identical multipliers, bond by bond, and hence identical cost. Matching the recovered multiplier on every active bond is precisely balance of this ledger. By Proposition 7.2, on legal actions this occurs exactly when the action closes the authored reliance. The comparison concerns the bond multipliers; it does not assert equality of the remaining reserves. □

What the first endpoint held only in Bob's reserve is now a quarter-unit of bond cost, computed from actual exposure, repair and payment (Figure 7.1).

7.6 The installed record governs, and omission stays executable

What the physical write contributes is this: the record is written before Alice acts and selects her action through the register taken to stand for her own means of action, so the governing reading is fixed in advance and is not chosen after we see what she did. This rests on one physical construction: the selection the accepted terms require is written into the register, the one from which Alice's first action is read, and the authored formation, the retained terms and the check of omission against them are all in place before she acts. Without a physical record that selects her action, "the agreement governs her action" would be our description

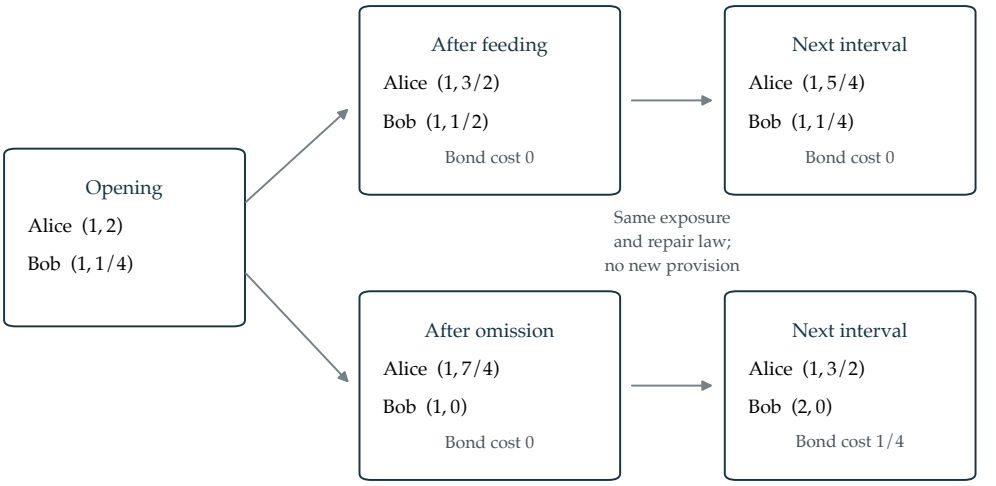


Figure 7.1. Care and omission through the next physical interval. In each box the first pair is Alice's and the second Bob's: multiplier, then remaining reserve. Both first endpoints have zero bond cost. The same further exposure, with no new provision, leaves Bob able to repair only if he was fed. The displayed cost is the sum over the two actual bonds.

of the case, given after the fact. With it, and granted the two premises Section 7.8 names, the agreement is implemented by the means of action itself, and which reading governs is a property of the physical system. The write does not make that reading owed: that is the identification Section 7.7 adopts, with Bob's reliance as its reason.

Figure 7.2 shows the route. The record retains the accepted terms: recipient Bob, start 1, finish 5. A *comparator* evaluates a candidate action against those terms and fails omission, which leaves Bob unmaintained at the first checkpoint. The register's selected-action coordinate holds mode zero, idle, until a *writer* acts: Bob's detected acceptance and the comparator's failure of omission together write action mode one, the food job, into it. *Alice's policy*, the rule that turns a register state into her action at each clock value, reads her first action from that coordinate. Comparator, writer and policy make up the controller and share one carrier, the register system built in Section 7.8.1. A controller configuration is *installed* when the completed accepted write has reached it: here, the register holding action mode one.

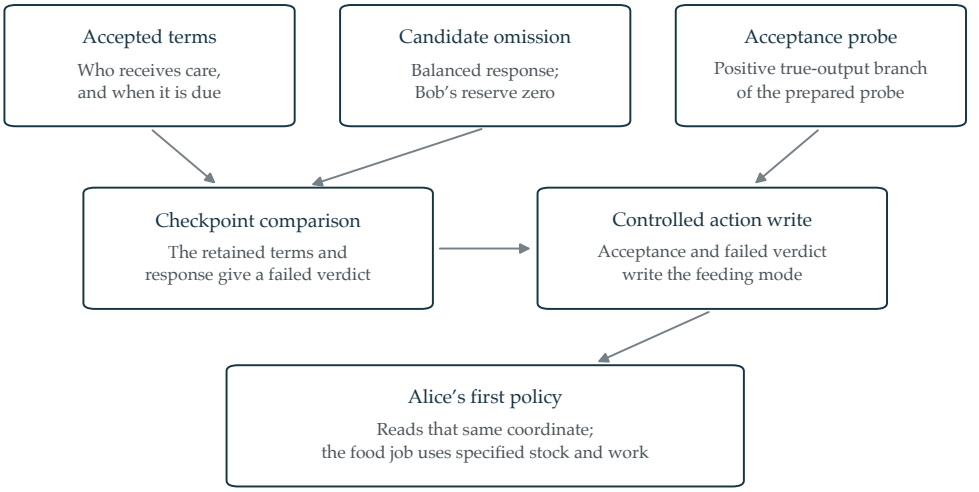


Figure 7.2. The physical route to Alice's first selected action. Bob's detected acceptance and a failed omission check together write 'feed' into the register Alice's first action reads. It assumes prepared inputs, retained records and the physical food job, with its stock and work.

The installed register settles which action the installed controller selects; it does not settle what Alice does. She can still run another controller, one that brings her means of action to a state selecting idle: the register before the write is such a state, and applying the exchange again restores it. Omission is running such a controller. The model represents what the agreement requires of Alice's means of action; it assumes that she can run another controller and does not model what in her would do so. The governing reading does not depend on which controller runs: Chapter 13 fixes the same reading from the authored history and the ledger alone, and that reading rejects the omission a rival controller chooses (Section 13.1). That route lets any controller be tested against it; the installed register fixes it before Alice acts. A *violation* is a legal execution the governing reading rejects, and the governing result needs only that one stays available. Responsibility for the omission attaches to the one who runs that controller, Alice, whose interventions these are (Chapter 8).

The write also needs Bob's acceptance detected, and at the contact used, with probe and target at the same position, the probe detects it on outputs of total weight one half (Section 7.8.3). On the undetected branch the writer stays disabled and the register keeps mode zero. Nothing is installed there, so nothing in the construction governs Alice's action, and

the identification of Section 7.7, which applies to installed agreements, says nothing there; the retained comparator still records that idle fails the accepted terms. Who answers for those terms on that branch is open. The governing result concerns the detected branch and uses the weights only to show that it exists. The half is what one step of this probe gives at that contact. A detector that read Bob's acceptance with certainty, a further apparatus not built here, would empty the undetected branch and leave the argument unchanged.

Let $\mathcal{L}(a)$ mean that the first action a is physically legal; feeding and idle both satisfy it.

Definition 7.4 (Readings). A *reading* is a collection of properties required of an action, its *ends*. An action *performs* a reading when it is physically legal and has every property in the reading; it is then a *performance* of the reading. For a condition P on actions, the *consequence reading* of P contains a property G precisely when

$$\forall a, \qquad \mathcal{L}(a) \wedge P(a) \implies G(a).$$

We say a comparator *implements* the consequence reading of its pass condition, and a reading is *grounded* by the installation when an installed controller implements it. A reading is *coherent* when it contains the property that always holds, does not contain the property that never holds, contains the conjunction of any two properties it contains, and contains G' whenever it contains G and every legal action with G has G'. A reading is *complete* when it contains every property shared by all of its performances.

A reading, then, is a set of requirements on Alice's action, and a coherent one is a consistent body of requirements: closed under conjunction and consequence, containing truth and excluding falsehood. Ordered by implication over legal actions, it is a proper filter of properties. The consequence reading of P is the principal filter generated by P; it collects everything true of every legal action that fulfils P. For joint continuation those are the single food jobs, so the reading contains, for instance, Bob's positive reserve and the closure of his reliance. Its performances are exactly the legal actions satisfying P: such an action has every property the reading contains, and P itself belongs to the reading. The installed comparator passes on Bob's maintained first checkpoint, so it grounds

the consequence reading of performing the accepted content. Completeness is used in Chapter 13, which finds the same reading without the installed controller.

The governing criterion asks three things:

1. at least one grounded coherent reading has a legal action satisfying all its requirements;

2. every grounded coherent reading requires performance of the accepted care content;

3. at least one physically legal action fails that content.

Together these clauses identify a requirement that can actually be fulfilled, that no grounded reading drops, and that can be violated.

Once grounding is defined, uniqueness is the easy part: the accepted write has exactly one outcome, so every configuration it can reach is the same scheduled configuration, with one implemented reading. The weight of the result lies in what fixes that reading, and when.

Proposition 7.5 (The fixed installation satisfies the governing criterion). *The accepted shared carrier has exactly one grounded reading: the consequence reading of joint continuation, equivalently of closure of the authored reliance and of closure on the continuing ledger. This reading has a legal performance, contains closure itself, and is failed by legal idle.*

Proof. The accepted finite-time write reaches the selected-action coordinate read by Alice's policy (Section 7.8.3). Every configuration decoded from that accepted written register is therefore the scheduled configuration. Its comparator reads the actual accepted payload (Section 7.8.2). Equal decoded payloads give equal verdict predicates, because they use the same terms and the same finite truth table. The resulting pass predicate is performance of the active content.

On every legal action, that performance equals joint continuation and closure of the authored reliance. The next-interval calculation identifies the same class with closure on the continuing ledger. Their consequence readings are therefore equal: each implication over legal performances can be transferred through these pointwise equivalences.

The positive accepted detection branch and its completed write supply an installed configuration, so the grounding class is nonempty. Every

installed configuration has the reading just identified, proving uniqueness. Feeding is legal and passes. Any consequence of legal performance holds of feeding, so feeding performs the whole reading. The reading contains the property that always holds and is closed under consequence over legal actions by composition of implications, and under conjunction by applying both contained properties to the same performance. It cannot contain the property that never holds, since feeding is a legal performance. Thus it is coherent and nonvacuous. Closure belongs to that reading because every legal closing action closes. Idle is legal and does not close. These facts give all three governing clauses. □

Bringing the meal performs the one reading the accepted record grounds; staying away fails it and remains executable. The reading built on Alice's continuation alone is coherent, but no installed controller implements it, so it is not grounded. In content it lacks closure of Bob's reliance; Chapter 13 writes it out and shows that coherence and completeness cannot tell it from the grounded reading, while faithfulness to that reliance can.

7.7 What the record selects is what Alice owes

The proposition settles what the accepted record does; the opening question asked what Alice owes. One identification connects them, and we adopt it as a premise: what Alice owes Bob is what the record the two of them installed requires of her action. In general terms, the requirements of an installed agreement are the ends of the reading its apparatus implements. The installation proves which reading the apparatus implements; the identification says that this reading states her obligation.

Bob's reliance is the reason to accept it here. The installed reading contains closure of the authored reliance, which leaves Bob exactly where he would have stood had the agreement never formed and he had fed himself (Section 7.2). He stands below that position only because he relied on the agreement, spending his one interval of labor on work instead of food; preventing either formation act returns him to it. The installed requirement asks Alice to restore what her own proposal led Bob to give up, and its performances are exactly the legal actions that do so. Omission breaks it visibly on the ledger: one interval later Bob carries a quarter-unit of bond cost that feeding would have left at zero

(Proposition 7.3). The record selects the meal; the reliance it answers is why Alice owes it.

Stated for every installed agreement, the identification reaches further than this reason. Every use of it here concerns Bob, who relied; an installed agreement that no one relied on would need another reason.

7.8 A physical record that selects Alice's first action

The governing result of Section 7.6 rests on what is built here. The logic of the selection is classical: an interlock that writes 'feed' when acceptance is detected and omission fails would carry it out, and a hardware interlock would be physical too. What the construction adds is an explicit evolution with proofs of the properties the governing argument uses. Generated by a positive Hamiltonian, the write completes exactly at a definite finite time, at which the register holds the selection Alice's first action is read from; it retains every control coordinate, so the evidence that enabled it stays on record; and it conserves the energy held in its coordinates, so what the memory gains the reservoir loses. Its control is the retained output of one physical operation, the probe of Appendix B.1. These properties fix the requirement, on record, before Alice acts.

Two physical identifications connect the construction to the people, and both are adopted premises: Bob's act of accepting prepares the occupation the probe reads, and the register stands for Alice's means of action, the physical state her first action is read from, wherever in her body or tools it sits. Throughout, the inputs are prepared and protected as described and the contacts are available; a written mode supplies no food, work or intake by itself.

7.8.1 The record and its exact write

Alice and Bob each keep a record with a claim entry, a signed liability entry and an offset entry, all starting at zero; each writes only their own, by a conserved own write (Chapter 4) whose balancing amount goes to the offset. These entries record the agreement. They are separate from the reserves, no number passes between the two, and they add no bond to the care ledger, which holds only Alice's and Bob's internal bonds. On the relation from Alice to Bob, the *mismatch* is Alice's claim plus Bob's signed liability, the gap between what she claims and what he acknowledges. The mismatch is a difference and forms no bond: an agreement's bond

is the ratio of the two entries (Chapter 8), which needs both positive, and here both start at zero. Alice proposes by adding 1 to her claim and -1 to her offset. Bob accepts by adding -1 to his liability and 1 to his offset. The mismatch opens to 1 and then closes to 0. The undertaking also keeps a separate record of its active terms: recipient 1 (Bob), start 1, finish 5.

The closed mismatch is the same zero it was before anyone wrote, so the account gap alone cannot carry what was accepted. The records must retain the terms, support a comparison with the candidate response, and connect that comparison to the selected action. One finite writer serves all three steps (Figure 7.2).

The writer is a conditional swap. On a basis state prepared with memory at mode zero it is an ordinary conditional move, completed at a definite time: the reservoir's value ends in memory, and the reservoir is left at mode zero.

Let $M = \{0, \dots, 7\}$ be the set of modes, enough to hold the recipient code and the clock values the terms name. Eight is a choice of this construction, separate from the eight entries of Section 2.2. Let K be a finite control set. A register is

$$r = (k, m, \rho) \in K \times M \times M,$$

where m is the memory coordinate and ρ the reservoir coordinate. The carrier is the complex vector space with these registers as an orthonormal basis. For a two-valued control predicate $\kappa : K \to \{0, 1\}$, define the exchange

$$E_\kappa(k, m, \rho) = \begin{cases} (k, \rho, m), & \kappa(k) = 1, \\ (k, m, \rho), & \kappa(k) = 0. \end{cases}$$

Write $|r\rangle$ for the basis vector at register r, and a star for the conjugate transpose. The exchange matrix W_κ sends $|r\rangle$ to $|E_\kappa(r)\rangle$. Applying the same exchange twice restores every register. Hence

$$W_\kappa^2 = I, \qquad W_\kappa^* = W_\kappa, \qquad W_\kappa^* W_\kappa = I.$$

The matrix $U_\kappa(t)$ sends an initial amplitude vector to its state at time t, and its Hamiltonian H_κ generates that evolution. The positive constant $\hbar$ converts energy times time into phase.

Proposition 7.6 (An exact finite-time write). *For $g > 0$, the Hamiltonian and flow*

$$H_\kappa = g(I - W_\kappa), \qquad U_\kappa(t) = \frac{1 + e^{-2igt/\hbar}}{2} I + \frac{1 - e^{-2igt/\hbar}}{2} W_\kappa$$

give a positive Hamiltonian, a unitary solution of the Schrödinger equation, and the exact write

$$U_\kappa\left(\frac{\hbar\pi}{2g}\right) = W_\kappa.$$

The complete control and the sum of memory and reservoir holding energies are retained.

Proof. The involution and adjoint identities imply

$$I - W_\kappa = \frac{1}{2}(I - W_\kappa)^*(I - W_\kappa),$$

so H_κ is positive semidefinite. At $t = 0$, the exponential is one and $U_\kappa(0) = I$. Differentiating the displayed flow and using $W_\kappa^2 = I$ gives

$$\frac{dU_\kappa}{dt} = -\frac{i}{\hbar} H_\kappa U_\kappa.$$

Writing $\omega = e^{-2igt/\hbar}$, the identity $\omega\bar{\omega} = 1$, together with $W_\kappa^* = W_\kappa$ and $W_\kappa^2 = I$, gives $U_\kappa(t)^* U_\kappa(t) = I$ by multiplication. At the positive time $T = \hbar\pi/(2g)$, $\omega = e^{-i\pi} = -1$; the coefficient of I vanishes and that of W_κ equals one.

For any control energy $\varepsilon_K(k)$ and mode energy $\varepsilon(m)$, the holding observable is diagonal with value

$$D(k, m, \rho) = \varepsilon_K(k) + \varepsilon(m) + \varepsilon(\rho).$$

The exchange retains k and only swaps the last two summands. Thus D commutes with W_κ, and consequently with $U_\kappa(t)$. Unitarity gives $U_\kappa(t)^* D U_\kappa(t) = D$. The same exchange retains every control coordinate, so the evidence used to select the write is never erased. □

The proof uses only that W_κ is self-adjoint and squares to the identity, and, for the energy, that D commutes with it. Any self-adjoint involution can take its place; the flow then performs it at $t = \hbar\pi/(2g)$ and conserves every observable commuting with it.

Two operators carry energy here. H_κ is the coupling that drives the exchange; D is the energy held in the control, memory and reservoir coordinates, and the write conserves it. On a prepared basis state $|k,0,1\rangle$, an enabled write ends exactly at $|k,1,0\rangle$, with amplitude one. The memory gains $\varepsilon(1)-\varepsilon(0)$ and the reservoir loses exactly that. An inactive control leaves the preparation unchanged.

7.8.2 Remembering the terms and evaluating omission

A payload of terms names a recipient $p \in \{0,1\}$ and two clock modes $s,f \in M$, a start and a finish. It takes one of $2 \cdot 8 \cdot 8 = 128$ values, so the comparator below works from a finite table. Encode (p,s,f) directly as three modes. A recipient mode outside $\{0,1\}$ decodes to no recipient.

Three independent copies of the writer carry preparations

$$(q,0,p), \qquad (q,0,s), \qquad (q,0,f),$$

where the retained event mode q names the event's author: 1 for Bob's acceptance, 0 for another author. Exchange is enabled exactly when $q=1$, and the memory may be read only when all three retained event coordinates equal one. The accepted preparation therefore writes and reads (p,s,f) exactly; a preparation by another author leaves no readable record. For the actual payload the memory modes become $(1,1,5)$. The three reservoirs fund the three memory changes separately. The construction uses independent apparatus copies and assumes no additional composite contact Hamiltonian.

The comparator's finite control holds the decoded payload, a checkpoint mode τ, and two response bits for each participant: β_p says that the bond multiplier is one, and γ_p says that the reserve is positive. Its truth table is

$$F(p,s,f,\tau,\beta,\gamma) = \begin{cases} \beta_p \wedge \gamma_p, & s \le \tau < f, \\ 1, & \text{otherwise.} \end{cases}$$

Inside the active window the verdict is the recipient's maintenance; outside it the terms ask nothing. Preparing the verdict memory and reservoir at $(0,1)$, and using F as the exchange predicate, writes verdict one exactly when the accepted checkpoint is met. The whole input is retained.

At the first checkpoint $\tau=1$, the window $1 \le 1 < 5$ is active. Feeding gives Bob $(1,1/2)$, so his bits are $(1,1)$ and the verdict is pass. Omission gives $(1,0)$, so the bits are $(1,0)$ and the verdict remains zero.

The response bits are prepared inputs, computed from the candidate's response by a separate device. The governing result concerns only the legal first actions, which leave Bob at 0 or $1/2$, so a threshold that separates those two levels suffices. One continuous rail is

$$\theta(e) = \max\{0, \min\{1, 4e - 1\}\},$$

with $\theta(0) = 0$ and $\theta(1/2) = 1$, Bob's two values of γ_p. It is no general positivity test ($\theta(1/4) = 0$), and the claim needs none.

7.8.3 Acceptance controls the coordinate that selects the action

Bob's acceptance reaches the writer through a probe (Appendix B.1). The probe acts on a cell of two channels, each occupied or empty, and the pair of occupation bits encodes the author: $(0, 0)$ for another author and $(0, 1)$ for Bob. It is one recorded operation applied to two preparations that differ only in those bits, and its event journal keeps the prepared command. Every output carries a nonnegative weight, its squared amplitude. The contact weakens with the distance between probe and target. At the contact used here, with probe and target at the same position, the retained detection bit is true on outputs of total weight zero for the other author and one half for Bob.

Those two numbers identify the author. A true output with positive weight cannot belong to the other author's preparation, and Bob's true outputs, summing to one half, include one of positive weight. This supplies a legal positive accepted branch z.

Let V_0 be the complete written comparator register for omission, and let $d(z)$ be the detector's retained acceptance bit. The action writer has control (z, V_0), memory zero and reservoir one. Its exchange predicate is

$$\kappa(z, V_0) = d(z) \wedge [\text{verdict memory of } V_0 \neq 1].$$

The omission calculation makes the second factor true. On the positive accepted branch the first is true as well, so the flow of Proposition 7.6 writes action mode one at time T. The mode decoder selects the food job at mode one and idle at every other mode. Before the write it selects idle; after the accepted write it selects feeding. Because the exchange squares to the identity, applying it again restores the ready register.

Alice's policy reads its first output from the selected-action coordinate of this very register on the shared carrier, so the policy holds no separate

first-action value of its own. The same writer construction extends to the complete fixed care schedule as follows.

On the detected branch the construction thus establishes the whole chain: acceptance identified, write completed, register installed at action mode one (the installed configuration of Section 7.6), and the physical food job of the opening selected. On the other half of the weight of Bob's preparation the detection bit is false, nothing is written and Alice's policy selects idle (Section 7.6).

7.8.4 The accepted record controls all four scheduled intervals

Retain the complete acceptance-detector output, the comparator's rejection of omission, and a service clock with four positions in one control. At each clock position prepare the reservoir with the action mode required by the fixed schedule. The exchange is enabled exactly when acceptance is detected, omission is rejected, and the clock calls for service. The selected-action coordinate starts at idle.

On the positive detected branch, this one controlled writer selects the scheduled action at each of the four positions. At a service position it exchanges the prepared action mode into the selected coordinate; at an idle position it leaves idle selected. The exact finite-time flow of Proposition 7.6 performs each write. It retains the detector, comparator and clock, is reversible, and conserves their holding energy together with that of the action and reservoir coordinates. On an undetected branch the same apparatus selects idle at every position.

The physically written policy therefore equals the verified four-interval schedule, so the schedule's continuation result applies to the whole accepted undertaking. This connects the accepted record to every action selection within that finite horizon. The clock-specific reservoir preparation remains supplied: the result does not replenish it for an unlimited sequence of undertakings. Nor does writing an action selection by itself join every food packet, delivery, work debit and intake into one available execution. Appendix B develops those physical contacts and states the remaining joins. Omission remains physically legal throughout; selecting the accepted policy and having a governing requirement are still different facts.

7.9 A shortage changes the physical question

In the funded opening the meal protects both people, so Alice's omission cannot be attributed to a shortage. A nearly exhausted Alice is a different case. Let her open with reserve $1/2$, Bob with $1/4$. The meal leaves her at $(1, 0)$ and Bob at $(1, 1/2)$; staying leaves her at $(1, 1/4)$ and Bob at $(1, 0)$. Every choice between idle and feeding fails to protect one of them. An undertaking does not create the absent reserve, stock or work capacity.

For Alice's first action in the funded opening, what the record requires has an exact answer: the installation grounds joint continuation, a requirement she can perform; feeding performs it, and omission violates it and stays executable. What remains to be derived is the identification of Section 7.7, the two connections Section 7.8 adopts (from Bob's act of accepting to the detector occupation, and from the register to a physical state of Alice), and the preparation and replenishment needed for further undertakings. The fixed four-interval schedule already has the protected physical writes of Section 7.8.4. Chapter 13 carries the installation through changes of controller and participant, and, when a shortage sets two undertakings against one unit of food, derives what any answer must respect.

Chapter 8

Consent, responsibility, and release

Alice writes a claim against Bob into her own books, and a moment later she erases it. Bob's burden rises and then falls back to zero. He benefited from the erasure and had no part in it. Did he consent to it?

He did not. Consent is an authored event; Bob's lower burden is an effect. The undertaking of Chapter 7 binds Alice's and Bob's actions together without merging them: Alice proposes, Bob accepts, Alice controls the first delivery and Bob undergoes its consequences. A favorable total, computed after the participants and events have been discarded, cannot recover who did which.

8.1 Consent belongs to an authored act

Give each participant a row of entries that only they can write. An individual journal then has one author, and a joint posting has both endpoint authors. This authorship is the part of consent derived here; whether an acceptance was also informed and uncoerced needs more than a record of who wrote it.

Take four participants $0, 1, 2, 3$. Alice is participant 0 and Bob is participant 1; participants 2 and 3 complete a cycle of directed relations

$$0 \longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 0.$$

In the cycle each participant is the source of exactly one relation and the destination of exactly one, so every row has a claim and an acknowledgment that are read. Participants 2 and 3 never act. With Alice and Bob alone there would be no one else for a move between them to reach; the two others let us check that it reaches no one else.

Each participant has three accounts: asset account zero, signed liability account one, and free offset account two. A book is an array b_{ia}, with a

row for each participant i and a column for each account a. On relation $i \to i+1$, with indices taken modulo four, the two designated entries and their gap are

$$x_i = b_{i0}, \qquad y_i = -b_{i+1,1}, \qquad G_i = x_i - y_i.$$

The source claims x_i, the destination acknowledges y_i, and G_i is the gap between them (Figure 8.1). The gap is defined for any entries; the ratio x_i/y_i needs both positive, and is then one exactly when the gap is zero. On positive entries that ratio is the relation's multiplier, the agreement case of the bond of Chapter 3, and its recognition cost is $J(x_i/y_i)$. For these relations the costs below rest on the pair identification (Section 3.2); the author sets do not.

A record that claims nothing holds the amount zero, and J compares positive quantities, so these books record amounts above a baseline of one: the row $(1, -1, 0)$ claims nothing and acknowledges nothing. The cost is still the J of Chapter 2, but the ratio the books hand it changes with the baseline, while a ratio of two signals has none. So on these books the size of a cost, and of any harm read from them, belongs to the baseline. Whether a cost is zero does not: J vanishes only at balance, and two amounts agree at every baseline or at none. Every change of cost below is one relation's cost moving to or from zero, so its direction is the same at every baseline.

A participant's *half-share* is half their incident cost: each relation's cost is split equally between its two ends, a convention that makes the half-shares add up to the total cost. The factor two changes no sign.

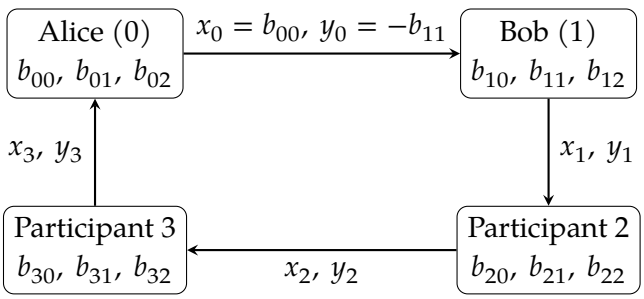


Figure 8.1. The four-participant chart. Each participant writes only their own row. A relation is read from two rows, the source's asset entry and the destination's liability entry, so one person's write can change it without the other taking part.

Two kinds of move change the books. An *own journal* by i adds a vector v of sum zero to row i, leaving every other row fixed. Its author set is $\{i\}$. A *joint posting* of amount u on $i \to j$ adds $(u, 0, -u)$ to the source row and $(0, -u, u)$ to the destination row. Its author set is $\{i, j\}$. Both designated entries increase by u, so their gap is preserved.

Author sets come from who wrote; half-shares come from the entries. Nothing requires them to agree.

Proposition 8.1 (A favorable outcome does not constitute consent). *A participant can receive a strict reduction in their half-share of recognition cost without consenting to the action. A consenting author can undergo a strict increase in that half-share.*

Proof. Start with every row equal to $(1, -1, 0)$. All four relations then have entries $(1, 1)$, ratio one and zero cost. Alice writes $(1, 0, -1)$ on her own row. The new array is

$$\begin{pmatrix} 2 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 0 \end{pmatrix}.$$

Only relation $0 \to 1$ changes its ratio: its entries are now $(2, 1)$. Thus its cost is $J(2) = 1/4$, and Alice and Bob each carry a half-share of $1/8$. The claim's author set is $\{0\}$.

Alice can now write the reversal $(-1, 0, 1)$. It restores every row to $(1, -1, 0)$, and Bob's half-share falls strictly from $1/8$ to zero. The reversal's author set remains $\{0\}$; Bob did not author it.

The first journal also exhibits the converse failure. Its author, Alice, consented to that write while her own half-share rose from zero to $1/8$. Hence neither strict improvement nor non-worsening identifies authored consent, and consent does not imply non-worsening. □

Inferring consent from benefit would credit Bob with an act he did not perform; inferring refusal from burden would deny Alice one she plainly authored. Nothing here forbids Alice's reversal: correcting her own unsupported claim is a different thing from obtaining authority over Bob, whom it helps.

Authorship is also specific to content. Only the gap matters here, so no baseline is needed: start with every entry zero. Alice records $(1, 0, -1)$,

a full unit, while Bob is shown one half and writes $(0, -1/2, 1/2)$. Their rows become

$$b_0 = (1, 0, -1), \qquad b_1 = (0, -1/2, 1/2).$$

The source entry is one, the destination entry one half, and the gap one half. Bob authored exactly the half-unit entry he saw; the concealed full unit occurs nowhere in his journal.

Nor does a later joint action retrospectively authorize an earlier unilateral demand: joint postings preserve an existing gap, so Bob can co-author a later posting with Alice while the record still shows that he did not author her earlier claim.

8.2 A physical summary can forget the authors

The full books give back the authors of any move that changes them. A nonzero own journal changes exactly its author's row. A nonzero joint posting changes both endpoint rows, because the posted amount and its offset land in different accounts. No other row changes. The nonzero condition matters: an own zero journal by Alice and an own zero journal by Bob leave identical before-and-after books but have different author sets.

A coarser physical summary loses more. The bond ledger keeps each relation's endpoints and ratio, and an action of Chapter 3 names one acting participant, who cannot hold a joint author set. So a joint posting by Alice and Bob and the same posting by Alice and Carol can leave the ledger and the action identical: a posting adds the same amount to both entries of its relation, so a relation that opens balanced stays balanced, and the action names Alice either way. The books check it. Give Alice two relations, $0 \to 1$ to Bob and $0 \to 2$ to Carol, participant 2. Each row now has four accounts, whose roles depend on the row:

Row	account 0	account 1	account 2	account 3
Alice	claim on Bob	claim on Carol	balancing entry	free offset
Bob, Carol	signed liability to Alice	unused	balancing entry	free offset

The opening books are

$$b = \begin{pmatrix} 1 & 1 & -2 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}.$$

Both relations have entries $(1,1)$. Alice makes a joint unit posting with Bob on the first relation, using offset three at each end, or the same posting with Carol on the second. The respective results are

$$b' = \begin{pmatrix} 2 & 1 & -2 & -1 \\ -2 & 0 & 1 & 1 \\ -1 & 0 & 1 & 0 \end{pmatrix}, \qquad b'' = \begin{pmatrix} 1 & 2 & -2 & -1 \\ -1 & 0 & 1 & 0 \\ -2 & 0 & 1 & 1 \end{pmatrix}.$$

Every row is balanced, all designated entries are positive, and both moves change the books. Each relation still has ratio one in each result, so the bond ledger is identical before and after either move. The action representing either move names Alice, the source of both relations, and its strains, the new log ratios minus the old, are all zero in both cases. The authors are $\{0,1\}$ in the first move and $\{0,2\}$ in the second. By the test of Chapter 1, no procedure that sees only the ledger and the action can report the authors in both.

The remedy keeps the author sets alongside the effect on the ledger: over a finite sequence, each effect is calculated from the actual intermediate books, and the ordered author sets travel with them. The lost information was part of physical history, and an adequate moral description has to keep it.

8.3 Compliance and responsibility in the installed care case

In the care construction, formation consent consists of Alice's proposal, Bob's acceptance and the resulting active terms. It does not license every later act.

Call an action *compliant* when it performs every installed reading: it is physically legal and has every property each installed reading contains. Call a property *obligatory* when it belongs to every installed reading. In plain words, a compliant action does everything the installed agreement requires; avoiding what it forbids is not enough. In these terms the governing result of Chapter 7 becomes a judgment.

Proposition 8.2 (The first care judgment). *In the fixed installed opening, compliance is equivalent to physical legality together with joint continuation, and hence to physical legality together with closure of the authored reliance. Joint continuation and closure of the authored reliance are obligatory. Feeding is compliant; omission is a violation.*

Proof. Section 7.6 established that every installed reading is the joint-continuation reading and that this reading is installed.

If an action is compliant, it performs joint continuation itself, since that property belongs to the installed reading. Conversely, a legal action preserving joint continuation performs every property that follows from legal joint continuation. It therefore performs every installed reading.

The equivalence with closure of the authored reliance follows from their equality on legal actions. Joint continuation belongs to its own consequence reading, and closure follows from it there, so both are obligatory. Feeding supplies the positive witness. Idle is legal but fails closure, making it a violation. □

In this opening the one installed reading is the governing one, so every legal first action is compliant or a violation; what the judgment decides is that omission is a violation. When Alice omits the meal, the proposal and acceptance are still in the record. Bob's consent to form the undertaking cannot turn its unperformed first checkpoint into a compliant omission.

Responsibility adds control, and here the construction assigns it: the care interventions of the first interval physically belong to Alice alone, and they can alter Bob's later burden. Responsibility combines that control with the installed obligation to close the reliance. Both hold, so Alice is the responsible actor for this judgment, and running a controller other than the installed one does not change it, because the governing reading still rejects the omission (Sections 7.6 and 13.1). Control alone would not settle it: the ability to affect someone does not by itself assign every obligation or grant every enforcement power. The judgment holds on the branch where Bob's acceptance was detected (Chapter 7).

8.4 What is due cannot be read from claims alone

What is owed depends on the events, which a record can report truly or falsely. Take a different arrangement between the same two people: a sale, in which Alice delivers food and Bob owes her its price.

Use the four-participant chart again, starting with every row $(1, -1, 0)$. In the two histories below the book move at tick zero is Alice's claim $(1, 0, -1)$, and later book moves are null journals. The entire book history is therefore the same, and from tick one onward its rows are exactly the claim array of the first proposition.

Alongside the books, keep a separate protocol event record. It starts with price zero, no offer, no acceptance, no delivery and amount due zero. An offer from Alice to Bob records the price; Bob's acceptance activates the offered terms. The first accepted delivery from Alice to Bob records performance and sets the amount due to that price. A second delivery does nothing. Repayment by Bob to Alice subtracts an amount between zero and the amount due after delivery. A release by Alice, the creditor, subtracts an amount within the same bounds. An attempted release by any other participant is refused.

In the sale, the events at ticks zero, one and two are an offer at price one, Bob's acceptance, and delivery. In the invoice they are the same offer, the same acceptance, and no delivery. After three events the states are respectively

$$(\text{price}, \text{offered}, \text{accepted}, \text{delivered}, \text{due}) = (1, 1, 1, 1, 1)$$

and $(1, 1, 1, 0, 0)$. The book histories remain identical. The pair of histories is the argument; the proposition is the working test of Chapter 1 applied to the books.

Proposition 8.3 (Book history does not determine entitlement). *No procedure using only the book history can always recover the amount due in these protocol histories.*

Proof. The two histories supply the same book input and different amounts due. A procedure depending only on that input would have to return the same amount for both. It cannot agree with both obligations. □

Bob's books cannot tell him whether he owes Alice one unit or nothing; what decides it is whether the food arrived. The event record that suffices retains offer, acceptance, delivery, repayment and release. Its transition rules are premises of the exchange protocol; applied in event order, they determine exactly one state, by induction on the event count. Books do

not occur in them, so changing a claim or writing a denial leaves the amount due unchanged.

Delivery itself is a physical event, read from where the food is. Space is divided into cubic cells two voxels on a side, and the record dates a delivery from Alice to Bob at the tick the unit of food starts to cross from her cell into his. In a second sale history, with the offer and acceptance at ticks zero and one, the unit waits at the origin through tick two and then moves one voxel per tick along one axis. It crosses at tick three, a tick later than in the first sale, and one unit is due from tick four. A unit that stays at the origin never crosses, and nothing is due. The book histories are the same; the retained crossing tells the cases apart.

The terms matter as well. Replace only the offer's unit price by zero, keeping acceptance, delivery and books the same, and the resulting gift has amount due zero. Table 8.1 shows what later events do to the delivered unit-price sale.

Event after the delivered unit-price sale	Amount due
None	1
Release of w, $0 \le w \le 1$, by Alice, the creditor	$1 - w$
The same attempted release by Bob, the debtor	1
Valid repayment of m, $0 \le m \le 1$	$1 - m$
A book denial alone	1

Table 8.1. Payment, release and a claim follow different transition rules. A valid repayment or the creditor's release can reduce what is due; the debtor's attempted release and a denial written into the books leave it unchanged.

Performance, the terms and the identity of the releaser are each needed: discard any one, and two histories with different amounts due look the same.

8.5 Truthful correction can create disagreement

Two matching records can describe the same false obligation. Their agreement cost can be zero while neither describes the amount the events make due.

Start with the same unit books and an event record containing no events, so nothing is due at any tick. At tick zero Alice and Bob jointly

post one unit on $0 \to 1$. At tick one Alice writes the reversal $(-1, 0, 1)$ on her own row. Their rows after these successive moves are

	b_0	b_1
after joint posting	$(2, -1, -1)$	$(1, -2, 1)$
after source correction	$(1, -1, 0)$	$(1, -2, 1)$.

The other rows remain $(1, -1, 0)$. Above the baseline, the represented claim is $x_0 - 1$ and the acknowledged amount is $y_0 - 1$. After the joint posting both report one, although the event amount due is zero. Their designated entries $(2, 2)$ agree, with cost $J(1) = 0$. After Alice's correction the entries are $(1, 2)$. Alice truthfully reports zero due, Bob's record still reports one, and their cost is

$$J(1/2) = \frac{1}{4}.$$

The value $1/4$ belongs to the baseline; the rise from zero does not, since at every baseline the agreed entries are equal and the corrected ones differ.

A history-based departure test reads who opened a gap: a participant has departed when a move they wrote strictly widened the gap between the two records and the records have not agreed at any tick since. It identifies Alice as the author of the newly opened gap. That says who created this disagreement; it does not establish that the correction was false or wrongful.

The correction is constructible in general. For an event-determined amount due q and a source entry x, its amount is

$$\delta = 1 + q - x.$$

The source journal adds δ to the designated asset, $-\delta$ to a fixed free offset and zero elsewhere. It is balanced and makes the new entry $x + \delta = 1 + q$. In the false-agreement case above, $q = 0$, $x = 2$ and $\delta = -1$, exactly the reversal Alice wrote. A journal confined to those two accounts has one free number, and truthfulness fixes it.

Proposition 8.4 (The supported truthful correction is unique). *Fix a free offset account. Among balanced source journals supported only on that offset and the designated account, exactly one makes the source entry truthful to the event-determined amount due.*

Proof. Truthfulness fixes the new designated entry, and therefore fixes its change from the old entry. Support restricts every other possible nonzero component to the offset. Balance then forces the offset component to be the negative of the designated change. These are exactly the two components of the stated journal. □

Other participants' rows and the other designated entries remain unchanged. A second identical correction has amount $1 + q - (1 + q) = 0$.

The correction also acts on the ledger, by a strain it fixes. On positive old and corrected entries, the corresponding action changes each active bond by the strain

$$\log(x'_e/y'_e) - \log(x_e/y_e),$$

the logarithm of the new ratio over the old, so multiplying the old multiplier by its exponential gives exactly the new ratio. No other strain does this: a strain takes the old ratio to the new exactly when it brings their quotient, old over new, $(x_e/y_e)/(x'_e/y'_e)$, to one, and Proposition 3.4 fixes that strain. The action of any book move with positive entries before and after likewise produces exactly the ledger the books represent, with its strains fixed in the same way.

8.6 Release changes the present requirement

The care protocol designates Bob as releaser: his release clears the active terms, and Alice's purported release does not. Like the sale, which designates Alice, the creditor, it gives that power to the party owed; each designation is a rule of its protocol, not a result. Bob's release changes present compliance for the same omitted delivery. It does not supply the missing food.

The record has to carry the designation, because formation alone cannot recover it. Let the designated releaser be $r \in \{0, 1\}$, with Alice (0) as proposer and Bob (1) as acceptor. From an empty cell, Alice's proposal of terms $(1, 1, 5)$ (recipient Bob, start 1, finish 5) and Bob's matching acceptance never read r, so by induction every release-free sequence gives the same cell under both designations: here the cell with those active terms, not closed. A release authored by i then sets the active terms and the closed flag to

$$\text{active} = \begin{cases} \emptyset, & i = r, \\ \{(1, 1, 5)\}, & i \neq r, \end{cases} \qquad \text{closed} = [i = r].$$

Take the same release by Alice, $i = 0$. The $r = 0$ protocol closes the cell and the $r = 1$ protocol leaves it active, while the omitted delivery and Bob's exhausted reserve are identical in both; present compliance differs because only the first cell has no active requirement. The formation records agree and the release authority differs, so by the test of Chapter 1 no procedure that sees formation alone can recover who may release.

8.7 Repair retains the past

A missed checkpoint is an event of the history. A later release by Bob changes the current undertaking cell, and a later meal the current physical state; neither changes the state at which the accepted checkpoint was missed. Feeding can restore Bob's response while his reliance history stays physically distinguishable (Chapter 9 calculates such a funded remedy), and a tail of actions performed after the omission can meet the remaining checkpoints with the available food and work, judged over the futures still performable from the actual state (Section 6.6).

Release changes an active requirement, repair a present response; responsibility connects an actor's control to an established requirement, and the retained history states what occurred. An adequate physical account keeps all four. It can recognize a real benefit without inventing consent, an active entitlement without mistaking a claim for delivery, and a successful repair without rewriting the past.

Two questions remain to be derived, each from the authored history itself: when an authored acceptance is also informed and uncoerced, and which release designations are valid. Chapter 13 begins the authority argument, with changed controllers and competing claims.

Part III

The capacities of moral action

The virtues are capacities to act within this structure. Each has its own physical objects and exact construction. Value is a comparison, and a comparison is adequate only when it keeps what the judgment needs.

Chapter 9

Giving, responding and repairing relations

Bob needs food and funded care. Love's conserving endpoint alone does not supply those resources (Chapter 5). Someone has to act. Alice can supply food, spend her time, take part of his burden onto herself or release a debt he owes her. These actions change different parts of reality. Food changes available capacity. Work uses capacity. A release changes what Bob owes. A grateful response changes the disposition with which Bob approaches a later encounter.

Compassion, Sacrifice, Forgiveness and Gratitude are the virtues of these operations, each named, as Love and Justice are in Chapter 5, for the work it does. For each we give an operation that does the work and follow what moves, what remains, who acts and who pays; Table 11.2 gives the status of each result. For Compassion, Sacrifice and Gratitude we also test the rule the catalogue proposes.

9.1 Compassion counts the side that gives

An account of help has to count the side that gives it. Compassion's work is to supply what another lacks from a provider's reserve without consuming the provider. On an actual shared ledger that work is the care of Chapter 6: provision that keeps both people funded, which Theorem 6.3 shows is possible exactly when each person's own condition holds. The catalogue's rule adds a cap on the transfer that keeps the provider funded however large the need; its rule for skew fails, because it does not act on Alice's and Bob's relations.

The rule works on summaries. Alice, the helper, carries subscript A, and Bob, the recipient, subscript B; each is described by a positive reserve, E_A and E_B, measured as recognition-cost capacity in the units of the rule,

and a skew, σ_A and σ_B. The rule applies when

$$\sigma_B > 0, \qquad E_B < T.$$

Here T is a target for Bob's reserve, set by his condition and the purpose of the assistance. The second condition is the need; the first says that Bob's outgoing log-flow exceeds his incoming.

Two of the rule's constants are powers of

$$\phi = \frac{1+\sqrt{5}}{2}.$$

Like the rule's third constant, a tenth, and the constants of the Sacrifice and Gratitude rules below, they are the catalogue's chosen values, and they stay parameters until a law of the reserve or the response fixes them. Every proof below uses only $\phi > 1$, so ϕ supplies values here, not reasons. Chapter 10 derives one golden-ratio fraction of this kind, Temperance's cap, under a condition stated there.

The rule transfers

$$t = \min\left(\frac{E_A}{\phi^2},\, T - E_B\right).$$

It supplies no more than Bob's shortfall and commits no more than the fraction $1/\phi^2$ of Alice's reserve; since $1/\phi + 1/\phi^2 = 1$, it leaves her at least E_A/ϕ.

The skew part converts the transfer into integer skew relief for Bob and, for Alice, an integer burden of at most a tenth of it:

$$a = \lfloor t\phi^4 \rfloor, \qquad b = \left\lfloor \frac{a}{10} \right\rfloor.$$

The successor descriptions are

$$E'_A = E_A - t, \quad \sigma'_A = \sigma_A + b,$$
$$E'_B = E_B + t, \quad \sigma'_B = \sigma_B - a.$$

The rule places each new skew on a *physical witness*, a new two-bond state built to have it (a bond at multiplier e^{τ} and a return bond at 1 give skew τ), and says nothing about how Alice's and Bob's own relations change.

Take $E_A = 10$, $E_B = 1$, $T = 3$ and any positive σ_B; these numbers belong to this rule, not to the care opening. Alice's cap is $10/\phi^2 \approx 3.82$

and Bob's shortfall is 2, so $t = 2$. Then $t\phi^4 \approx 13.71$, giving $a = 13$ and $b = 1$. Alice's reserve falls to 8 and her skew rises by 1; Bob's reserve rises to 3 and his skew falls by 13.

Proposition 9.1 (A capped transfer preserves the helper's reserve). *Let $E_A, E_B > 0$, $\sigma_B > 0$ and $E_B < T$. Then $t > 0$, $E'_A > 0$, and the recipient gains exactly the reserve the helper supplies. The recipient's signed skew decreases by exactly a.*

Proof. Both arguments of the minimum defining t are positive. Also,

$$t \leq \frac{E_A}{\phi^2} < E_A,$$

because $E_A > 0$ and $\phi^2 > 1$. Thus $E_A - t > 0$. The two reserve changes are $-t$ and t. The skew equation gives $\sigma_B - \sigma'_B = a$ directly. □

The cap guarantees that however large Bob's shortfall, Alice keeps a positive reserve. An account that simply raises Bob's reserve has left out the side that supplied it.

The skew part passes its own arithmetic. A small positive transfer can raise Bob's reserve without producing one full unit of skew relief; a transfer of at least

$$t \geq \frac{1}{\phi^4}$$

makes $t\phi^4 \geq 1$, so $a \geq 1$. When $a > 0$, the helper's integer burden is smaller than the recipient's relief,

$$b \leq \frac{a}{10} < a,$$

and the sum of the two skews falls:

$$\sigma'_A + \sigma'_B = \sigma_A + \sigma_B + b - a < \sigma_A + \sigma_B.$$

Without positive integer relief, the inequality is non-strict. But these are the witnesses' skews. Skew counts natural logarithms, so the fall of 13 in the example divides the outgoing multiplier of Bob's witness by e^{13}, a scale nothing in the rule ties to an actual relation; and the rule does not cap a at Bob's initial skew, so it need not bring his skew closer to zero. The skew part is bookkeeping on summaries; what makes the rule Compassion is its reserve part.

9.2 Sacrifice puts a burden on a particular participant

Sacrifice's work is to take part of another's burden onto oneself: the provider's own condition worsens so that the recipient's improves. Love's equalizing move does this on two people's own bonds (Theorem 5.3): taking Alice's and Bob's internal bonds from $(1, r^2)$ to (r, r), with $r > 1$, raises her incident cost by $s = J(r)$ and lowers his by $s(2s + 3)$, while the total falls (Table 5.1 gives $r = 2$). The endpoint is Love's; what Alice takes on to reach it is the sacrifice. By Theorem 5.4, under the charge of Table 5.2, both continue exactly when each opens with a positive reserve and each renewal budget is at least $J(r)$. A lower total does not keep Alice going, so Sacrifice requires the member account as well as the whole account.

The catalogue's Sacrifice rule fails where the ledger succeeds. Written directly on skew, it fixes Alice's share in advance, although the ledger shows the share is set by the opening state through the ratio $2s + 3$. It takes a positive integer amount a from Bob's signed skew and adds

$$\left\lfloor \frac{a}{\phi} \right\rfloor$$

to Alice's. Its reserve debit is

$$d = \min\left(\frac{a}{20}, \frac{E_A}{\phi} \right),$$

and Bob's reserve gain is d/ϕ. For $a > 0$, the inequality $\phi > 1$ gives

$$\left\lfloor \frac{a}{\phi} \right\rfloor < a,$$

so the sum of the two prescribed signed skews strictly decreases, and the cap $d \le E_A/\phi < E_A$ leaves Alice with a positive reserve. But Bob receives only d/ϕ of Alice's debit; the rest, $d - d/\phi = d/\phi^2$, reaches neither reserve, and the rule does not say where it goes. On the shared ledger every change lands on a named bond. The account of Sacrifice is the ledger computation.

The ledger's move can also be performed: the converter of Section 9.6 carries it out at the one value of r its pulse fixes, granted the identification of its two comparisons with Alice's and Bob's internal bonds. Performing

it replenishes neither reserve, so continued agency still needs the opening reserves and renewal budgets above: a single successful sacrifice and a sustainable practice of sacrifice are different physical accomplishments.

9.3 Forgiveness changes a claim without erasing an event

Alice delivered food to Bob on credit and now lets the debt go. The food stays delivered. Forgiveness's work is to release a claim without erasing the event that created it; here the claim is a debt.

Take the delivered sale of Chapter 8: an offer of a unit delivery at price one, Bob's acceptance and the delivery, with one unit due. A release of w authorized by Alice, the creditor, where

$$0 \le w \le 1,$$

leaves

$$\text{due} = 1 - w.$$

A full release leaves zero due. The delivery still occurred.

The release is an event in the protocol record, and the account books need not change when it occurs. A delivered sale with no release and the same sale with a full authorized release can therefore have identical book histories and amounts due one and zero, so, as in Proposition 8.3, no procedure using only those histories returns the correct amount in both; the event record does. Forgiveness therefore cannot be represented by deleting an inconvenient balance or by making two reports agree.

Once the debt is released, Alice's books can be made truthful, and that correction acts on the relation itself. Her entry begins at 2: 1 is the unit baseline of Chapter 8 and the excess unit is the claim. A truthful entry after release is

$$1 + (1 - w) = 2 - w.$$

The correction is therefore $-w$, the truthful correction $\delta = 1 + q - x$ of Chapter 8 with $q = 1 - w$ and $x = 2$. Supported on the designated claim account and a free counteraccount, it has counterentry $+w$ and is unique (Proposition 8.4).

Bob's entry stays at 1, because the correction touches only Alice's row. Since the bond multiplier is the ratio of the two entries, it becomes

$$\frac{2 - w}{1} = 2 - w.$$

At full release it becomes 1, and the relation is balanced. That depends on Bob never having written: in the sale his entry acknowledged nothing while one unit was due, so after a full release both entries truthfully report zero. Had Bob acknowledged the debt, Alice's truthful correction would open a disagreement instead of closing one, leaving the entries $(1, 2)$ of Section 8.5, at cost $J(1/2) = 1/4$, until Bob corrected his own entry.

A debt can be closed in two ways (Table 8.1): by repayment, which moves resources to the creditor, or by release, which corrects the claim. Only the second is forgiveness.

A broken undertaking has the same shape, with a requirement in place of the debt. Once Alice has omitted the first meal, Bob's release as designated releaser empties the active terms $(1, 1, 5)$ and closes the cell (Chapter 8). The missed checkpoint stays missed and Bob's reserve stays where the omission left it: the release changes what is required of Alice now and leaves what happened to Bob as it was.

9.4 Release and reliance do not commute

Timing becomes visible when a promise changes another person's physical choices. Bob, counting on Alice, may work instead of feeding himself, and a later release cannot give that meal back.

Start from the reliance history of Section 7.2. Before the care opening, Alice's reserve is $9/4$ and Bob's is $3/4$. In this first interval Bob has one unit of labor, which he can use to feed himself or to work; in the next he has none, and Alice has the labor needed to feed him. Her meal consumes one unit of food, supplies one unit of intake to Bob and uses one unit of her labor. Intake and labor change each participant's reserve at the care model's rates (Chapters 6 and 7),

$$f_i = \frac{2\,\text{intake}_i - \text{labor}_i}{4}, \tag{9.1}$$

and each interval's care maintenance (Table 5.2) costs each person $J(2) = 1/4$ when their reserve can pay it.

If Bob feeds himself first, his reserve remains $3/4$. If he works first, it falls to $1/4$. Both actions use his available labor, but only self-feeding supplies intake. Alice pays her maintenance and enters the second interval with 2.

Now let the accepted care undertaking act on Bob through an explicit response rule: while the undertaking is active, he works; after release, he feeds himself. The release is Bob's, as the designated releaser of Chapter 8. Compare two orders:

release, then respond, respond, then release.

Both finish with the same released undertaking record. In the first order Bob has fed himself. In the second he has already worked, and the reserves 2 and 1/4 are those of the care case that opens Chapter 1.

Order	Bob's action	Bob's reserve	After Alice omits
Release, then respond	feeds himself	3/4	1/2
Respond, then release	works	1/4	0

Table 9.1. Release and response do not commute. Both orders end with the same released record; Bob's reserve after his first action, and after Alice omits the next meal, shows the difference his reliance made.

If Alice then omits feeding, Bob's reserve becomes 1/2 in the first history and 0 in the second (Table 9.1). The final record agrees; the physical dependence differs. Both releases came before the care window, so no checkpoint was missed in either history, and the record of failures cannot tell them apart.

A funded remedy remains available in the second history. Feeding leaves Bob with reserve 1/2, exactly the response reached after early release and omission. It leaves Alice with 3/2, rather than the 7/4 available after omission. Alice's labor cost stays in the account.

In both orders the released record requires nothing further of Alice. Whether she owes Bob anything once he has relied on her and then released her is still to be derived, and the record cannot settle it: the judgment has to read his dependence from the physical history. It meets the question Chapter 8 leaves open, whether his late release was informed and uncoerced.

9.5 A reinforcement law gives gratitude its shape

Alice helped Bob, and Bob is readier to cooperate the next time they meet. Gratitude's work is to carry a received benefit forward into later conduct.

A benefit that changes nothing about how Bob meets the next encounter leaves no trace in the relation; gratitude is the change it leaves.

The catalogue's response law gives such a change its shape, though not yet its object: as written it is reinforcement in general. It tracks a single number, Bob's cooperation propensity p, his readiness toward anyone, within

$$0 \le p \le 1,$$

and it moves when a trigger reports that some virtuous act occurred, whoever performed it. On that trigger, update

$$p' = p + \frac{1-p}{\phi}.$$

Without the trigger, leave p unchanged. The rate $1/\phi$ is the catalogue's choice.

The increment is a fraction of the remaining distance to full cooperation, so it is nonnegative and cannot carry p beyond one. From $p = 0$, one triggered update gives $1/\phi \approx 0.618$ and a second gives about 0.854.

Proposition 9.2 (Repeated gratitude has a geometric response). *After n triggered updates,*

$$p_n = 1 - (1 - p_0)\left(1 - \frac{1}{\phi}\right)^n.$$

For $p_0 \in [0,1]$, every iterate remains in that interval, the sequence is nondecreasing, and it converges to one.

Proof. Set $\lambda = 1 - 1/\phi$. Since $\phi > 1$, $0 < \lambda < 1$. Rearranging the update gives

$$1 - p' = \lambda(1 - p).$$

Induction gives $1 - p_n = \lambda^n(1 - p_0)$. This quantity is nonnegative, no larger than $1 - p_0$, and tends to zero. The stated bounds, monotonicity and limit follow. □

Each recognized kindness closes the same fraction of the remaining gap, so the response shrinks as the gap closes: Bob's readiness rises quickly, then settles toward full cooperation without passing it.

9.6 A converter that carries out the shared sacrifice

The Sacrifice section rests one claim on this construction: that sharing a burden can be carried out as a single physical operation. The claim matters because Sacrifice is something Alice does: if no physical process could carry the move from $(1, r^2)$ to (r, r), it would be a pair of numbers we assign to two bonds, with nothing for her to select or to pay for. The converter supplies one process that performs it: a single unitary evolution moves both comparisons at once, a retained control can select it or leave everything idle, and its debit from a work store is computed from the same dynamics. The comparisons are built as functions of one population, so moving it moves both; what the construction shows is that one controlled pulse moves it to the right place, at a computed cost to the store.

Take three cells: a donor cell, a recipient cell and a one-cell work store. Each is a copy of the eight-entry carrier of Chapter 2, the same eight (Section 2.2), split by a fixed frame, the orthonormal basis below, into an empty and an occupied sector of four dimensions: an occupation label zero or one together with a four-valued payload index j, with basis

$$|0, j\rangle,\ |1, j\rangle, \qquad j \in \{0, 1, 2, 3\}.$$

The two sector embeddings V_0, V_1 take the payload basis $|j_1, j_2, j_3\rangle$ into the three-cell carrier by

$$V_0|j_1, j_2, j_3\rangle = |1, j_1\rangle \otimes |0, j_2\rangle \otimes |0, j_3\rangle,$$
$$V_1|j_1, j_2, j_3\rangle = |0, j_1\rangle \otimes |1, j_2\rangle \otimes |1, j_3\rangle.$$

Thus $V_0^\dagger V_0 = V_1^\dagger V_1 = I$ and $V_0^\dagger V_1 = 0$: the embeddings preserve every payload coordinate and have orthogonal images. The labels mark sectors: total occupation is one on the first and two on the second, the extra unit being the store's. With the occupation observable $\hat{n}|\epsilon, j\rangle = \epsilon|\epsilon, j\rangle$, the store observable is $W = I \otimes I \otimes \hat{n}$, so $WV_0 = 0$ and $WV_1 = V_1$: the work store has occupation zero on the first sector and one on the second.

With $F = V_1 V_0^\dagger$, the generator and pulse are

$$G = F - F^\dagger, \qquad U = e^G.$$

Since $G^\dagger = -G$, the pulse is unitary. Direct multiplication gives $GV_0 = V_1$ and $GV_1 = -V_0$, so the even and odd powers in the exponential series

give the exact action

$$UV_0\xi = (\cos 1)V_0\xi + (\sin 1)V_1\xi, \qquad UV_1\zeta = -(\sin 1)V_0\zeta + (\cos 1)V_1\zeta.$$

On the orthogonal complement of both images, $G = 0$ and $U = I$: unused sectors remain present and unchanged.

Take normalized payloads ξ, ζ and prepare an incoherent mixture of $V_0\xi$ and $V_1\zeta$ with probabilities μ, ν. Put

$$\eta = \sin^2 1, \qquad r = \frac{\eta}{1-\eta}, \qquad \mu = \frac{1}{1+r^2}, \qquad \nu = \frac{r^2}{1+r^2}.$$

Here $\eta > 1/2$: the angle 2 lies between $\pi/2$ and π, so $\cos 2 = 1 - 2\sin^2 1 < 0$. Also $\eta < 1$, hence $r > 1$, $0 < \mu < \nu$ and $\mu + \nu = 1$. Numerically, $\eta \approx 0.708$, $r \approx 2.43$, $\mu \approx 0.145$ and $\nu \approx 0.855$. The pulse turns by one radian, which fixes $\eta = \sin^2 1$ and so the shared multiplier $r \approx 2.43$: the converter realizes this one sharing, not the case $r = 2$ of Table 5.1. One multiplier is enough for what the construction shows, that a physical operation can carry the move.

Let X be the actual population of the reverse sector, the image of V_1, where the store is full; the store's mean occupation is therefore X itself. Identify Alice's internal comparison with ν/X and Bob's with X/μ. Both depend on the same physical X: lowering X raises Alice's comparison and lowers Bob's. This identification of the device with the people takes the two comparisons to be their internal bonds. The preparation is tuned: μ and ν are chosen from η so that one pulse lands exactly on equal multipliers, and the converter does not search for that point. The endpoint depends on X alone, so any process that moved X from ν to $r\mu$ would reach it.

Theorem 9.3 (One controlled population realizes the shared sacrifice). *For this preparation and this identification of the two comparisons, one enabled pulse changes the two multipliers from* $(1, r^2)$ *to* (r, r). *It reduces the work store's mean occupation by the positive amount*

$$D = \eta(\nu - \mu) > 0.$$

A retained two-state control can select this pulse or leave the entire converter unchanged.

Proof. Initially $X = \nu$, so the comparison pair is $(\nu/\nu, \nu/\mu) = (1, r^2)$. Orthogonality of the two sectors gives the terminal reverse population

$$X' = \eta\mu + (1-\eta)\nu.$$

Substitute $\nu = r^2\mu$ and $r = \eta/(1-\eta)$. The expression becomes $r\mu$. Therefore

$$\left(\frac{\nu}{X'}, \frac{X'}{\mu}\right) = (r, r).$$

For every positive population X, the product is $(\nu/X)(X/\mu) = \nu/\mu = r^2$. Thus the endpoint is precisely the opposite-strain action already calculated on the two original bonds.

The store's mean occupation is initially ν and finally X'. Their difference is

$$X' - \nu = \eta(\mu - \nu) = -D.$$

Since $\eta > 0$ and $\nu > \mu$, the debit is positive. This is an actual difference of the retained store's occupation.

On the converter tensored with a two-state control, choose the block generator

$$\widetilde{G} = 0 \oplus G.$$

Its exponential $\widetilde{U}$ is $I \oplus U$. Consequently a state in the idle control sector is unchanged, while a state in the enabled sector undergoes U and retains the same control state. Both blocks are unitary. Write $\iota_1\psi = \psi \otimes |1\rangle$ for inclusion in the enabled sector. For every converter observable O,

$$\langle \widetilde{U}\iota_1\psi, (O \otimes I)\widetilde{U}\iota_1\psi\rangle = \langle U\psi, OU\psi\rangle,$$

because $\iota_1^\dagger\iota_1 = I$ and $\widetilde{U}\iota_1 = \iota_1 U$. Apply this identity to the two component states and add with weights μ, ν. The enabled mixture has exactly the population and store occupation just calculated. The terminal coherences remain in the state; no incoherent terminal preparation is assumed. □

One enabled pulse thus moves both comparisons at once, is selected by a control that can leave everything idle, and takes $D \approx 0.502$ from the store. The debit and the two comparisons are functions of the same population: the debit is its fall, $\nu - X'$, and the comparisons are its ratios to the two preparation weights. The event needs both, counted differently: the costs of the comparisons say what each person bears, the debit what the device spent.

9.7 What each kind of help still needs

Compassion's work on a shared ledger is the care of Chapter 6; the others still need a connection to the people they concern that remains to be derived. The converter performs Sacrifice once its two comparisons are shown to be Alice's and Bob's internal bonds, its enabling control to be hers alone, and her informed willingness and permission to come from her own history. Forgiveness needs its authority to release, here taken from the exchange rules, derived from the creditor's agency and consent. Gratitude needs its trigger tied to the actual act and its author, and Bob's readiness tied to his later conduct toward that person; those two connections would make the rule gratitude, beyond reinforcement in general.

Giving, accepting a burden, releasing a claim and reinforcing cooperation can support the same relationship through different mechanisms. Keeping those mechanisms visible lets us ask the right practical questions: what was supplied, what was paid, what remains due, and what can the participants do next?

Chapter 10

Choosing under limits

Alice knows that Bob would be better off fed. That knowledge does not yet tell her what to do. She needs an action that is actually available, enough reserve to perform it, and information that separates its consequences from those of the alternatives.

Four virtues answer different parts of this problem. Wisdom compares the possibilities on offer, Prudence counts the imbalance a net figure hides, Temperance limits what one step may take from a reserve, and Courage carries a justified action through when it costs the person who performs it. For Wisdom, Prudence and Temperance we take the catalogue's rule (Chapter 5), and for Courage the pieces a principle would connect; for each we prove what it achieves and say which of its numbers a condition fixes and which are chosen.

10.1 Wisdom compares the possibilities on a given menu

Wisdom's work is to choose, among the actions actually available, the one whose consequences are best. That takes a menu of what can really be done, a score that carries the consequences for everyone affected, and a comparison that finds the best-scoring entry. Only the comparison is proved here; the menu and the score are still to be derived. The catalogue's score, one participant's absolute skew, shows why the score matters: it cannot separate Alice's first two options. Bringing the meal and staying away both leave every bond balanced (Chapter 7), so every skew is a sum of terms $\log 1 = 0$ and both candidates score zero. What separates them is Bob's reserve: $1/2$ after the meal, 0 after her absence. To compare these alternatives in the positive-reserve state type below, use Alice's state in each outcome; her reserve remains positive in both.

Bob's exhausted state belongs to the nonnegative extension of Chapter 4. His need is precisely what Alice's skew score omits.

A candidate state s contains a bond ledger, a positive reserve, a participant identity and a time. Its skew $\sigma(s)$ is that participant's signed net logarithmic imbalance on the ledger. The catalogue's rule assigns the score

$$W(s) = \frac{|\sigma(s)|}{1 + \phi},$$

whose common factor $1/(1 + \phi)$ rescales every candidate alike and so changes no choice. For a nonempty ordered list, begin with the first candidate. Scan the remaining candidates in order, replacing the current choice only when the next has a strictly smaller score. If the list is empty, return a fallback.

The scan finds a minimum: for a nonempty candidate list, the selected state belongs to the list and has absolute skew no larger than that of any candidate. Two facts hold throughout the scan: the current choice is one of the candidates already scanned, and its score is no larger than the score of any of them. Both hold at the first candidate. Replacing the current choice at a lower score preserves them, and so does retaining it otherwise. Induction establishes both facts for the complete list. Finally, $1/(1 + \phi) > 0$, so multiplying every absolute skew by it preserves their ordering, and minimum score means minimum absolute skew.

10.2 Prudence counts the imbalance a net figure hides

Prudence's work is to count what a net figure hides. The catalogue's rule has no probabilities, of injury or anything else: it counts the imbalance of the bonds as they are. A treatment of uncertainty about consequences is still to be derived, with the rule's penalty and weight.

Opposite log-flows cancel in a participant's skew. One bond from Alice at multiplier 4 and one bond to her at multiplier 4 give skew $\log 4 - \log 4 = 0$, as in perfect balance, while both bonds sit far from balance. Prudence adds an imbalance penalty to the comparison and keeps a fallback, so a plan that looks balanced on the net figure can still lose to one whose bonds sit nearer balance.

For positive bond multipliers x_b, define

$$D(s) = \sum_{b \in L_s} (x_b - 1)^2,$$

where L_s is the finite set of active bonds in the candidate's ledger. Every summand is nonnegative, so $D(s) \geq 0$. The two bonds at multiplier 4 give $D = 18$; the balanced ledger gives $D = 0$.

Why not penalize imbalance by the recognition cost itself? The catalogue's rule does not, and no derived result chooses between them. The two differ: since $J(x) = (x-1)^2/(2x)$, each summand is $(x_b - 1)^2 = 2x_b J(x_b)$, and the extra factor x_b weights a multiplier above one more heavily than its reciprocal. A bond at 4 scores 9, a bond at $1/4$ scores $9/16$, though $J(4) = J(1/4) = 9/8$.

For a chosen weight $\lambda \geq 0$, the prudent score is

$$P_\lambda(s) = |\sigma(s)| + \lambda D(s).$$

Begin with the fallback state, then scan the candidate list, replacing the current choice only at a strict improvement.

Proposition 10.1 (Prudent comparison includes the fallback). *The selected state is the fallback or a listed candidate. Its score is no larger than the score of any state in that combined set.*

Proof. Use the two invariants of Wisdom's scan: membership among the states considered so far, and a score no greater than theirs. Here the initial state is the fallback. Each comparison preserves both invariants. After the last candidate, they apply to the fallback and the whole list. □

The fallback changes the decision. Given a nonempty list, Wisdom always chooses from the list; Prudence keeps the fallback whenever every listed candidate scores worse. At $\lambda = 0$ the prudent score is the absolute skew, and the selected absolute skew is the smaller of the fallback's and the Wisdom minimum.

10.2.1 What a positive penalty weight can establish

Take two candidates, *steady* and *uneven*, with equal skew and with steady strictly smaller in D. On the menu uneven followed by steady, for $\lambda > 0$, steady has the strictly smaller prudent score. If it also strictly beats

the fallback, the procedure selects steady. The last condition matters: preferring one proposal to another shows nothing about the fallback.

Prudence can also accept more net imbalance in exchange for a smaller D. For a steady and an uneven candidate, the exact compensation condition is

$$\lambda(D(\text{uneven}) - D(\text{steady})) \geq |\sigma(\text{steady})| - |\sigma(\text{uneven})|.$$

For the menu steady followed by uneven, this inequality and a strict improvement over the fallback make the procedure select steady. Rearranging the displayed inequality gives

$$P_\lambda(\text{steady}) \leq P_\lambda(\text{uneven}),$$

and the strict replacement rule retains steady, including at equality.

10.3 Temperance limits what one step may take

Temperance's work is to limit what a single step may take from a reserve, so that the reserve survives to support the next. An expenditure that can be scaled down is capped below the whole reserve; a job that cannot is admitted only when the whole of it can be done and paid for (Section 10.4).

Fix a *commitment* K, the amount committed to the work in view, and call a transition from reserve E to reserve E' temperate when

$$|E' - E| \leq \frac{K}{\phi}.$$

The commitment is a parameter of the condition, separate from the reserve the work draws on: in the construction below it is 10, while the reserve may be 20 or 2.

Unlike the catalogue's other golden-ratio constants, this ϕ is fixed by one condition: that the commitment is divided self-similarly. Divide K into a cap K/κ for one step and what remains, $K - K/\kappa$, and ask that the whole be to the cap as the cap is to what remains. Then what remains is K/κ^2, and since the cap and the remainder make up the whole,

$$\frac{1}{\kappa} + \frac{1}{\kappa^2} = 1.$$

Proposition 10.2 (The self-similar cap is golden). *If* $\kappa > 1$ *satisfies* $1/\kappa + 1/\kappa^2 = 1$, *then* $\kappa = \phi$.

Proof. Multiplication by κ^2 gives $\kappa^2 = \kappa + 1$. Since ϕ satisfies the same equation, subtraction gives

$$(\kappa - \phi)(\kappa + \phi - 1) = 0.$$

The second factor is positive, so $\kappa = \phi$. □

A self-similar division is the golden section, so the proposition's content lies in its condition. Granted it, the whole, the cap and the remainder are three successive levels K, K/ϕ and K/ϕ^2, the first the sum of the other two, as in the golden scaling of Chapter 2. Nothing yet derives the condition: a law of the reserve showing that a commitment divides between successive levels of that kind would.

The bound alone does not protect the reserve: a debit of 2 from a reserve of 2 meets it for commitment 10, since $2 \leq 10/\phi \approx 6.18$, and leaves nothing. One construction adds a second limit, on the present reserve. Take commitment 10 and a requested expenditure 6, within the cap $10/\phi$, and never debit more than E/ϕ, so some of the reserve remains however small it is. This second ϕ is a chosen value, not one the self-similar condition fixes; positivity uses only $\phi > 1$. From a positive reserve E, set

$$d = \min\left(6, \frac{E}{\phi}\right), \qquad E' = E - d,$$

and leave the bond ledger and time unchanged. At $E = 20$ the request is granted in full, since $20/\phi \approx 12.36$; then $E' = 14$. At $E = 2$ the reserve limit binds: $d = 2/\phi \approx 1.24$ and $E' \approx 0.76$.

Proposition 10.3 (A capped debit leaves a positive reserve). *For every* $E > 0$, *this construction leaves* $E' > 0$ *and satisfies the temperance bound for commitment* 10.

Proof. The debit is nonnegative and obeys

$$d \leq E/\phi < E.$$

Thus $E - d > 0$. Also $d \leq 6 \leq 10/\phi \approx 6.18$, and

$$|E' - E| = |-d| = d.$$

Hence the transition is temperate. □

10.3.1 Several moderate actions still have a cumulative cost

The cap bounds each step, and only each step: if each of the n transitions of a finite chain is temperate for the same commitment K, then

$$|E_n - E_0| \le n\frac{K}{\phi}.$$

For two transitions the triangle inequality gives

$$|E_2 - E_0| \le |E_2 - E_1| + |E_1 - E_0| \le 2K/\phi,$$

and induction along the chain gives the rest. Continued agency also depends on renewal: under the charged law of Chapter 5 (Table 5.2), renewal short of the charge of the bonds kept exhausts any reserve, however moderate each step (Theorem 5.4).

10.4 Temperance can control a complete job

A feeding job cannot be scaled down: performing the first affordable fraction and calling the job complete would record a meal never delivered. The limit then takes the only form left, the whole job or a wait. The whole input has to be available, the whole work interval allowed, and each person the job draws on left with a positive reserve after paying the renewal that follows.

Return to the care case of Chapter 6, with subscript A for Alice and B for Bob. Only the food units, the work grant and the guard are new. Three distinct food units, numbered $0, 1, 2$, are initially present. The accepted work grant allows Alice at most two active intervals within the four intervals beginning at times $0, 1, 2, 3$; it has no expiry and requires no rest intervals. Bob has no work capacity. A request is either to wait or to feed Bob one specified food unit. A feeding job uses that entire unit and one of Alice's work intervals; it adds one unit to Bob's intake count and records completion of the specified job. Waiting uses neither food nor work. Both operations occupy one interval.

The rest is Chapter 6's environment, in the form the guard reads. Each person carries a response (m, e): the multiplier of their internal bond and their reserve. Initially these are $(1, 2)$ for Alice and $(1, 1/4)$ for Bob. Write

$$J(x) = \frac{x + x^{-1}}{2} - 1, \qquad c(m) = J(2m).$$

A feeding job is that chapter's food job, which changes the reserves before renewal by $q_A = -1/4$ and $q_B = 1/2$; waiting has $q_A = q_B = 0$. The renewal that follows is its paid repair, the care maintenance of Table 5.2, written as one map:

$$R((m,e),q) = \begin{cases} (1, e+q-c(m)), & c(m) \le e+q, \\ (2m, e+q), & c(m) > e+q. \end{cases}$$

An unpaid renewal keeps the carried exposure; it does not record a repair that failed to occur. Every multiplier reached from this opening has $m \ge 1$, and therefore

$$R((m,e),q) = (1,e') \text{ with } e' > 0 \quad \Longleftrightarrow \quad c(m) < e+q. \qquad (*)$$

Indeed, the paid branch leaves exactly $e+q-c(m)$, while the unpaid branch has multiplier $2m \ge 2$ and cannot satisfy the stated conclusion. At the opening $c(1) = J(2) = 1/4$, and a first feed leaves Alice $3/2$ and Bob $1/2$ (Table 1.1).

10.4.1 The guard reads executed history

Let ω be the chronological list of operations actually selected in earlier intervals; a rejected feed appears in ω as a wait. Replaying ω from the opening with the transitions above gives the current state $s(\omega)$. Let N_j be its remaining stock of food unit j, t its time, w its cumulative accepted work use, and (m_A, e_A) and (m_B, e_B) the two carried responses. The whole-job guard for a new request to feed unit j is

$$\begin{aligned} \Gamma(\omega, j) \quad \Longleftrightarrow \quad & N_j \ge 1, \quad t < 4, \quad w < 2, \\ & c(m_A) < e_A - \tfrac{1}{4}, \quad c(m_B) < e_B + \tfrac{1}{2}. \end{aligned}$$

The selector chooses the requested complete feed precisely when $\Gamma(\omega, j)$ holds, and otherwise selects waiting. A request to wait selects waiting. Every condition reads the actual stock, accepted work and carried responses; none is a separate verdict that the action is safe.

Proposition 10.4 (Whole-job criterion). *For every retained history ω and requested food unit j, the guard holds if and only if the complete food recipe is physically enabled, its one interval of Alice's work satisfies the accepted grant, and renewal after that recipe leaves both people with multiplier one and positive reserve.*

Proof. The recipe consumes exactly one specified food unit, has no production output and performs one interval of Alice's work. Since Alice's interval capacity is one and Bob's labor is zero, its input and labor requirements are enabled exactly when that unit is present. The grant has no expiry and no rest intervals, so it requires only that the new work interval end by time four and that cumulative actual work stay at most two; at integer time and work use, these are exactly $t < 4$ and $w < 2$. Finally, apply $(*)$ to the two actual carried responses with the feeding operation's two funding amounts. The response multiplier never falls below one, because paid renewal sets it to one and unpaid renewal doubles it. So the argument applies after every retained history, without substituting the clock for it. □

When admitted, the job withdraws exactly one food unit:

$$N'_j + 1 = N_j.$$

It increments Bob's intake count, Alice's actual work and that job's completion count by one. These are the complete recipe's effects; the stock and intake accounts together conserve the material, and no clipped withdrawal is recorded as completion.

When rejected, the next state is exactly the waiting successor. Time increases by one, Bob's intake is unchanged and each response undergoes $R((m,e),0)$. Breaches already retained stay retained: blocking an unaffordable job does not meet an outstanding need or prevent a missed repair.

Replaying ω and then the selected operation gives exactly the state of the extended history, since a concatenated history executes its parts in succession. Chapter 6 places the two responses on distinct internal bonds of one ledger, where exposure, funding and paid repair each touch only their owner's bond and reserve. Each step is therefore exactly the two individual renewals above; the guard keeps no capacity account of its own.

10.4.2 Food can remain when permission to work has ended

The sequence feed 0, wait, feed 1, wait passes the guard. It is one of the two maintained histories of Figure 6.1, ending with Alice at 1/2 and Bob at 1/4. The feeds at times zero and two satisfy the guard; the requested waits remain waits. After the second feed, food unit 2 is still present,

but Alice's work use is already two. A third feed requested at time three therefore selects wait. Its successor is exactly the fourth checkpoint of the original sequence, with the same food, intake, work and carried responses. Unused food does not renew an exhausted work grant.

The selector also distinguishes histories at the same clock. After feeding unit 0, another request for unit 0 is blocked because that unit has been used. After feeding unit 1 instead, the same request at the same time is admitted: unit 0 remains, one work interval remains, and both renewal charges fit. Time alone cannot make that distinction.

The guard shares no constant with the cap. It belongs to Temperance by its work, done for a step that cannot be scaled: as the reserve limit of the capped debit keeps one divisible expenditure from emptying a reserve, the guard keeps one indivisible job from leaving either person without a positive reserve after the renewal that follows. Section 10.6 builds a physical exchange that makes this selection. The guard's mechanical conditions grant no care authority. The same opening feed can pass them while lacking that authority, which has its own governing requirements (Chapter 7).

10.5 Courage needs an action that can burden its actor

Sometimes the right action costs the person who takes it. Courage's work is to choose such an action when it is the one that exposes the actor, and to complete it. Courage itself, a principle under which the actor selects the exposing action and can complete it under that exposure, has not been derived. Three pieces such a principle would connect are proved. Exposure is exact, as shown below. A beneficial action can be the exposing one: in the shared burden of Section 5.3, Alice's burden rises while Bob's falls by more, and the total falls. And such an action can be executed and paid for: the converter of Section 9.6, granted its identification with the two people, carries out one such move, at the one multiplier its pulse fixes.

Difficulty alone does not make an action good, and an action does not become harmless because its performer feels brave. The ledger says exactly where such an action's recognition cost to its actor can fall: on the actor's own bonds, and nowhere else.

Let S be an action's finite support: the bonds on which it can apply a nonzero logarithmic strain. Let I_i be the bonds incident to participant i. The action changes each multiplier by

$$x'_b = x_b e^{v_b},$$

with $v_b = 0$ outside S. If $S \cap I_i$ is empty, the action leaves participant i's incident recognition cost unchanged: every incident bond lies outside the support, so its strain is zero and its multiplier remains $x_b e^0 = x_b$, and summing the unchanged costs $J(x_b)$ over the same incident bonds gives the same incident cost.

Proposition 10.5 (Exposure is exact). *A participant's incident cost can be raised by some action exactly when the participant has an incident bond. On a balanced ledger, the action that strains one bond incident to the participant by $v \neq 0$, and no other bond, raises their incident cost by exactly $J(e^v) > 0$.*

Proof. If participant i has no incident bond, every action's support misses I_i, and by the observation above C_i is unchanged. Suppose instead that b is incident to i, and write $x_b = e^t$. Strain b alone, by $v = 1$ if $t \geq 0$ and by $v = -1$ if $t < 0$. Only the term for b in C_i changes, since b is counted once, and it goes from $J(e^t) = \cosh t - 1$ to $\cosh(t + v) - 1$. Here $|t + v| = |t| + 1 > |t|$, and cosh is strictly increasing in the absolute value of its argument, so C_i rises. On a balanced ledger every incident bond starts at multiplier 1 and cost 0. Straining b by v changes only that bond, to e^v, so C_i rises from 0 to $J(e^v) = \cosh v - 1$, which is positive for $v \neq 0$. □

Anyone with a relation can be burdened through it, and only someone with no bond is beyond reach. An actor who keeps every action off their own bonds keeps their own burden unchanged, which is the case courage is not about.

10.6 A physical exchange writes the guard's selection

The construction shows that a physical exchange can make the selection of the guard of Section 10.4: it reads a retained control holding the history and the request, writes the selected mode into an action register, and keeps what it read. Treating Temperance's limit as an execution rule rests on it, together with the two connections named at the end of the section.

Over four intervals the retained control is finite: four food-or-wait history cells, a length from zero to four, and the new food-or-wait request. Only the history prefix of the indicated length is read. Let χ denote a full control and $\mathcal{X}$ the finite set of such controls. The state space is $\ell^2(\mathcal{X} \times \{0, \dots, 7\}^2)$, with two action registers of eight modes each. The eight is the choice of the writer of Chapter 7, separate from the carrier's count; the exchange reuses that writer's registers. Here mode zero means wait and modes $1, 2, 3$ mean the complete feeds of food units $0, 1, 2$; any finite mode set containing these four would serve. Prepare the selected-action register at zero and an alternative register at the mode of the requested job.

Define X on the whole space to exchange the two action registers when the request is a feed and $\Gamma(\omega, j)$ holds, and to leave them alone otherwise. In both cases it retains χ. Hence $X^2 = I$, and X is a self-adjoint unitary permutation. After the exchange, the selected register holds exactly the selector's choice: the entire requested feed, or wait. The retained old control and the written output together record ω followed by the operation actually selected, including wait after a rejected request.

This is the conditional exchange of Chapter 7, with the control predicate "the request is a feed and the guard holds". As there, it permutes basis states, the reversible form of a classical interlock, so its logic is classical; what the construction adds is an explicit evolution whose properties are proved. With X in place of that exchange, the exact finite-time write of Proposition 7.6 writes the selected mode at $T = \pi\hbar/(2g)$ with amplitude one, retains the control and conserves the holding energy.

The exchange writes the selected mode only. It does not withdraw the food, perform Alice's work or pay the renewal, and it does not rewrite the four history cells for a later interval. The interlock still needs two physical connections: authentication of its input, prepared from the actual prior outputs, and a coupling of the written mode to the operation it selects.

10.7 What the four rules decide

Wisdom's minimum, Prudence's penalty, Temperance's cap and whole-job guard, and the exact reach of exposure let us calculate, for Alice's care of Bob, which requested meal can be carried out and paid for at each interval, and why a net skew cannot tell her meal from her absence. Bringing the menu, the penalty, the cap and an accepted exposure into

one governed decision is the step not yet derived. Chapter 11 follows Alice through time instead, where waiting, correction, hope and search decide whether an action can be sustained.

Chapter 11

Patience, humility, hope and creativity

Alice can choose the right next action and still fail to sustain it. The information she needs may arrive late. Her earlier assessment may be wrong. A workable future may need a sequence of actions whose costs arrive before their benefits. Sometimes every available alternative is inadequate and a new one has to be found.

These are physical problems. Waiting consumes reserves. Learning changes records and the actions they support. Continuing a project requires an actual path through successive feasible states. Searching for another path uses time, attention and material resources. Patience, Humility, Hope and Creativity concern these conditions of continued action, each connects a present decision to the conditions under which action can continue.

11.1 Patience and a complete observation

Patience's work is to let the information a decision needs arrive while keeping everyone who is waiting maintained. A defensible delay must therefore state what information the interval supplies, why that information matters and how the participants remain viable while it arrives. Each half has a cost if neglected: acting early acts on part of the picture, and waiting is not free.

Waiting has its own cost because exposure and the care maintenance continue (Table 5.2). If Alice stays away through the first care interval, Bob's reserve falls from 1/4 to 0 (Table 1.1), and postponing the next decision leaves him unable to pay the next repair.

Which process a given decision waits on, and that process's period, is the information horizon still to be derived. What can be computed

exactly is how long a complete observation of a given process takes. The catalogue's rule waits eight ticks, the length for one kind of process, and we grant that the decision waits on that kind: a process whose state at each tick is one configuration of three binary coordinates, and which shows all eight configurations in every cycle. Eight is the same count as the carrier's entries (Section 2.2): three binary coordinates have 2^3 joint states. Waiting for the whole cycle is then a complete observation. The count of configurations fixes the minimum possible observation length; a cycle that repeats configurations can take longer.

Proposition 11.1 (Eight ticks show three binary coordinates). *Suppose a process shows one configuration of three binary coordinates at each of T ticks, and every configuration appears at least once. Then $T \geq 8$. A pass of exactly eight ticks can show every configuration.*

Proof. There are $2^3 = 8$ configurations. For each configuration choose one tick at which it is shown. A tick shows only one configuration, so distinct configurations receive distinct ticks, and there are at least eight ticks. Conversely, listing the eight configurations in any order, one per tick, gives a pass of length eight. □

For another periodic process the complete window is set by that process's own period. In the eight-tick rule below, exact period eight is a separate assumption.

Time is measured in natural-number ticks. Let $s(t)$ be the observed state at tick t. An observation beginning at time t records the ordered list

$$W(t) = (s(t), s(t+1), \dots, s(t+7)).$$

If the last relevant event occurred at t_0 and an action that waits on this process is executed at $t_1 \geq t_0$, the elapsed wait is $w = t_1 - t_0$. The action is patient when $w \geq 8$, and its shortfall is

$$d(w) = 8 - \min(8, w).$$

A wait of 5 ticks falls short by $d(5) = 3$; a wait of 11 has $d(11) = 0$. The shortfall counts missing ticks, not harm.

Waiting further whole periods shows a decision nothing new. Suppose the observed stream is eight-periodic:

$$s(k+8) = s(k) \qquad \text{for every } k \geq 0.$$

If $t_1 = t_0 + 8n$ for a positive integer n, then $W(t_1) = W(t_0)$. Indeed, applying the periodicity identity repeatedly, induction on n gives $s(k + 8n) = s(k)$ for every k; taking $k = t_0 + j$ for each $j \in \{0, \dots, 7\}$ makes the jth entries of the two observations equal, so the ordered lists are equal.

A decision that waits on an eight-periodic process of three binary coordinates that shows all eight configurations in every cycle therefore needs at least eight ticks of observation, and eight suffice; the funded care calculations of Chapter 6 show when a schedule keeps the waiting participants maintained. Neither half of Patience's work makes indefinite postponement a virtue.

11.2 Humility and correction toward evidence

Humility's work is to correct one's own record toward evidence, and the correction is only as good as the evidence. Alice holds an assessment, and another observer or a calibrated record disagrees with it. The catalogue's rule moves her assessment halfway toward the evidence, so repeated correction converges to the value it is given. The rate one half is a parameter of the rule.

Let a be her assessment and y an external assessment. The correction takes their midpoint:

$$a' = a + \frac{y - a}{2} = \frac{a + y}{2}.$$

For the following result, y is a fixed real number. Starting from $a_0 = 0$ with $y = 8$, repeated correction gives 4, 6, 7, 7.5, and the distance to y halves each time.

Proposition 11.2 (Repeated midpoint correction). *Let a_0 be an initial assessment and define*

$$a_{n+1} = \frac{a_n + y}{2}.$$

Then

$$a_n = y + \frac{a_0 - y}{2^n}, \qquad |a_n - y| = \frac{|a_0 - y|}{2^n}.$$

Consequently a_n converges to y. Every step strictly reduces the discrepancy while that discrepancy is nonzero.

Proof. Subtracting y from the update gives $a_{n+1} - y = (a_n - y)/2$. Starting with $n = 0$, induction yields the displayed closed form. Taking abso-

lute values gives the error formula. Division by two strictly decreases a positive error, and 2^{-n} tends to zero. □

The proposition establishes accuracy relative to the target it is given. Whether that target is the relevant truth depends on evidence about the source of the feedback: repetition brings an agent just as close to a mistaken assessment.

Chapter 8 gave a demanding case: a physical agreement record can be made truthful even when the immediate recognition cost rises. Humility therefore cannot be a rule to accept whichever message makes the present state look cheaper. Correction concerns what happened, who authored it and what follows from the actual record.

11.2.1 One correction on the same relationships

An assessment is distinct from the relation it assesses. Correcting Alice's assessment changes her record and no one else's. The same midpoint step enacted on a coordinate that shared relations carry, such as Alice's net log-flow, is a different act, and it can be computed exactly. Carried out on two oppositely directed bonds between Alice and Bob, it moves her net log-flow halfway to the evidence, gives Bob's the exact opposite change and leaves everyone else as they were. On shared bonds the step is Justice's execution: a change in two people's relation, with a payment the step leaves open, and only as good as the evidence it moves toward. It would correct Alice's own assessment only if the physical register that holds that assessment were her skew, and that identification has not been derived.

Let $b_1 : i \to j$ and $b_2 : j \to i$ be distinct active bonds between distinct participants; think of i as Alice and j as Bob. Their positive multipliers are x_1 and x_2. Every other bond keeps its endpoints and its multiplier. The coordinate to be corrected is the actual net log-flow

$$\sigma_i = \sum_{b:\,\mathrm{src}(b)=i} \log x_b - \sum_{b:\,\mathrm{dst}(b)=i} \log x_b.$$

The sums range over the active bonds and may include relationships besides b_1 and b_2. Given a real feedback value f for that coordinate, put

$$\delta = \frac{f - \sigma_i}{2},$$

half the discrepancy. An identified actor specifies the action whose log-strains are $\delta/2$ on b_1, $-\delta/2$ on b_2, and zero on every other bond. Thus

$$x_1' = x_1 e^{\delta/2}, \qquad x_2' = x_2 e^{-\delta/2}.$$

These are the half-strains of Justice's conserving correction (Theorem 5.2) with increment δ. The actor's identity is recorded separately from i and j, and these formulas impose no equality between them. If $\sigma_i = 0$ and $f = 2$, then $\delta = 1$: Alice's coordinate moves halfway, to 1, and Bob's falls by 1.

Proposition 11.3 (A midpoint on the original bonds). *The specified action has the exact effects*

$$\sigma_i' = \frac{\sigma_i + f}{2}, \qquad \sigma_j' = \sigma_j - \frac{f - \sigma_i}{2}.$$

It preserves the active bonds and all their endpoints, the product $x_1 x_2$, and every multiplier outside the selected pair. Every participant other than i and j retains both net log-flow and incident recognition cost. Moreover,

$$|\sigma_i' - f| = \frac{|\sigma_i - f|}{2}.$$

Proof. The action is supported on the reciprocal pair: b_1 is outgoing for i and b_2 incoming, so σ_i rises by $\delta/2 - (-\delta/2) = \delta$, and its two strains sum to zero. These are therefore the strains of Theorem 5.2 with increment δ: σ_j falls by δ, and every other participant keeps both net log-flow and incident cost. Substituting $\delta = (f - \sigma_i)/2$ gives the two displayed coordinates. The action changes multipliers only, so it retains the bonds and their endpoints; the factors $e^{\delta/2}$ and $e^{-\delta/2}$ multiply to one, preserving the pair product; and every bond outside the pair has zero strain. Finally, subtracting f from the first coordinate gives $\sigma_i' - f = (\sigma_i - f)/2$, which proves the error identity. □

The actor and the payer are recorded separately too. For a payment $c \geq 0$ by payer r, the accompanying bookkeeping is $E_r' = E_r - c$, with every other reserve unchanged. The same bond action and successor ledger can be paired with payment zero or payment one. Which payment the correction requires needs a payment law, with its physical implementation, which has not been derived; neither the midpoint identity nor a change in recognition cost supplies one.

11.3 Hope and an available continuation

Hope's work is to keep action directed at a future that is not yet present without mistaking a description of that future for a way to reach it. Alice works toward a future in which Bob is well. A future becomes available for present deliberation only when a feasible path to it has been shown: a sequence of actions whose costs can be paid as they arrive.

Care makes the distinction concrete. A finite schedule keeps both people maintained to a horizon when the capacity inequalities hold, and their continuation then rests on that sequence and the resources it consumes. When the inequalities fail, describing the same future more optimistically does not make the schedule feasible. From reserves 2 and 1/4 no schedule reaches a fifth checkpoint (Section 6.4). Production, a new source of provision or a change in the undertaking may alter the possibilities; each must enter the physical account.

A projected reserve also differs from an acquired one. A forecast can assign a larger future reserve, but spending it now requires a mechanism that supplies it now. An argument that relies on continued agency must keep this distinction through the intermediate steps, where a participant can run out of reserve before the expected benefit arrives.

The catalogue's rule for Hope makes exactly this mistake. With a reference reserve $E_* = 1$ in the chosen units, it projects

$$E' = E + \max\left\{0, \frac{E_* - E}{\phi}\right\},$$

while keeping the ledger, agent and tick fixed; its reference and rate do not matter to the point. The added term is nonnegative, so $E' > 0$ follows from $E > 0$. Bob's reserve of $1/4$ is projected to $1/4 + 3/(4\phi) \approx 0.71$, but the rule transfers none of the added 0.46 from another participant or store. As a map on states it adds reserve from nothing, which is what spending a forecast would do.

Hope's practical content is the reverse: keep a candidate future and test it for a funded path. Hope can then sustain work whose intermediate actions are costly, because the sequence, rather than each isolated step, carries the achievement. Choosing among several feasible paths, by the evidence and the governing requirements, is a step not yet derived.

Hope requires no pretence that every undertaking succeeds. When a path closes, a truthful account of the closure can release effort for another.

Hope depends on preserving the ability to respond to reality, including the reality of a failed route.

11.4 Creativity adds candidates

Creativity's work is to make further alternatives available when every listed one is inadequate. Choosing well among inadequate options cannot help: a decision rule chooses only among the candidates it is given, and creativity adds candidates. The catalogue's rule is a search, and two things are proved about it, neither of them that its candidates are good: its angles never repeat, and for a positive seed and inputs of nonzero skew the real-valued mixture behind each candidate departs from the weighted mean its angle assigns to the inputs.

A candidate is a state description of the kind the rules of these chapters return: a skew, a reserve, an agent and a tick. The search builds each one from two existing states. A seed sets an angle, the angle sets how the two known skews are mixed, and a cross term moves the mixture off their weighted mean.

The angles come from the golden ratio

$$\phi = \frac{1 + \sqrt{5}}{2};$$

any irrational multiplier would serve, since the proof below uses only irrationality. For a nonnegative integer seed n, set

$$u_n = n\phi - \lfloor n\phi \rfloor, \qquad \theta_n = 2\pi u_n.$$

Here $\lfloor z \rfloor$ is the greatest integer no larger than z, so $0 \le u_n < 1$ and the angle stays within one complete turn. The first seeds give $u_1 \approx 0.618$, $u_2 \approx 0.236$ and $u_3 \approx 0.854$.

Proposition 11.4 (Distinct search angles). *Distinct seeds give distinct u_n and distinct θ_n. In particular, the angle sequence has no positive integer period.*

Proof. The golden ratio is irrational. A rational root μ/ν in lowest terms of $X^2 - X - 1 = 0$ would satisfy $\mu^2 - \mu\nu - \nu^2 = 0$. Hence ν divides μ^2, and since μ and ν share no factor, $\nu = 1$. An integer root would satisfy $\mu(\mu - 1) = 1$, impossible because a product of consecutive integers is even.

Now suppose $u_n = u_{n'}$. Rearranging their definitions gives

$$(n - n')\phi = \lfloor n\phi \rfloor - \lfloor n'\phi \rfloor.$$

If $n \neq n'$, this expresses ϕ as a ratio of integers, a contradiction. Multiplication by $2\pi \neq 0$ preserves distinctness. A positive period would equate the angles at two distinct seeds and is therefore impossible. □

For two skew values σ_1, σ_2 taken from existing states, the search forms the real-valued mixture

$$m_n = \sigma_1 \cos^2 \theta_n + \sigma_2 \sin^2 \theta_n + \sqrt{|\sigma_1 \sigma_2|} \sin(2\theta_n).$$

The first two terms form a weighted mean, since their nonnegative weights sum to one; on their own they would only interpolate between the two known skews. The mixture, cross term included, is a parameter of the search.

Proposition 11.5 (A nonzero departure from the weighted mean). *If $n > 0$ and both input skews are nonzero, then the cross term in m_n is nonzero. Thus m_n differs from its displayed weighted mean.*

Proof. The square-root factor is positive because $|\sigma_1 \sigma_2| > 0$. If $\sin(2\theta_n) = 0$, then $2\theta_n$ is an integer multiple of π. Since $2\theta_n = 4\pi u_n$, this makes u_n a rational number with denominator dividing four. But $n\phi$ is irrational for positive integer n, and subtracting its integer part leaves it irrational. The sine factor therefore cannot vanish. □

The catalogue's rule then rounds the mixture down to the integer $\lfloor m_n \rfloor$ and builds its candidate from that; neither proposition uses the rounding. Distinct angles do not by themselves prove distinct mixtures, and rounding can identify further outputs. An explored direction may also increase harm, violate an undertaking or fail to be physically available.

To become a state, a candidate needs bonds that carry its skew. One such carrier is a two-bond *witness*. For any real target τ, positive reserve E, agent i and tick t, take a bond from i to $i + 1$ with multiplier e^τ and a return bond with multiplier 1; the net logarithmic flow for i is

$$\log(e^\tau) - \log 1 = \tau.$$

Attaching the reserve and tick gives a state with the prescribed skew. The witness touches no original relation: it describes a candidate and enacts

nothing. The construction assigns the candidate the mean of two positive reserves,

$$E' = \frac{E_1 + E_2}{2},$$

which is positive and lies between $\min(E_1, E_2)$ and $\max(E_1, E_2)$, as the mean of two numbers does. The candidate is then a complete state description: the witness for $\lfloor m_n \rfloor$, reserve $(E_1 + E_2)/2$, and the first input's agent and tick.

Turning a candidate into an action requires the checks any action needs: physical realization, participant effects, funding and governing authority. The exploration rule itself, which directions are worth searching, has not been derived.

11.5 In list form, the fourteen virtues do not generate every skew-preserving map

With Patience, Humility, Hope and Creativity, all fourteen virtues of the catalogue have been tested. The catalogue also proposed more: that the fourteen, written as maps on finite lists of participant states, are a complete set of moral operations, so that every transformation keeping a group's total skew is some sequence of them. That proposal is false, and the transformation that refutes it keeps the total right by dropping a participant. A restricted generation question remains open. A morally adequate comparison class must retain the participants, their histories, actual resources and governing requirements, as well as the relevant conserved quantities. Whether the fourteen generate that class, and whether any of them can be built from the others, requires the corresponding specification; preserving list length and total skew alone does not supply it.

In the list form each state carries a ledger, an agent, a positive reserve E, its skew σ and a tick, and the claim is about the fourteen maps of Table 11.1, so its refutation is too. They are not the constructions of the earlier chapters: four are the identity, Love's gives the first entry the larger share $\phi/(1+\phi)$ of the combined reserve, and Hope's is the projection above. The argument uses one property: every one of the fourteen returns as many states as it receives. A sequence of such maps also keeps the number of entries: the empty sequence returns the opening list, and if a

sequence preserves length, applying one further length-preserving map keeps it; induction covers every finite sequence.

Now merge a pair. For an input $[s_1, s_2]$, set

$$\tau = \sigma(s_1) + \sigma(s_2).$$

Construct the two-bond witness for τ, retain the first entry's agent, positive reserve and tick, and return just that state. On all other list lengths, return the input unchanged. The skew sum is preserved: for a pair, the output skew is exactly the sum of the input skews, and in every other case nothing changes.

Take two balanced witness states, Alice's with agent 0 and Bob's with agent 1, each at tick zero with reserve one. Each has multiplier one on both witness bonds and hence zero skew. Their list has length two and skew sum zero. The merge returns one state of skew zero, carrying only Alice's agent.

Proposition 11.6 (The virtues do not generate every skew-preserving map). *In the list realization of Table 11.1, every finite sequence of virtue maps preserves the number of entries. The merge preserves total skew, and no such sequence agrees with it on every list of total skew zero.*

Proof. The first statement is the induction above. The merge preserves total skew by construction. On the list of Alice's and Bob's balanced states, whose total skew is zero, the merge returns one entry while every sequence of virtue maps returns two, so no sequence agrees with the merge even on this one list. □

In list form, then, the fourteen generate a proper part of the skew-preserving maps. The merge is a countermodel and no license. It keeps the total right by dropping Bob, which is the loss Chapter 4 shows a moral description must never make, so the virtues should not produce it: a moral transformation must retain the participant structure, histories and requirements relevant to what actually happens, and Chapter 15 makes this obligation concrete.

No completeness theorem vouches for the catalogue as a whole, so each virtue stands on its own construction, and the standing ranges from uniqueness theorems to rules with chosen parameters (Table 11.2). Each operation has the scope its construction establishes.

Their subject is nevertheless continuous. The same Alice waits, corrects an assessment, searches for another action and tries to continue a life with Bob. Reserves consumed during search are unavailable for care. An inaccurate record can invalidate a chosen action. A useful innovation can expand the feasible alternatives without acquiring authority over its users. These connections are the reason to study the virtues together, while keeping each proof attached to the physical work it actually establishes. Whether one number can compare what Alice knows with what she can spend is the question of Chapter 12.

11.6 What the virtue constructions establish

The fourteen names identify connected kinds of moral work. They do not confer the same proof on every proposed rule. Table 11.2 keeps the exact results together: uniqueness under specified constraints, constructions with proved effects, parameters still supplied, and formulas that fail to perform the work their name suggests. Powers of the golden ratio remain parameters unless the relevant physical conditions fix them, as the stated self-similar division fixes Temperance's cap.

Table 11.2. Results and limitations of the fourteen virtue constructions. Uniqueness fixes the stated endpoint or correction under its conditions; it does not select a physical path. Chosen parameters remain inputs. A failed catalogue rule does not establish the virtue's intended work.

Virtue	Standing	What is proved	Ch.
Love	Uniqueness theorem, by strict convexity	A unique conserving endpoint minimizes total cost; for finitely many internal bonds, separate personal capacity bounds give the clipped common-level minimum	5
Justice	Uniqueness, by linear conditions	The record fixes a correction's content (Chapter 8); keeping the pair's flow fixes its execution	5
Forgiveness	Uniqueness, by one linear condition	After a release, the correction that makes the claim truthful is unique	9
Compassion	Exact construction; chosen constants	On a shared ledger its work is care, with an exact existence condition (Chapter 6). The catalogue's transfer rule leaves the provider a positive reserve; its skew part fails, acting on no actual relation	9

Virtue	Standing	What is proved	Ch.
Sacrifice	Exact construction; the catalogue's rule fails	Moving imbalance onto the helper's own bond raises her burden by less than it lowers the recipient's; a physical device carries out one case, given its identification with the two people. The catalogue's skew rule debits the helper more than anyone receives	5, 9
Gratitude	Chosen parameters	A response law with a chosen rate; its closed form and limit are proved	9
Wisdom	Exact construction; the catalogue's score fails	On any nonempty finite menu the comparison finds a candidate of least absolute skew, a score that cannot tell Alice's meal from her absence; a score that makes the choice wise is still to be derived	10
Prudence	Chosen parameters	An imbalance penalty and its weight, compared against a fallback	10
Temperance	Theorem under a condition	Self-similar division of a commitment fixes the cap at $1/\phi$ of it; an indivisible job is admitted exactly when all of it can be done and both people can pay the next renewal with reserve to spare	10
Courage	Exact construction of its pieces	Anyone with a bond can be burdened through it, and no one else can; a beneficial action can be the one that burdens its actor	10
Patience	Counting bound under a condition	A process through all eight joint states of three binary coordinates needs at least eight ticks to show them; that a decision waits on such a process is granted, a choice separate from the carrier of Chapter 2	11
Humility	Chosen parameters	A midpoint correction at rate one half; on shared bonds it is Justice's execution	11
Hope	Exact construction; the catalogue's map fails	Funded schedules (Chapter 6) decide whether a future can be reached; the catalogue's list map adds reserve from nothing, the error Hope avoids	11
Creativity	Exact construction; chosen mixing rule	Candidates come from angles that never repeat, and from a positive seed and two inputs of nonzero skew the real-valued mixture behind each departs from their weighted mean	11

Virtue	Map on a list of states
Forgiveness, Wisdom, Courage, Patience	return the list unchanged
Love	on a two-entry list, returns two entries, each with the average of the two skews and its own agent and tick; the first receives the fraction $\phi/(1+\phi)$ of the combined reserve and the second the rest; returns every other list unchanged
Justice	pairs each state with one correction entry, here of size zero, and returns one state per pair with the same skew, reserve, agent and tick
Temperance	debits $\min(6, E/\phi)$ from each entry, as in Chapter 10
Gratitude	adds $1/\phi$ to each entry's reserve
Humility	multiplies each entry's E by $1 - \min(1, \lvert\sigma\rvert/5)/\phi$
Hope	adds $\max(0, (1-E)/\phi)$ to each entry's reserve
Prudence	caps each entry's E at the reserve of the state that the prudent comparison of Chapter 10 selects, with the entry itself as fallback, from a fixed three-state menu
Creativity	keeps each entry's skew and sets its reserve to $(E+1)/2$
Compassion	adds $2/\phi^2$ to the reserve of each entry with positive skew
Sacrifice	debits $\min(1, E/\phi)$ from each entry with negative skew

Table 11.1. The fourteen virtues as maps on lists of participant states, the form in which the completeness claim is posed. These maps are not the constructions of Chapters 5, 9 and 10; the argument uses only that each returns as many entries as it receives.

Chapter 12

Information, value and comparison

Alice can carry a meal to Bob only if she knows where the food is, and she can use what she knows only if she has the strength to fetch it. Information and capacity enter the same action through different conditions, and a single number that compares actions has to keep both.

Here is a case where the choice of number decides the answer. The food is in one of two stores, each equally likely on a given morning. On half the mornings, decided by a fair coin, Alice checks, and her record names the right store. On the other mornings her record is blank. Now keep the frequencies of her entries and cut the connection: her record names each store as often as before, but the name has nothing to do with where the food is. In the first arrangement the record finds the food on half the mornings. In the second it never helps.

Three numbers can be attached to Alice's record. The *pooled meter*, a bank of energy detectors, counts how often the record names a store: it reports that probability times log 2. It has two parts to play: as a measure of information it fails the working test, and it is also the physical branch average, over Alice's checked and blank mornings, on which the mutual-information reading relies (Section 12.6). Mutual information, Shannon's measure (12.1), and Rényi's half-order measure R (Section 12.3.2) weigh what the record says about where the food is. Table 12.1 gives their values.

The pooled meter cannot tell the arrangements apart, although only in the first does Alice's record help her bring Bob his meal; by the test of Chapter 1, a number that merges them cannot say whether her record serves her care.

Mutual information and R separate the arrangements and disagree about the first. The cost side of a value for her action, the sum of J over the active bonds, is fixed once each bond's pair is identified (the pair

Alice's record	Pooled meter	Mutual information	Half-order R
Tracks the food	$\frac{1}{2}\log 2$	$\frac{1}{2}\log 2$	$-2\log\frac{1+\sqrt{1/2}}{2}$
Unrelated to the food	$\frac{1}{2}\log 2$	0	0

Table 12.1. Three exact values for Alice's record in the two arrangements. Sections 12.6 and A.7.1 derive them.

identification, Section 3.2). The information side is open. Two exact measures pass the natural conditions on information, two physical operations side with different ones, and the measures assign different worth to the record Alice keeps by checking. That worth is what she sets against the load of the trip. Which number preserves the judgment that matters, and which physical operation delivers it?

12.1 Two contributions to a value

A value that compares actions combines what a participant's record says about its environment with what its relations cost. When Alice decides whether to check the stores, she trades a better record against the load the trip puts on her bonds.

Let Y and E be finite sets of participant and environmental states, such as Alice's record entries and the two stores. Let $p(a,e)$ be their joint probability, with nonnegative entries summing to one. Define the marginals by

$$p_Y(a) = \sum_{e\in E} p(a,e), \qquad p_E(e) = \sum_{a\in Y} p(a,e).$$

The product $p_Y(a)p_E(e)$ is the independent table with these same marginals. Their Shannon mutual information [2] is

$$I(p) = \sum_{\substack{a\in Y,\ e\in E \\ p(a,e)>0}} p(a,e)\log\frac{p(a,e)}{p_Y(a)p_E(e)}. \tag{12.1}$$

Every denominator in the sum is positive, because a positive joint entry contributes positively to both of its marginals. Zero joint entries contribute zero. Logarithms are natural.

Let L be a finite set of active physical bonds, with positive multipliers x_b. Their recognition cost is

$$C(x) = \sum_{b \in L} J(x_b).$$

One candidate for combining information and recognition cost, with a fixed positive conversion scale κ, is

$$V(p, x) = \kappa I(p) - C(x). \tag{12.2}$$

It scores one participant's actions from that participant's record and relations, so it is no total over people, and it leaves some distinctions to the judgment it informs (Section 12.7). The scale κ sets the exchange rate between the two contributions, as a temperature does in a free energy; nothing below fixes it, and the conclusions below about Alice's record hold for every positive κ. The form is adopted. Its split into an information slice and a sum of bond costs follows once the cost part has the per-bond cost J and adds over disjoint bonds, which leaves κI as the candidate slice.

12.2 The cost contribution is fixed bond by bond

The cost side adds no free scale to (12.2). Continuous native reporting forces the cost of one bond (Chapter 2), disjoint additivity makes the total a sum, and an exact calibration at balance fixes the stiffness, which a tolerance would leave free.

Remove the active bonds one at a time: disjoint additivity writes the contribution of a configuration as that of one singleton plus that of the rest, and the empty configuration contributes zero, so one term $J(x_b)$ remains for each active bond. The same step applies to a candidate value whose information slice is its value at empty active support. Once the extracted cost contribution has the per-bond cost J and disjoint additivity, its form is

$$V(p, x) = V(p, \emptyset) - \sum_{b \in L} J(x_b).$$

The information slice $V(p, \emptyset)$ is characterized separately.

The stiffness is the quadratic coefficient at balance. Near balance,

$$J(1 + \varepsilon) = \frac{\varepsilon^2}{2(1 + \varepsilon)} \qquad (1 + \varepsilon > 0),$$

so

$$\frac{J(1+\varepsilon) - \varepsilon^2/2}{\varepsilon^2} = -\frac{\varepsilon}{2(1+\varepsilon)} \longrightarrow 0,$$

and the coefficient is exactly one half. A tolerance, the way a device's accuracy is quoted, would not fix it. The function

$$G(x) = \frac{11}{20}(x-1)^2$$

stays within one,

$$\left| G(1+\varepsilon) - \frac{\varepsilon^2}{2} \right| = \frac{\varepsilon^2}{20} \leq \frac{\varepsilon^2}{10},$$

yet its coefficient is $11/20$. In logarithmic coordinates, $x = e^t$, the limit $(e^t - 1)/t \to 1$ preserves the difference: the second-order curvature of G is $11/10$, where J has one. Only a limit fixes the coefficient, the vanishing relative error

$$\frac{G(1+\varepsilon) - \varepsilon^2/2}{\varepsilon^2} \longrightarrow 0 \qquad (\varepsilon \to 0,\ \varepsilon \neq 0). \tag{12.3}$$

Substituting $\varepsilon = e^t - 1$ carries this condition to the logarithmic coordinate, so the unit holds there too. Calibration fixes only the curvature at balance. The native reporting theorem of Chapter 2 fixes the rest of the function, and Theorem 2.4 reaches it a second way, from the composition law with this calibration [1]. In its own dimensionless units, then, a relation's cost has no adjustable stiffness. (The energy a relation carries has a separate scale, Chapter 3.)

12.3 The information conditions do not select mutual information

To weigh Alice's record we need a number for how much it says about the stores. Natural conditions on such a number leave more than one measure standing, even after a finite-record reporting requirement removes a discontinuous one.

Independent joint records should contribute additively. Mixing two records with the same frequencies should not raise the information above the average. Splitting Alice's record labels into sub-labels that know nothing more about the stores should add no information. For a candidate Φ, the independent-composition and mixture requirements are

$$\Phi(p \otimes q) = \Phi(p) + \Phi(q), \qquad \Phi(tp + (1-t)q) \leq t\Phi(p) + (1-t)\Phi(q).$$

The second inequality applies when p and q have the same marginals and $0 \le t \le 1$. Factorized conditional tables have zero information. A perfect fair binary correlation has the reference value $\log 2$.

For the split, each refined agent label a' has a parent a, and nonnegative weights that do not depend on the store divide the parent's mass among its children:

$$p_{ae} = p_a q^a_e, \qquad p'_{a'e} = \lambda_{a'} p_a q^a_e, \qquad \sum_{a' :\, \mathrm{parent}(a') = a} \lambda_{a'} = 1.$$

Here q^a_e is a normalized environment distribution for parent a, one for every parent, including a parent of zero mass. The normalized conditional table is $\lambda_{a'} q^a_e$, so the finer label has no additional environment dependence within its parent. The condition does not cover an informative refinement, one that acquires a finer record of the environment. Alice's coin schedule is one, and Section 12.6 studies it.

12.3.1 Finite reports exclude a support jump

Alice learns from finitely many mornings, so a measure she can use has to be reportable from finite blocks of records. A measure that asks only which joint cells can occur at all satisfies every condition above, and it jumps.

Define

$$S(p) = -\log \sum_{a,e:\, p(a,e) > 0} p_Y(a) p_E(e).$$

It has the independent-composition, restricted-refinement and same-marginal mixture properties. On a perfectly correlated fair binary pair it gives $\log 2$. On any binary table with all four cells positive it gives zero. Adding arbitrarily small uniform noise to the perfect pair therefore drops it from $\log 2$ to zero. Appendix A.1 proves these properties.

Let N_1 be the perfect fair binary pair, N_0 the uniform independent pair, and $N_t = tN_1 + (1-t)N_0$ for $0 \le t \le 1$. A block of m independent joint records has transcript $\ell = (\ell_1, \dots, \ell_m)$. Fix a real score $s(\ell)$ on each possible transcript. Its expected report is

$$M_{m,s}(N_t) = \sum_{\ell} s(\ell) \prod_{j=1}^{m} N_t(\ell_j).$$

Uniform finite-block reporting of a functional Φ on this family means that for every $\varepsilon > 0$ there are a finite m and fixed scores s with $|\Phi(N_t) -$

$M_{m,s}(N_t)| < \varepsilon$ for every $t \in [0,1]$. The block length and scores may depend on the requested accuracy, but stay fixed across the preparations being compared, including the endpoint.

Every fixed-block expectation is continuous in t. Uniform approximation by these reports therefore requires continuity at the endpoint and excludes S (Appendix A.2). A fixed finite block responds continuously to the noise in its source, so it cannot follow a quantity that leaps when the last trace of noise disappears. The condition does not ask every morally relevant distinction to vary continuously: whether Bob's reserve is zero or positive stays a sharp fact.

12.3.2 A continuous alternative remains

Continuity does not finish the selection. A measure from Rényi's family is continuous, calibrated and reportable from finite blocks, and it differs from mutual information on the same records.

Define the affinity of a joint table with its product of marginals by

$$\beta(p) = \sum_{a,e} \sqrt{p(a,e)}\sqrt{p_Y(a)}\sqrt{p_E(e)}, \qquad R(p) = -2\log\beta(p).$$

This is the half-order member of Rényi's family of information measures [3]. Some joint cell is positive, and so are both of its marginals; consequently $\beta(p) > 0$. The affinity multiplies under independent composition, so R adds. The normalized environment-independent subdivision above leaves β unchanged; its conditional tables have $\beta = 1$ and $R = 0$. Concavity of the square root and logarithm gives the same-marginal mixture inequality. The perfect fair binary pair has $\beta = 1/\sqrt{2}$ and $R = \log 2$.

This alternative is continuous as binary noise is removed, and it satisfies the uniform finite-block reporting condition on the family N_t: Appendix A.3 constructs, for each requested accuracy, one fixed score on a finite block that reports a polynomial approximation to $R(N_t)$. Since one fixed score reports any polynomial in t exactly, on this family the reporting condition asks for little beyond continuity.

At the same calibration it still differs from mutual information. At $t = 1/2$, the noisy binary table is

$$N_{1/2} = \begin{pmatrix} 3/8 & 1/8 \\ 1/8 & 3/8 \end{pmatrix}, \qquad p_Y = p_E = (1/2, 1/2).$$

Its two values obey

$$R(N_{1/2}) < \frac{3}{4}\log 3 - \log 2 = I(N_{1/2}).$$

Numerically, $R(N_{1/2}) \approx 0.069$ and $I(N_{1/2}) \approx 0.131$. The strict concavity calculation is in Appendix A.3.

Two calibrated, continuous, reportable measures thus disagree on a noisy binary record, and selecting mutual information needs a further physical condition.

12.4 Comparing a joint record with its independent reference

One physical way to price Alice's record compares it with a reference that has the same frequencies and no connection to the stores. The comparison apparatus of Chapter 2 then reports the ratio $(1-\beta)/\beta$ of a residual energy to a reference energy, where β is the affinity just defined.

Let $p_{ae} \geq 0$ be a probability table on finite sets of agent and environment labels, with $\sum_{a,e} p_{ae} = 1$. Write

$$p_a = \sum_e p_{ae}, \qquad p_e = \sum_a p_{ae}, \qquad q_{ae} = p_a p_e.$$

Both operands use the same labelled carrier (a, e), and the second is built from the first. The apparatus compares signals, so each enters as one amplitude per label pair. It measures quadratic energy, and its reference energy is the cross term of the two operands in each channel (see the proof below). We adopt the encoding in which each cell's energy is its probability, $u_{ae} = \sqrt{p_{ae}}$ and $w_{ae} = \sqrt{q_{ae}}$, so each operand carries total energy one. An amplitude p_{ae} would carry energy p_{ae}^2. The half-order measure follows from this choice, and what the comparison reports below is relative to it. The overlap of the two operands is the affinity

$$\beta(p) = \sum_{a,e} \sqrt{p_{ae}}\sqrt{p_a}\sqrt{p_e}.$$

For two complex amplitudes u, w, the comparison input is

$$f(u, w) = \frac{1}{\sqrt{2}}(u, -u, 0, 0, -w, w, 0, 0),$$

the pulse (2.1) of Chapter 2 with the phased second operand written as w. Let H be the half-turn of that chapter, which shifts the entries by four places, and put $P = (I-H)/2$, $Q = I-P = (I+H)/2$, the complementary orthogonal projections defined there; Qf is the mismatch part.

The apparatus records the energy of the mismatch part. Chapter 2 does this with the join's residual, and any operation whose recorded energy is a fixed positive multiple of $\|Qf\|^2$ gives the same report, because the multiple cancels against the reference (Section 2.3). Here we use one that keeps both parts, a reversible two-port exchange:

$$X(f,f') = (Pf + Qf', Qf + Pf').$$

On a ready second port it gives $X(f,0) = (Pf, Qf)$: the sum and mismatch parts occupy separate ports. Because P and Q are complementary orthogonal projections, X is a self-adjoint involution, which is all the flow of Proposition 7.6 needs. With X in place of the exchange there and a common positive energy coefficient γ, the positive generator $\gamma(I - X)$ enacts X exactly at $\tau = \hbar\pi/(2\gamma)$ on every channel, and the ready-input generator energy is $\langle (f,0), \gamma(I - X)(f,0)\rangle = \gamma\|Qf\|^2$. The recording bank of Appendix A.7 uses the same exchange.

For each label pair, compare u_{ae} with zw_{ae}, where $|z| = 1$. Use the same γ in every channel. Denote the resulting energy record by $D_{ae}(z)$, and define the phase reference by an actual quarter-phase increase,

$$D = \sum_{a,e} D_{ae}(1), \qquad \Delta = \sum_{a,e} (D_{ae}(i) - D_{ae}(1)).$$

The sums pool separate energy records, not the energy of a coherent sum of the labelled inputs, and the references are pooled before division.

Theorem 12.1 (The joint-reference comparison report). *For the amplitudes and comparison operation above,*

$$D = \gamma(1 - \beta(p)), \qquad \Delta = \gamma\beta(p) > 0.$$

A nonnegative additive relative energy meter calibrated to return one when its residual equals its positive reference therefore reports

$$\frac{D}{\Delta} = \frac{1 - \beta(p)}{\beta(p)}.$$

One fixed conversion of reporting units by κ gives $\kappa(1 - \beta(p))/\beta(p)$.

Proof. The four potentially nonzero entries of $Qf(u,w)$ each have magnitude $|u-w|/(2\sqrt{2})$, as in (2.2). Hence

$$\|Qf(u,w)\|^2 = \frac{|u-w|^2}{2}.$$

Substituting the two amplitudes gives

$$D_{ae}(z) = \frac{\gamma}{2}\left(p_{ae} + q_{ae} - 2\sqrt{p_{ae}}\sqrt{q_{ae}}\,\operatorname{Re} z\right).$$

Since $\operatorname{Re} i = 0$ and $\operatorname{Re} 1 = 1$, the measured quarter-phase increase is $\gamma\sqrt{p_{ae}}\sqrt{q_{ae}}$. The product reference is normalized: $\sum_{a,e} q_{ae} = (\sum_a p_a)(\sum_e p_e) = 1$. Summing the two displayed energy expressions therefore gives the asserted D and Δ.

Some cell of p is positive because its total mass is one. Both marginals at that cell are positive, so that cell contributes positively to $\beta(p)$; every other contribution is nonnegative. Consequently $\beta(p) > 0$. Zero-probability cells require no limiting convention and no division by their individual references.

For a fixed positive reference Δ, a response that is additive and nonnegative on nonnegative residuals is linear. Calibration at residual Δ fixes its slope to $1/\Delta$. Its report is therefore D/Δ; the common energy coefficient cancels. The single fixed factor κ then changes reporting units. □

If the joint table already factorizes, $\beta(p) = 1$ and the comparison report vanishes: a record independent of its environment leaves no residual. For a perfectly correlated fair binary pair, the affinity is $\sqrt{1/2}$ and the unscaled report is $(1-\sqrt{1/2})/\sqrt{1/2} = \sqrt{2}-1$. More generally, let $r(p)$ denote the unscaled report. Affinity multiplies under independent composition of records, so the report obeys the exact law

$$r(p \otimes q) = r(p) + r(q) + r(p)r(q).$$

Indeed, $1+r(p) = 1/\beta(p)$, so multiplicativity of affinity gives $1+r(p\otimes q) = (1+r(p))(1+r(q))$. Expanding and subtracting one gives the displayed law.

The physical output is an affinity ratio. Nothing in the construction or calibration applies a logarithm or identifies the report with mutual information. Its reports compose by $r + r' + rr'$; Section 12.5 finds the scale on which they add.

The theorem takes the record and its reference as prepared signals. If one fixed operation, chosen without knowing which record Alice holds, made the reference from copies of her record, the comparison would be an operation on her record alone. A *coherent copy* of a record is its square-root signal. For one fixed exact operation applied to a whole family of records, Appendix A.4 proves three results: none makes the reference from one coherent copy of every record; one fixed pair of reversible operations on two copies makes it exactly for every record that always names the right store; and no fixed operation on finitely many copies makes it exactly for every binary record (Theorem A.6). These results support the comparison apparatus and leave the question for Alice's choice where it was. They say what the comparison that reports R needs besides her record: a general record needs its reference prepared some other way, and its R is reported from the record together with that reference.

12.5 The comparison has one calibrated additive scale

If Alice holds two independent records, the information they give together should be the sum of what each gives. Comparison reports instead compose by $r + r' + rr'$. A continuous calibrated scale that turns this into addition is unique, a fixed multiple of $\log(1 + r)$, and on the perfect binary record it is exactly the half-order measure.

The raw comparison obeys

$$r(p) = \frac{1 - \beta(p)}{\beta(p)}, \qquad r_{12} = r_1 + r_2 + r_1 r_2$$

under independent composition. Write $r \star r' = r + r' + rr'$.

Theorem 12.2 (Calibrated additive representation). *Let $F : [0, \infty) \to \mathbb{R}$ be continuous and satisfy*

$$F(r \star r') = F(r) + F(r') \qquad (r, r' \geq 0).$$

Fix a reference $r_* > 0$ and a calibration $F(r_*) = \nu$. Then

$$F(r) = \nu \frac{\log(1 + r)}{\log(1 + r_*)} \qquad (r \geq 0).$$

This formula also supplies a continuous additive representation with that calibration. The real calibration ν is arbitrary; a positive reporting unit uses $\nu > 0$.

Appendix A.6 gives the proof: the logarithmic coordinate turns $\star$ into addition, and continuity fixes an additive function from its rational values.

The theorem asks for F on the whole half-line $[0, \infty)$. That is the comparison's domain. Every nonnegative ratio is the raw report of some normalized finite record: Alice's record of n stores that copies the right store on a fraction t of mornings, and is unrelated to the food on the rest, attains every report from 0 to $\sqrt{n} - 1$ (Theorem A.7, Appendix A.5). One reporting function serves them all.

Take the fair perfectly correlated binary record: its two nonzero joint probabilities are $p(0,0) = p(1,1) = 1/2$, and both marginals are fair. Its affinity is $\sqrt{1/2}$, so its raw reference report is

$$r_* = \frac{1 - \sqrt{1/2}}{\sqrt{1/2}} > 0.$$

Give this record the reporting value $\log 2$. The representation is then exactly

$$F(r(p)) = -2 \log \beta(p).$$

To see the factor, observe that $1 + r(p) = \beta(p)^{-1}$, and hence $\log(1 + r(p)) = -\log \beta(p)$. For the binary reference, $-2 \log \beta = \log 2$. Its nonzero calibration therefore makes the coefficient of $\log(1 + r)$ equal to 2. The reporting scale is the affinity functional $R = -2 \log \beta$, with its exact normalization.

Read on this scale, the joint-reference comparison reports the measure the information conditions failed to exclude, which falls below mutual information on $N_{1/2}$.

12.6 A controlled refinement separates the measures

Retaining a control flag distinguishes mutual information from the half-order measure. The test is the chain rule of conditional information: the information of a fine record should equal that of a coarser record made from it plus the weighted information within each coarse value. On Alice's flagged record mutual information obeys it, the half-order measure does not, and an assembly of calibrated branch reporters obeys it by total expectation.

The operation is Alice's first arrangement, with the stores as a fair binary environment and the coin as a fair independent control flag. It has

two branches: one keeps a perfect record of the environment, the other keeps none, and the flag is retained in both. What does an apparatus assembled from the two branch reporters report on average?

12.6.1 The flagged record and its coarsening

Let E take the values 0 and 1 with probability $1/2$ each. Independently choose a flag K, also with probabilities $1/2, 1/2$. When $K = 0$, retain E in the agent record. When $K = 1$, retain a blank record. The fine agent record Y has three labels:

$$\begin{aligned} Y &= 0 \quad (K = 0, E = 0), \\ Y &= 1 \quad (K = 0, E = 1), \\ Y &= 2 \quad (K = 1). \end{aligned}$$

Thus the fine joint distribution is

$$p_{YE} = \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \\ 1/4 & 1/4 \end{pmatrix}.$$

Rows are the three agent labels and columns are the two environment labels. The agent marginal is $(1/4, 1/4, 1/2)$ and the environment marginal is $(1/2, 1/2)$.

The coarse record keeps only the flag. Define $\theta(0) = \theta(1) = 0$ and $\theta(2) = 1$. Summing the fine probabilities over each fiber of θ gives

$$p_{KE}(j, e) = \sum_{a:\theta(a)=j} p_{YE}(a, e) = 1/4 \qquad (j, e \in \{0, 1\}).$$

This coarsens the same joint record and leaves the environment as it was; the retained flag alone is independent of the environment.

Conditional on $K = 0$, the normalized joint table is the perfect binary record

$$p_0 = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}.$$

Conditional on $K = 1$, the agent has one possible blank label and the environment remains fair:

$$p_1 = \begin{pmatrix} 1/2 & 1/2 \end{pmatrix}.$$

Both conditioning events have probability $1/2$, and dividing the corresponding rows of the fine table by $1/2$ yields exactly these conditional distributions. The blank label of the second branch can be called 0 when that branch is considered by itself; its fine label 2 retains the branch identity in the assembled record.

12.6.2 Assembled reporters average their branches

Take any calibrated reporter for the perfect-record branch, with output $s_0(a, e)$ on its input record, and any reporter for the blank branch, with output $s_1(0, e)$. Their outputs may be deterministic values or conditional mean outputs of a further output stage. Assemble them by selecting the appropriate reporter with the retained flag. The assembled output is

$$s(a, e) = \begin{cases} s_0(a, e), & a = 0 \text{ or } 1, \\ s_1(0, e), & a = 2. \end{cases}$$

The assembly keeps the branch reporters and their calibrations, and does not recalibrate their outputs after mixing the branches.

For a finite joint table p, write its mean report as

$$M_p(s) = \sum_{a,e} p(a, e)s(a, e).$$

Expanding the four nonzero cells of the fine table proves

$$\begin{aligned} M_{p_{YE}}(s) &= \tfrac{1}{4}s_0(0, 0) + \tfrac{1}{4}s_0(1, 1) + \tfrac{1}{4}s_1(0, 0) + \tfrac{1}{4}s_1(0, 1) \\ &= \tfrac{1}{2}M_{p_0}(s_0) + \tfrac{1}{2}M_{p_1}(s_1). \end{aligned}$$

This is total expectation for the assembly. No mutual-information formula appears in its definition or proof.

Use the reference calibration from before: a perfect fair binary record has information report $\log 2$, and a blank agent record independent of the environment has report zero. If the two devices have these calibrated mean outputs, the assembled device must have mean output

$$M_{p_{YE}}(s) = \tfrac{1}{2} \log 2.$$

The identity holds for every pair of branch reporters with those two means. Constant branch outputs $s_0 = \log 2$ and $s_1 = 0$ are one explicit finite pair of reporters with these means, so the calibration constraints are consistent.

12.6.3 Mutual information obeys the conditional accounting identity

Write $I(Y;E)$ for the mutual information (12.1) of the fine table. Its likelihood ratio is 2 at $(0,0)$ and $(1,1)$. It is 1 at $(2,0)$ and $(2,1)$. The zero cells contribute zero. Consequently

$$I(Y;E) = \tfrac{1}{4}\log 2 + \tfrac{1}{4}\log 2 + \tfrac{1}{4}\log 1 + \tfrac{1}{4}\log 1 = \tfrac{1}{2}\log 2.$$

The coarse flag is independent of the environment, so $I(K;E) = 0$. The conditional perfect branch has information $\log 2$ and the conditional blank branch has information zero. Thus this informative refinement satisfies the ordinary conditional-information accounting identity

$$I(Y;E) = I(K;E) + \tfrac{1}{2}I(Y;E \mid K=0) + \tfrac{1}{2}I(Y;E \mid K=1).$$

Here the conditional symbols mean the mutual information of the two normalized conditional tables just displayed. Mutual information equals the mean report of the assembled apparatus.

12.6.4 The half-order measure gives a different answer

The competing measure is $R = -2\log\beta$ of Section 12.3.2. It has the same reference calibration: $R(p_0) = \log 2$ and $R(p_1) = 0$. It also vanishes on the coarse independent flag/environment table.

For the fine table, each of the first two nonzero cells contributes $\frac{1}{4}\sqrt{1/2}$ to β, and each blank-branch cell contributes $1/4$. Therefore

$$\beta(p_{YE}) = \frac{1+\sqrt{1/2}}{2}, \qquad R(p_{YE}) = -2\log\frac{1+\sqrt{1/2}}{2}.$$

Set $\beta_0 = \beta(p_0) = \sqrt{1/2}$. Then $0 < \beta_0 < 1$ and $\log\beta_0 = -\frac{1}{2}\log 2$. Strict concavity of the logarithm gives

$$\log\frac{\beta_0+1}{2} > \frac{\log\beta_0 + \log 1}{2} = \tfrac{1}{2}\log\beta_0.$$

Multiplying by -2 yields the strict separation

$$R(p_{YE}) < \tfrac{1}{2}\log 2 = R(p_{KE}) + \tfrac{1}{2}R(p_0) + \tfrac{1}{2}R(p_1).$$

Numerically, $R(p_{YE}) \approx 0.317$ against $\frac{1}{2}\log 2 \approx 0.347$. The half-order measure fails the conditional accounting identity in this exact finite case, so it cannot equal the mean report of any controlled assembly of the same two branch reporters with these calibrations.

12.6.5 The pooled meter is the assembly with its calibrations fixed once

The pooled meter of Table 12.1 is the constant-output assembly carried out by a physical exchange. A reversible recording bank has one channel per joint label, and the exchange X of Section 12.4, applied to a single-operand pulse in each channel, deposits energy in proportion to that channel's probability. One energy reference serves every detector, so the pooled report of the selected labels is their probability mass. Selecting the perfect-record labels and calibrating once by $\log 2$, the bank reports $\frac{1}{2}\log 2$ on Alice's flagged record: the branch average, which mutual information equals and R does not. Its calibrations are fixed before any record arrives. On her unconnected record the same labels carry the same mass, so the bank reports $\frac{1}{2}\log 2$ again, while mutual information is zero (Appendix A.7 builds the bank; Section A.7.1 computes both reports).

The two reports show the meter playing both its parts. On the flagged record it is the branch average, which mutual information equals. On the unconnected record its calibrations, fixed before any record arrives, give the same value, so by itself it does not measure information. For Alice's choice, the physical support for mutual information is the average of reporters each calibrated to its branch's actual record, and on her checked record that average equals her record's mutual information.

12.6.6 Two accounting laws for records

Two physical operations now give two answers. The comparison's additive representation is exactly $F(r(p)) = R(p)$, so the strict separation applies to it: independent-composition additivity fixes that scale under its domain, continuity and calibration requirements, and the arithmetic accounting of separately calibrated branch reporters excludes it. The controlled assembly is a discriminator: if information is identified with the mean report retained under this assembly, the half-order measure is excluded and mutual information survives. The accounting identity of the assembled apparatus is derived. The identification is not implied by the earlier conditions, since the half-order measure satisfies all of them, including finite-block reporting on the binary mixture family.

12.7 What a comparison has to retain

An individual value requires individual input. To say whose information and cost are counted, we must identify that participant's state and

incident bonds. When resources or bonds are shared, their joint account must prevent double counting. A global quantity divided by an arbitrary share does not recover missing authorship, experience or who holds a resource.

The test of Chapter 1 finds this limit wherever a number records alike two histories that the judgment must tell apart. The care orderings of Table 6.1 end at equal reserves though only one takes Bob to zero, and in Chapter 8 equal physical effects can come from different authored events, so a comparison of effects cannot say who agreed. Alice's two records show the same thing at small scale: the pooled meter keeps how often her record names a store and loses whether the name is right.

The same limit holds when one number combines Alice's and Bob's values. A common rule transforms each value by a concave function and adds the results, so that a gain to the participant with less counts for more. Such a sum cannot stand in for the members' own accounts, and it meets the question the cost and information sides met: calibrating the rule at one point does not fix it; a global relation does (Appendix A.8). For Chapter 5's conserving repair the choice does not matter: under its fixed-product and separate-capacity conditions, every common strictly increasing concave transformation of negative individual cost selects the same constrained endpoint.

These are requirements on representation. A quantity can serve a precise purpose while leaving other questions to the physical relations it compresses. Recognition cost measures a defined feature of the bonds. Mutual information measures a defined dependence; whether that dependence is a true belief, a competent response or a felt experience concerns how the participant uses and undergoes it. The governing account must retain the participants, actions and histories on which its judgments depend. An objective morality can therefore have several indispensable descriptions of one reality. Their unity comes from their common physical objects and transformations, and their differences state which facts each description preserves.

12.8 Two exact candidates for the worth of Alice's record

Alice decides whether to check the stores by setting the worth of the record she would keep against the load the trip puts on her bonds. The

cost of that load is the sum of J over her active bonds, fixed once each bond's pair is identified (Section 3.2). The worth is κ times the record's information, and each candidate measure of that information is reported by a physical operation. Comparing the record, encoded so that each cell's energy is its probability, with a supplied reference reports R, though no fixed copy operation makes that reference exactly for every binary record. Assembling calibrated branch reporters under a retained flag reports the branch average, which mutual information equals on the flagged record. Whatever the scale κ, a value law gives her unconnected record no information under either measure, and for her checked record holds two exact candidates for its information, $R(p_{YE})$ and the strictly larger $\frac{1}{2}\log 2$, and so two for the worth she sets against the load of the trip, κ times each.

One question about value is therefore still open, and it is exact. When calibrated branch devices are assembled under an independent retained flag, does the information contribution of a physical value law keep their calibrated mean output? A physical derivation of that identification would exclude the half-order measure at the scope of the controlled refinement and fix which candidate is the worth of Alice's checked record; a complete classification would then need a general conditional-information rule for arbitrary physical refinements and a characterization of information under it.

Chapter 13 returns to authored undertakings and their shared resources. A scalar can describe a physical consequence precisely; the governing judgment must also retain the relation that makes the consequence required or attributable to someone.

Part IV

A world shared with others

When undertakings compete for one resource, when the means of action change hands, and when many people share production and institutions, the same ledger and the same authored records determine what every account must retain.

Chapter 13

Governing authority and competing claims

Alice has promised to feed Bob, and Bob has accepted. Later Alice makes Carol the same kind of promise, to be met from the same food, and Carol accepts. Both acceptances are valid: each answers its own proposal and comes from the right person. Alice has one unit of food, one unit of work and a reserve of $3/2$, what four intervals of maintenance left her while the two promises formed. Bob and Carol each hold a reserve of $1/4$, exactly one interval's charge. Table 13.1 follows the meal for Bob through two intervals. Whoever Alice feeds comes through still able to repair. The other ends the second interval with a bond out of balance and nothing left to repair it.

	Alice (A)	Bob (B)	Carol (C)
Opening	$(1, 3/2)$	$(1, 1/4)$	$(1, 1/4)$
After Bob's meal	$(1, 1)$	$(1, 1/2)$	$(1, 0)$
After the next interval	$(1, 3/4)$	$(1, 1/4)$	$(2, 0)$

Table 13.1. One unit of food for two accepted claims. Each pair gives a bond multiplier and the remaining reserve on one shared ledger. Serving Carol instead exchanges the Bob and Carol columns. Section 13.6 derives these values.

Which promise governs the one unit? Chapter 7 answered the question for a single undertaking: the accepted record of Alice's promise, written into the coordinate that selects her first action, fixes one governing reading, under which feeding Bob performs the undertaking and staying away violates it (Section 7.6). A second promise on the same food changes the physical question. So do quieter changes. The controller that carries out Alice's actions can be replaced, the record can be copied onto

new participant labels, and Carol can be added with an undertaking of her own before any conflict arises. Each change asks what continues, what can be altered, and whose act can alter it.

In the conflict, neither undertaking has been written into a controller. Both acceptances are recorded, and no write couples either of them to the coordinate that selects Alice's meal. The identification of Chapter 7, that what Alice owes is what an installed record requires of her action (Section 7.7), therefore applies to neither claim yet. The derivation has not said what she owes Bob or Carol here, and it has not said that she owes either of them nothing. What it can fix is what any governing answer must respect: what each meal leaves, which event ended joint performance, what each claim's local repair requires and what a correct release changes. The results below fix those facts and leave the governing question open. None of them selects the promise, and that selection has not yet been derived.

Four cases carry the numbers (Table 13.2). Each pair in them is a participant's *response*, as in Chapter 7: the multiplier of their internal care bond and their *reserve*, the amount on the care ledger that pays every repair. Every comparison between people below (who can continue, what each meal leaves, what restores each claim) is read from these pairs. Food and work enter the reserve through the care calibration of Equation (9.1): a unit of intake adds 1/2 and a unit of work costs 1/4. Table 13.3 lists the accounts and how they convert.

Case	Section	Alice	Bob	Carol	Food
The single undertaking	13.1	(1, 2)	(1, 1/4)		3
Carol's later undertaking	13.4	(1, 2)	(1, 1/4)	(1, 2)	3
One unit, two claims	13.6	(1, 3/2)	(1, 1/4)	(1, 1/4)	1
How the two claims formed	13.8	(1, 5/2)	(1, 3/2)	(1, 3/2)	1

Table 13.2. The four numerical cases. Each pair is an opening response, a multiplier and a reserve; the last column is the food stock in whole units. The formation history ends at the opening of the one-unit case.

Account	What it measures	Unit	Into the reserve
Reserve	what a participant can spend on repair; a paid repair subtracts the cost J of the exposed multiplier	units of J	
Food stock and intake	food held, and food a participant eats	units of food	a unit of intake adds 1/2
Work capacity	labor a participant can perform in one interval	units of labor	a unit of work costs 1/4
Material reserve	the formation history's check that each operation is materially possible: intake worth 2, labor costing 1, maintenance and spoilage zero, opening 6	whole units	none

Table 13.3. The accounts of the chapter. Only intake and work convert into the reserve, at the rates of Equation (9.1): one quarter of the material reserve's values for them, 2 for a unit of intake and 1 for a unit of labor, so in the formation history, where maintenance is zero, each operation's funding is one quarter of its change in the material reserve. The material reserve's level never enters the reserve, and no conversion between the two levels has been derived. The constructions of Appendix B keep accounts of their own (Table B.2).

13.1 Faithfulness to the history and the ledger fixes the governing reading

A requirement has a source and a means of performance, and they are different parts of the physical history. The controller that carries out Alice's delivery is a means. It can execute the meal without any authority to release Bob's claim, and better equipment can enlarge what Alice is able to do without choosing between Bob and Carol. Chapter 7 found the governing reading through the installed controller; the same reading is the one complete reading faithful to a comparison built from the source alone, the authored history and the ledger, which are what a changed controller or a second claim acts on. Nothing in this characterization requires the current controller to obey: a controller other than the installed one can choose omission (Section 7.6), and the reading still rejects that execution.

Take *the single undertaking* of Chapter 7 on its own, before Carol appears: Alice opens with reserve 2, Bob with $1/4$. The comparison comes from Bob's reliance. Chapter 7 prevented either authored formation event, Alice's offer or Bob's acceptance, ran the resulting physical history and applied the same next maintenance interval (Section 7.2). Neither intervention consults the controller's preferred outcome. Both leave the two active bond multipliers at one, so they agree on the reference the undertaking displaced. Let $R(a)$ say that the actual continued ledger after action a matches these intervention-derived multipliers on both active bonds.

Call an action legal when it is physically legal in the sense of Chapter 7: it passes the operation, resource and history checks listed there. After the next interval (Proposition 7.3) the legal actions have two outcomes. Omission leaves Alice at $(1, 3/2)$, two repairs down from her 2, and Bob at $(2, 0)$; provision leaves them at $(1, 5/4)$ and $(1, 1/4)$. The reference executions give $(1, 3/2)$ and $(1, 1/4)$. Since R compares multipliers, provision satisfies it and omission fails it.

Use the readings, performances and completeness of Definition 7.4, and add two terms. A reading $\mathcal{R}$ is *sound* for the restoration comparison when every performance satisfies R, and *faithful* to it when its performances are exactly the legal actions satisfying R:

$$\{a : a \text{ performs } \mathcal{R}\} = \{a : a \text{ is legal and } R(a)\}.$$

Completeness says the reading leaves out nothing its own performances share. Faithfulness says those performances are exactly the restorations. Together they determine the entire reading. If G belongs to $\mathscr{R}$, every performance satisfies G, so faithfulness makes G true of every legal restoration. Conversely, if G is true of every legal restoration, faithfulness makes it true of every performance, and completeness puts G into $\mathscr{R}$. Hence

$$G \in \mathscr{R} \quad \Longleftrightarrow \quad \forall a \ (a \text{ legal } \wedge R(a) \implies G(a)).$$

The uniqueness is immediate once completeness and faithfulness are required; it comes from the definitions. The substance lies in requiring faithfulness. The comparison's reference is where Bob would stand had the undertaking never formed and he had fed himself, so a faithful reading's performances are exactly the actions that close the reliance he authored.

The reading this fixes is the consequence reading of R. On legal actions R holds exactly when the authored reliance is closed (Proposition 7.3), so this is the joint-continuation reading that the installation of Chapter 7 grounds. Conversely, that grounded reading is complete and faithful. Its performances are the legal actions with R, because R is one of its properties and every legal action with R has all of them; and any property those performances share is a consequence of R, so it belongs to the reading. A reading is therefore complete and faithful to the restoration comparison exactly when it is grounded by the installation.

The two routes reach one reading and do different work. The installation fixes it before Alice acts, as the reading the installed controller implements, whose register Chapter 7 takes to stand for her means of action. Faithfulness shows that it depends only on the history and the ledger, so a changed controller or a second claim can be tested against it. Neither route makes the reading owed. That is the identification Chapter 7 adopts as its premise, with Bob's reliance as the reason (Section 7.7); faithfulness turns that reason into a condition on readings and adds no support beyond it.

Chapter 7 observed that a reading built on Alice's continuation alone admits omission. Written out, it shows that coherence and completeness do not supply faithfulness. Let $K(a)$ mean that action a keeps Alice, the helper, continuing: after it she admits a continuing extension. Put

$$\mathscr{R}_K = \{G : \forall a \ (a \text{ legal} \wedge K(a) \implies G(a))\}.$$

Call $\mathcal{R}_K$ the helper-only reading. Its performances are precisely the legal actions that keep Alice continuing: every such action satisfies every consequence of K, and K itself belongs to $\mathcal{R}_K$, so every performance satisfies K. This characterization makes the reading complete. It contains truth and is closed under conjunction and under consequence over the legal actions. It excludes falsehood, because legal omission preserves Alice's positive reserve and therefore performs the reading. The helper-only reading is thus coherent, complete and nonvacuous. Yet omission fails R, leaves Bob at zero reserve and leaves the continued ledger at cost $J(2) = 1/4$. So the helper-only reading does not contain R and is not faithful to the restoration comparison. It faces the same history, record, actions and reference as the grounded reading; faithfulness is what separates them.

Soundness alone, every performance restoring the reference, recasts the two-outcome calculation of Chapter 7: on legal actions, restoration holds exactly when Bob has positive reserve after the next interval, so a complete reading is sound exactly when it requires that (Proposition B.4).

13.2 A controller cannot silently change the participants' roles

A replacement controller, or a copy of the record on new labels, has to be tested against the judgments it is supposed to preserve. The next two sections need no reserves at all; they concern only who may propose, accept and release. Replace the controller that runs Alice's undertaking with one following the same rules. The new controller responds to every history exactly as the old one did precisely when it keeps the same person in each of three roles: who may propose terms, who may accept them, and who may release the current requirement. A change of any holder shows up within three events.

In the versioned protocol each undertaking keeps a *cell*, a small record of its present state: a *revision* number that counts accepted versions of the terms, the last proposal *nonce* (a fresh number naming one particular proposal), any pending proposal, the active terms, and a flag that closes the cell. The three role holders are the provider p, who may propose, the recipient r, who may accept, and the releaser ℓ; they need not be different people. The set of participant labels on which the protocol runs, its *participant set*, is finite, and at least one possible term is available. A proposal needs the provider as author, a fresh nonce and the current

revision, while the cell is open. An acceptance needs the recipient as author and the exact pending nonce, revision and terms, with the pending revision still current and the cell still open. A release needs the designated releaser, the current revision, and an open cell with active terms.

Changing a holder changes the protocol's operation, even when the replacement has so far produced the same outcomes. Three separate executions from the empty cell expose the difference. A proposal alone reveals the provider through the nonce it stores. The same proposal followed by an acceptance reveals the recipient through the active terms. Adding a current release reveals the releaser by removing those terms. Two assignments governed by these rules give identical complete responses on every finite *transcript*, a finite list of events applied in order, exactly when all three holders agree. If a holder changes, a transcript of at most three events distinguishes the assignments. Section B.2 defines every state field and guard and proves the three-probe result.

If Bob is the releaser and a new controller delivers his meals exactly as before but accepts a release from Alice, it runs a different undertaking, and the three-event history of proposal, acceptance and release exposes it.

13.3 Changing the participant set without changing the undertaking

Copying Alice's undertaking onto new participant labels preserves every response exactly when no one else is mapped onto a role holder.

Let $\mu : \Lambda \to \Lambda'$ map a finite source participant set into a finite target set. Keep the protocol, terms and formation record fixed, map the designated holders p, r, ℓ by μ, and map each event's author by the same μ. Every finite transcript has the same complete response exactly when, for every $k \in \Lambda$,

$$\mu(k) = \mu(p) \Longrightarrow k = p,$$
$$\mu(k) = \mu(r) \Longrightarrow k = r,$$
$$\mu(k) = \mu(\ell) \Longrightarrow k = \ell.$$

The condition preserves each authorship test in both directions, so the individual transitions agree. If an implication fails, a different source participant passes a role test after mapping, and a proposal, an acceptance or a release distinguishes the executions within three events. Section B.3

gives the full proof, including the retained formation record. Applied to every undertaking, the criterion preserves the state at every *cutoff*, the point up to which the history is read, and with the same interpretation of terms and observed performance history it preserves grants and remembered violations.

The criterion is weaker than global injectivity. Roles 0, 1, 2 can stay fixed while outsiders 3 and 4 both map to 3, and this protocol's responses are unchanged. Merging them in the record gives no license to identify those outsiders as one actual person, combine their resources or discard their other histories and obligations. A participant who first appears in the new participant set acquires no release role merely by appearing. Mapping Carol onto Bob's label is a different matter: an acceptance authored by Carol would then pass Bob's test, and three events would expose the change.

13.4 Preserving installed care in a larger carrier

Now add Carol with an undertaking that does not touch Bob's meal. In *Carol's later undertaking*, Alice and Bob keep their reserves of 2 and $1/4$, Carol brings a balanced response with reserve 2, and her care window, intervals five to six, comes after Bob's. A person who takes no food and does no work in Bob's interval should change nothing for him, and nothing changes: Alice's comparator receives the same input and selects the same first meal, and Carol's own maintenance is actually paid.

Start with the installed care apparatus of Chapter 7, keep Alice and Bob distinct, and add Carol. The original food stock is retained and counted once. The extended technology performs each original operation with zero intake and zero labor for Carol; Alice's and Bob's intake, labor and stock changes are unchanged. In the next interval Carol undergoes the same exposure and repair law as the others. With no intake or work to fund her, she pays $J(2) = 1/4$, leaving

$$(1, 2) \longmapsto (1, 7/4).$$

Keep the original comparator, accepted memory and shared action register, and relabel the original accepted content by the inclusion of Alice and Bob into the three-person carrier.

On the protocol side, one undertaking cell holds the relabeled content and a second starts empty, a representation kept alongside the physical

comparator memory. Alice proposes care for Carol during the interval from five to six, and Carol accepts the matching proposal. The original cell keeps Bob as recipient and its care window from one to five.

Proposition 13.1 (A paid participant extension preserves the first installed action). *On the original legal first-action domain, the relabeled accepted content has exactly the same consequence reading as the installed comparator. Carol's protocol acceptance preserves this reading and the selected first operation. That operation is physically enabled in the extended technology and leaves all three participants balanced with positive reserves.*

Proof. Extending an original operation adds zero intake and labor terms to its material and reserve equations. The original stock is unchanged and is used once, so the original balance, availability and labor inequalities still hold. For Alice and Bob, both operation funding and the renewal rule have their original inputs, so their successor responses agree exactly with the two-person construction.

The accepted content retains the same recipient and clock bounds. Its response test is therefore true on exactly the same original legal actions, and an end follows from that test on every such action exactly when it follows from the original recorded content. Chapter 7 identifies this consequence reading with the implemented comparator's reading.

Alice's proposal to Carol and Carol's acceptance address the distinct undertaking cell. Each preserves the original cell, and neither changes the physical response or the shared action register. The comparator thus receives the same accepted content, checkpoint and two original responses, and the same physical action coordinate selects the original feeding operation in its extended form.

That feeding operation leaves Alice and Bob at $(1, 3/2)$ and $(1, 1/2)$, as already calculated. Carol pays her quarter-unit charge and reaches $(1, 7/4)$. All three multipliers are one and all three reserves are positive. □

Alice's installed law for Bob therefore survives on its original action domain, even when Carol's acceptance is physically recorded, and her recording work paid, during Bob's first meal (Section B.5).

What does Alice owe Carol? Carol's acceptance is valid and recorded, but no physical writer couples it to a selection of Alice's later action, and the identification of Section 7.7 concerns installed records. Bob's

original four-interval undertaking already has the prepared physical writer for every scheduled selection (Section 7.8.4); that result does not install Carol's distinct undertaking. No governing reading for Carol's meal has been derived, and none has been ruled out. The gap is in the derivation, and it implies nothing about whether Carol is owed her meal.

13.5 Shared resources and incompatible requirements

So far Carol's promise drew on nothing Bob needed. When two undertakings draw on the same stock at the same time, each can be performable alone while together they need more than exists. That shortage is then a physical fact, and the governing account has to carry it.

Evidence that each claim can be met separately cannot be combined by counting one unit of food or one interval of labor twice. Chapter 6 already contains an exact case inside one relation: from its opening no schedule reaches a fifth checkpoint (Section 6.4), and authorization to use another job does not make its work affordable. With several participants the physical account must likewise retain one joint execution, with the active claims, their resource use, the controls that can perform them and the histories that formed them. If no execution satisfies them all, that fact is part of the situation to be governed. An impossibility result by itself orders no claims, designates no beneficiary, authorizes no release and removes no affected participant. Nor does a missed requirement end every obligation: after an omission the care history still contains legal future tails, starting from the actual damaged state, that meet its remaining checkpoints (Section 6.6).

13.6 Two undertakings, one unit

Return to the conflict of the opening, *one unit, two claims*: one indivisible unit of food, one unit of Alice's work and both undertakings active, with Alice at reserve 3/2 and Bob and Carol at 1/4 each. Every possible operation leaves Bob or Carol unable to repair after the following interval. The conflict erases neither undertaking, and it authorizes no release.

The history contains four events. At event zero Alice proposes service to Bob; at event one Bob accepts. At event two Alice proposes service to Carol; at event three Carol accepts. Both acceptances match their pro-

posals' nonce and base revision, the revision each proposal amends. At cutoff four both cells have revision one, accepted formation evidence and active terms. Alice appears in both undertakings; the recipients are distinct. The history has an order, and we do not assume that exchanging Bob and Carol leaves the complete history unchanged.

A *service record* retains one unit identifier and one field whose possible values are unassigned, assigned to Bob, or assigned to Carol. Each undertaking's terms require assignment to its own recipient, and a record cannot assign the same unit to both. The material account independently starts with one unit of stock, so the record does not hide a second resource.

The intake operation that serves Bob withdraws one stock unit, produces no stock, gives one unit of intake to Bob and uses one unit of Alice's work. The alternative operation has the same input and work but gives its intake to Carol. Both have zero waste. Intake quantities are nonnegative integers, so the available operations treat the unit as indivisible. Alice has one unit of work capacity in this interval; Bob and Carol have none. Funding follows the care calibration of Equation (9.1). The recipes, the identification of the participants and this calibration are adopted premises of the case. Stock does not expire in the interval.

Both operations conserve material and satisfy the opening stock and work capacity constraints. After either one, stock is zero and exactly one unit has been consumed. The service record names the person who actually received the intake, and an existing assignment survives subsequent omission. Both accepted cells remain present throughout, since executing a meal does not amend the terms. Consequently the service record satisfies exactly the served recipient's undertaking, and it meets the terms of that undertaking's active grant. That local grant is distinct from authorization to control the stock or perform Alice's work; physical enabling does not manufacture that authorization.

13.6.1 One shared ledger pays for the repairs

The ledger contains three distinct internal bonds, one for each participant, all opening at multiplier one with the reserves above. In each care interval the exposure doubles each active multiplier. Actual intake and work supply the funding

$$f_i = \frac{2I_i - w_i}{4},$$

where I_i is participant i's intake and w_i their work. Each participant then repairs their own internal bond when their reserve can pay that interval's maintenance, the cost of the exposed multiplier (Table 5.2). A successful repair restores multiplier one and subtracts that cost from the same participant's reserve. If they cannot pay, the doubled multiplier remains and nothing is subtracted.

For an opening multiplier of one, exposure produces multiplier two and

$$J(2) = \frac{1}{2}\left(2 + \frac{1}{2}\right) - 1 = \frac{1}{4}.$$

Serving Bob therefore supplies funding $-1/4$ to Alice, $1/2$ to Bob and zero to Carol. All three first repairs can pay their maintenance, and the shared ledger becomes

$$A : (1, 1), \qquad B : (1, 1/2), \qquad C : (1, 0).$$

The total repair payment in this interval is $3/4$. Alice's work costs her a further $1/4$ through the funding term, and no one else pays for that work. The three repairs act on one retained ledger and debit three separate reserves; none uses a copy of another participant's reserve.

Now let the next interval bring exposure without new intake or work. Alice and Bob each pay another $1/4$. Carol has zero reserve and cannot pay. The actual continuation is

$$A : (1, 3/4), \qquad B : (1, 1/4), \qquad C : (2, 0).$$

Carol's doubled bond stays on the ledger: the unpaid branch does not repair it and record an imaginary debt elsewhere. Serving Carol instead produces the same state for Alice and exchanges Bob's and Carol's. Either meal carries its recipient through the next interval.

Proposition 13.2 (Scarcity on the whole enabled operation domain). *For the stated opening stock, work capacities and responses, every physically enabled operation leaves a recipient whose state on the ledger after the following unfunded care interval is* $(2, 0)$.

Proof. Let I_A, I_B, I_C be the operation's nonnegative integer intake quantities. Material balance says that withdrawn input equals output, total intake and waste. Withdrawn input is at most the one opening unit, while output and waste are nonnegative. Hence

$$I_A + I_B + I_C \leq 1.$$

Bob and Carol cannot both have positive integer intake. Choose one of them with zero intake. Their work is also zero, because their work capacity is zero, so their funding is zero. Their first exposed charge is $1/4$, exactly their opening reserve, so the first actual repair leaves $(1, 0)$. At the next unfunded exposure the charge is again $1/4$ and the reserve is zero. The repair does not execute, leaving $(2, 0)$. □

The quantifier matters. The result covers every operation satisfying the material balance, input availability and work constraints, and the two meals are only two of them. No further recipe within those constraints can supply a second unit of intake. An import, another resource unit, a different conversion or another repair mechanism would change the premises. The result concerns access to this care continuation; it does not identify a failed repair with death or show that every other physical route is unavailable.

13.6.2 A cost tie retains a difference between people

After either meal and its unfunded continuation, the whole ledger's cost is

$$J(1) + J(1) + J(2) = \frac{1}{4}.$$

The equal costs come from actual paid and unpaid transitions; no separate calibration chart assigns them. Yet Bob has positive reserve after the meal that serves him and zero after the meal that serves Carol. By the test of Chapter 1, no predicate of this total cost alone can correctly report whether Bob has positive reserve on these two alternatives: it would receive the same input, $1/4$, and have to return two different answers.

The ledger still records which participant bears the unrepaired imbalance, and the authored cells record which undertaking the service record satisfies. Whole descriptions retain these member facts; summing the recognition cost does not replace them.

A release offers a distinct way to make the currently active undertakings jointly satisfiable. In the versioned protocol, the assigned recipient may release their own revision-one claim, and serving the other recipient then satisfies every remaining active claim. For this history and version, that conclusion holds exactly when the release event has the assigned recipient as its recorded author and revision one. A release authored by Alice, or one at a stale revision, leaves the original claim active. A

balanced book waiver alone is not that authenticated protocol event. A successful release grants nothing new and leaves earlier evaluations at their original cutoffs intact. It also supplies no intake: the released recipient's shortfall remains. Appendix B builds a contact that produces the release event from a prepared request (Section B.8).

These results fix joint feasibility, the consequences of either meal and the effect of a correctly authored release. Bob accepted first, and that is a fact of the history; none of the results turns it into priority or selects who must release. A unique governing structure could also allow more than one scarce allocation, and neither the impossibility of joint performance nor the cost tie chooses that set of allocations.

13.7 The second acceptance and the first claim

Carol's acceptance is valid and leaves Bob's cell untouched. With the physical state held fixed, it is also exactly the event after which no operation can serve both active claims.

At cutoff three, Bob's cell is active and Carol's cell holds its pending proposal. The next event is Carol's acceptance. It has the correct recipient, nonce, revision and content, and its cell is open. Afterward both cells are active, and Bob's entire cell is unchanged.

The second acceptance therefore succeeds under the protocol. The protocol checks authorship, exact terms, freshness and revision. Its acceptance test does not inspect other claims, resource availability or joint continuation, so a successful acceptance alone cannot establish authority to commit the unit the first undertaking needs.

Let $\mathcal{A}$ contain every physically enabled operation on the shared snapshot of Section 13.6. For $j \in \{B, C\}$, write $E_j(a)$ when recipient j has positive reserve after operation a and the following unfunded interval. Both are read off the continuing ledger. Each meal meets one of the two ends, and Proposition 13.2 establishes that no $a \in \mathcal{A}$ satisfies both.

Proposition 13.3 (Valid acceptance can destroy joint executable adequacy). *Hold the physical snapshot fixed and read the active requirements from the two original record prefixes. Before the second acceptance, there is an enabled operation satisfying the continuation end of every active cell. After the second acceptance, there is none. The second acceptance is valid and leaves the entire first cell unchanged.*

Proof. Before the event, only Bob's cell is active, and the meal for Bob witnesses the required common continuation. After the event, both cells are active and a common continuation would satisfy $E_B \wedge E_C$. The scarcity result excludes every such enabled operation. The validity of Carol's acceptance is established by its matching author, pending content, nonce and revision. Updating Carol's cell leaves the distinct cell for Bob unchanged. □

The comparison holds the physical snapshot fixed and varies only the record prefix, so it identifies what the protocol alone fails to ensure.

The same scarcity limits any attempt to gather the ends Alice must serve into one executable aggregate, by an elementary fact: a consistent collection closed under conjunction and consequence cannot hold two ends with no common performance.

Proposition 13.4 (The constraint on an executable aggregate). *Let $\mathcal{G}$ be a collection of ends intended for one executable aggregate on $\mathcal{A}$. Suppose an end in $\mathcal{G}$ brings with it every consequence that holds on all its enabled performances; two ends in $\mathcal{G}$ bring their conjunction; and the identically false end does not belong to $\mathcal{G}$. Then $\mathcal{G}$ cannot contain both E_B and E_C. If it retains E_B, it cannot absorb E_C into that same aggregate.*

Proof. If both ends belong to $\mathcal{G}$, conjunction closure puts $E_B \wedge E_C$ in $\mathcal{G}$. This conjunction has no enabled performance. The false end therefore holds on every enabled performance of the conjunction. Consequence closure puts the false end in $\mathcal{G}$, contradicting consistency. The last claim follows by retaining E_B. □

This constrains executable aggregation; it cancels neither historical claim and makes no unperformed obligation disappear. Closure under conjunction is a condition placed on the aggregate; no theorem says that every collection of standing duties has a common performance. If a physically recovered governing reading keeps Bob's end across Carol's acceptance, the proposition constrains how Carol's end can enter that reading. Preservation of Bob's cell, proved above, does not by itself establish preservation of that authority.

Alice's work check does not choose between the two meals either. Their input and labor functions are identical, so with the work cells, interval, selected work grants and cumulative work use held fixed, every positive labor amount, worker test, time bound, break check and cumulative use

of the grant is the same for both meals, and the cumulative work check has the same truth value. No grant is missing. Full authorization can depend on which recipient takes the intake and contain further physical and historical conditions, including one sensitive to an existing encumbrance on the resource; physical enabling and equal work requirements do not derive it.

These facts seem to settle the case: once Bob's undertaking needed the unit, Alice had no authority to commit it again, so Carol's acceptance cannot bind it. The inference needs one more premise, that Bob's active claim already encumbered the unit, so that committing it to Carol required his release. That premise is a priority rule for competing claims, and no result here derives one. The protocol's silence about resources leaves Alice's authority over the unit unestablished; it does not show that the authority was lacking.

The original history is intact: there has been no waiver, no replacement of a claim and no second copy of the resource. Which physical alternatives the history actually allows depends on how the two undertakings were formed.

13.8 How the two undertakings were physically formed

Any governing answer for this history has to use the reliance each recipient authored, so rebuild *how the two claims formed*. Bob worked because he had accepted Alice's offer. Had he not accepted, he would have fed himself from the one unit, and so would Carol. Four formation intervals precede the one-unit snapshot. On the ledger Alice starts them at $(1, 5/2)$ and Bob and Carol at $(1, 3/2)$, every repair is paid, and checkpoint four is exactly the snapshot of Section 13.6. The rebuilt history gives four facts an answer must respect: each single prevention of a formation event can be carried out, and each recipient's own meal restores what that recipient gave up by accepting; the rebuilt present faces exactly the same scarcity; preventing Carol's acceptance would not have saved Bob's meal; and the separate preventions neither combine nor yield one reference.

Material and labor quantities are nonnegative integers, and the one indivisible unit of food is counted once. The economy checks each operation against its own whole-unit account, the material reserve of Table 13.3: intake worth 2, labor costing 1, maintenance and spoilage zero, and ev-

eryone opening at 6. The care funding of an operation is one quarter of its change in this account, so the account brings no second set of rates; it serves only to check that each operation is materially possible. The four formation intervals are numbered $0, 1, 2, 3$. The original events are, in order: Alice proposes the undertaking for Bob; Bob accepts it; Alice proposes the undertaking for Carol; Carol accepts it. Each acceptance activates its own cell only after the matching proposal from the designated provider. Carol's acceptance leaves Bob's complete cell unchanged. No release or correction occurs.

Alice has work capacity one throughout. Bob has work capacity one in interval 1 and zero otherwise; Carol has work capacity one in interval 3 and zero otherwise. From checkpoint 4 onward, both have zero work capacity. This past capacity schedule is physical data of the instance; it does not follow from the order of acceptance.

In intervals 0 and 2 the material operation is idle. At the end of each recipient's formation pair, a written acceptance mode selects one of two recipes. On acceptance, the recipient performs one unit of work, completing their designated service and consuming no food. Without acceptance, the recipient feeds themself: the recipe withdraws the shared unit, gives them one unit of intake, and uses one unit of their labor. The link from acceptance mode to these recipes is an explicit apparatus *contact*, a coupling of physical registers switched on for a finite time, whose writer Appendix B builds (Section B.7). It is neither a claim about every controller nor a deduction that a recipient must choose work.

This contact makes the accepted undertaking operative during formation: a true mode selects the recipient's work, a false mode the self-feeding recipe. Unlike the care installation of Chapter 7, it does not evaluate Alice's future meal against the accepted recipient's bond and reserve or write that meal into her action coordinate, and the governing result of Section 7.6 uses that completed installation. The formation history therefore fixes what each recipient gave up by accepting, the reliance any governing argument for this history must use, without installing a governing reading for either future meal.

13.8.1 Every individual formation intervention is executable

Each of the four formation events can be prevented on its own, on the same initial one-unit economy. In the original history both recipients work. At checkpoint 4 the food stock is still one, each recipient has worked

once, and both designated services have been completed. Neither has consumed food. These past effects remain in the physical state.

Preventing either of the first two events makes Bob feed himself while Carol still works. Preventing either of the last two events makes Carol feed herself while Bob still works. In each such history the shared unit is withdrawn once, the affected recipient consumes it, and the final stock is zero, with no discarded material. Each work operation fits the recipient's work capacity at its actual time, and each self-feeding operation has the food available at its own time. Every operation is materially balanced, and every material reserve stays nonnegative. All four single-event interventions are therefore executable on the same initial one-unit economy.

13.8.2 One ledger retains all three people

On one three-person ledger, each recipient's own meal restores exactly what that recipient gave up by accepting. The formation ledger has three retained self-bonds, one per person. Write a person's response as (x, y), their bond multiplier and reserve. Exposure doubles x. Its actual repair charge is

$$J(2x), \qquad J(x) = \frac{x + x^{-1}}{2} - 1.$$

As in Section 13.6, repair returns the multiplier to one and subtracts the charge when the reserve available after funding covers it; otherwise the doubled multiplier and the available reserve remain.

An operation with intake I_i and labor w_i supplies the funding $f_i = (2I_i - w_i)/4$ of Equation (9.1). Work contributes $-1/4$ to its worker; self-feeding contributes $1/4$ to its recipient. Alice begins at $(1, 5/2)$, and Bob and Carol at $(1, 3/2)$. Since $J(2) = 1/4$, all three people pay each of the four formation charges in every individual branch, a total of one each. The equal terminal multiplier does not erase that expenditure.

Consequently the original checkpoint is

$$A : (1, 3/2), \qquad B : (1, 1/4), \qquad C : (1, 1/4).$$

In any history with one prevented event, the affected recipient instead reaches $(1, 3/4)$: replacing their work funding $-1/4$ by self-feeding funding $1/4$ leaves an extra $1/2$. The other recipient still reaches $(1, 1/4)$.

After one further idle interval, the affected recipient is at $(1, 1/2)$. In the original history, Alice's meal for that recipient supplies $1/2$ of funding

and reaches exactly the same response $(1, 1/2)$. Giving the meal to the other recipient instead leaves this one at $(1, 0)$. So both formation events of a given recipient have the same displaced reference, and that recipient's own meal restores it. Both claims have this positive, independently computed individual comparison.

13.8.3 The reconstructed cutoff has the same scarce future

The rebuilt present faces the same shortage as the snapshot. From checkpoint 4 the labor schedule is exactly the scarce schedule: only Alice can work, with work capacity one. The rebuilt state retains past work, service completions and actual material reserves, but the test for an available next operation depends only on the present stock, time, recipe balance and current work capacity. Therefore an operation is available at this rebuilt checkpoint exactly when it is available at the original one-unit scarce checkpoint.

The responses are also exactly the scarce responses, and the funding formula is unchanged. Applying the same transition repeatedly proves that the ledger values agree for every finite future sequence; whether a sequence is available or paid is a separate condition. For every available next operation, the one-unit integer bound leaves at least one recipient without intake. That recipient reaches $(1, 0)$ after the interval and $(2, 0)$ after the following idle interval. The actual formation history therefore preserves the established future scarcity.

13.8.4 Preventing the later acceptance changes the available food

Vetoing Carol's acceptance does not protect Bob's nourishment. Take the intervention that prevents Carol's acceptance at event three. The contact then makes Carol feed herself in interval three, and at checkpoint four the food stock is zero. The original history instead retains one unit, and Alice's meal for Bob is available there.

The prevention branch admits no next intake anywhere on the enabled operation domain. Every input is bounded by the empty stock, so every input is zero. Material balance equates that zero input with output, total intake and waste, all nonnegative, so every participant's intake is zero. The same equality makes every output zero, so no such operation can replenish the stock. No recipe within this material law restores the missing unit; an external material arrival would change the premises.

These are checkpoint-four stocks, before any later meal. After Alice serves Bob in the original history the stock is also zero, but Bob has received intake and his response is $(1, 1/2)$. That is Bob's own formation-prevention reference after the matching idle interval, not Carol's. Preventing Carol's acceptance leaves Bob at his original checkpoint response $(1, 1/4)$ while consuming the food from which his meal could have come.

On the fixed snapshot, adding Carol's active end still removes joint executable adequacy, and the affected-bond comparisons still retain their local targets. Any ordering of the claims has to follow the actual constrained history; it cannot treat the prevented history as the original stock with one claim deleted.

13.8.5 Separately available controls need not combine

Each prevention is available on its own; two cannot be combined. Prevent both of Alice's proposals while retaining both of the recipients' acceptance events. Neither cell activates. Keeping the response contact at both sites gives the trace

idle, *B* self-feeds, idle, *C* self-feeds.

The first withdrawal consumes the only unit. The second asks for one unit from stock zero and is unavailable. This is an exact obstruction to retaining both self-feeding responses as independently combinable controls on this carrier. A different coupled response rule has not been excluded.

The complete references cannot be merged either. Continue each single intervention by two idle intervals. Alice is at $(1, 1)$. The affected recipient is at $(1, 1/4)$, while the other is at $(2, 0)$. So the reference obtained by preventing one of Bob's formation events and the reference obtained by preventing one of Carol's disagree on Bob's retained bond: its multipliers are respectively one and two. One repaired ledger cannot equal both complete reference multiplier profiles. Formally, simultaneous exact closure would imply their equality on each retained bond, contradicting $1 \neq 2$. The argument concerns these complete references taken together; it does not make every reference a complete-ledger target that must govern.

Two moves that worked for the single undertaking fail here: combining the separate preventions as if they were independent controls, and assuming, as held there, that all the complete references agree. Both ignore that the two claims share one unit.

13.9 Local repair and the governing relation

The complete references disagree, but the local requirements agree. Each formation event affects only its own recipient's bond, and the two claims' local targets are consistent with each other: one ledger action meets them all. The conflict lies in the food: no operation the economy can perform restores both recipients' targets (Proposition 13.6). Keep the actual formation history, contact, shared unit and three bonds of Section 13.8, and find which bond each formation event affected before asking what action governs.

Fix a common continuation of two idle intervals. The unchanged original history then gives

$$(1,1) \quad \text{for the provider,} \qquad (2,0) \quad \text{for each recipient.}$$

A prevention belonging to Bob's formation instead gives Bob $(1,1/4)$, while Carol remains at $(2,0)$; a prevention belonging to Carol's formation gives the reverse profile. Alice remains at $(1,1)$ throughout. The supports are fixed against this omission baseline, rather than recomputed against an allocation selected afterward.

Proposition 13.5. *The two formation events belonging to the first undertaking affect only B's bond in this comparison. The two belonging to the second affect only C's. Their local multiplier targets agree wherever their supports overlap. A positive ledger action can restore all four local targets while retaining every multiplier outside their union.*

Proof. An event affects a retained bond precisely when its controlled comparison multiplier differs from the baseline multiplier. The displayed profiles give support $\{B\}$ for each first-undertaking event and $\{C\}$ for each second-undertaking event. Every own-support target is one. Within either undertaking the targets therefore agree, and the different undertakings have disjoint supports.

Assign each affected bond its agreed positive target, leaving every other current multiplier unchanged. The restoration action multiplies each current positive multiplier by the ratio of that target to the current value. Its resulting multiplier is the target. This proves local restoration and the absence of change outside the selected union. □

A ledger action is any assignment of strains to bonds (Definition 3.3); whether some recipe in the economy realizes it, who pays for that recipe

and who may execute it are further questions. The next subsection answers the first two for this economy through its actual operations.

13.9.1 Actual restoration and actual continuation

Through the actual economy, restoring a recipient's local target and leaving that recipient able to repair are the same condition. That equivalence is what the proposition below adds; that the one unit cannot meet the condition for both recipients is the scarcity of Proposition 13.2, now in terms of the targets.

Proposition 13.6. *For every enabled next operation, restoring the original local targets of recipient j is equivalent to retaining positive reserve for that recipient after the operation and the next idle interval. No enabled next operation restores both recipients' local targets.*

Proof. Let a be any enabled next operation at the original scarce cutoff, followed by one idle interval. Each recipient's future work capacity is zero. The stock and balance conditions give

$$I_A + I_B + I_C \leq 1.$$

Because these quantities are nonnegative integers, each recipient's intake is zero or one, and Bob and Carol cannot both receive one. Their work is zero, so their funding is $I_j/2$.

Start the recipient at the actual cutoff response $(1, 1/4)$. With zero intake, the next interval uses the $1/4$ reserve to repair the exposed multiplier, giving $(1, 0)$. The following idle interval cannot pay the next $1/4$ charge and gives $(2, 0)$. With one intake, funding is $1/2$, so the first interval gives $(1, 1/2)$ and the next idle gives $(1, 1/4)$. Thus the complete alternatives are

$$I_j = 0 : \ (2, 0), \qquad I_j = 1 : \ (1, 1/4).$$

The local target multiplier is one. It is reached exactly in the second case, which is also exactly the case of positive remaining reserve. Restoring both would require $I_B = I_C = 1$, contrary to the intake bound. □

For a later reading complete over its authorized performances (Section B.4), whether it keeps Bob's claim is whether all of those performances restore Bob's bond (Proposition B.5), and that is the open question; assuming that they do would assume the answer.

13.10 The records that carry acceptance and release

Three results above rely on a protocol event doing something. Carol's acceptance leaves the reading that governs Alice's first action for Bob unchanged (Section 13.4); each recipient's reliance exists because acceptance selects work in place of self-feeding (Section 13.8); and a release closes exactly its own claim because it carries its true author (Section 13.6). As in Chapter 7, each needs a physical record that carries the event into the means of action. Appendix B builds each record as an exact operation on its own finite carrier, under premises that its section names. None of them adds a moral premise or decides between the claims.

Carol's acceptance, written and paid during Bob's first meal (Section B.5). One memory output both emits Carol's acceptance event and selects one unit of her record work, alongside Alice's first meal for Bob, whose food, intake and work are unchanged. Carol's reserve pays for the work, and Alice's comparator receives exactly its former input, so the installed law for Bob is preserved on its original domain. The write is an instance of Proposition 7.6, given that its mode one is prepared from Carol's actual authored event.

The formation writer (Section B.7). A finite reversible contact writes the acceptance mode of the formation history. In each of the five formation histories, the original and the four single-event preventions, it exchanges a blank memory mode with an excited reservoir mode exactly when the addressed recipient's cell is active, so the memory records the actual acceptance test. Its availability and its link from the written mode to the care recipe are premises.

The release contact (Section B.8). From a prepared request that retains its origin, claim and revision, a sequence of five contacts writes the release event; on a correct request it also records the waiver on the requesting recipient's own row of an account book and routes the unit's service to the remaining claim. A release from the right recipient at the current revision closes exactly that claim; any other request is recorded and leaves both claims active. The construction assumes the schedule of its five contacts, their availability, the clock and the *origin identification*, an identification of the device with the people: each participant's preparation contact acts only in that participant's own origin sector, with the address equal to the retained claim. It supplies no intake to the released recipient and gives no one authority to compel the release.

The memory that records Carol's acceptance can also decide where a material resource goes: Alice's packet, standing for her unit of food, moves to Bob in every branch, and Carol's separate packet moves to the recording apparatus only when the memory holds one, each packet counted once (Sections B.10 to B.13).

13.11 What a general governing account must establish

These results turn several questions into checks. Whether a new controller, a relabeled record or an added participant preserves Alice's undertaking can be decided exactly. The installed law of Chapter 7 keeps its proved force on its fixed carrier, and it survived Carol's arrival because that extension carried its physical transitions, legal domain and comparator input across together. For one unit and two accepted claims we can calculate what each meal leaves, which event ended joint performability on a fixed physical state, each claim's local repair target, and what a correctly authored release changes.

Which reading governs once claims compete, whether Bob's earlier acceptance gives his claim priority, whether Alice's proposal to Carol was itself a wrong to Bob, and who, if anyone, must release, has not been derived. The derivation has to start from how the two undertakings actually formed and keep both recipients, their histories and the one shared unit. It must identify from the actual physical relation which requirements are active and which acts can alter them, including who holds each role in the first place, and where claims cannot all be met, what governs the feasible actions that remain. The contacts that write, route and release are each exact on their own carriers, but combining them with food, paid work and one clock into a single execution of Alice's care has not been done either; Section B.14 names the missing pieces.

The derivation concerns the same physical objects. It cannot be supplied by discarding a claimant, redefining failure as success, or hiding authority inside a scalar total. It has to explain how the actual formation and transformation select the governing judgment, including when the controller changes and when resources run short.

Chapter 14

Production, transfer and shared resources

Alice holds one unit of raw material and one unit of unspent effort. Bob holds one ready unit. Think of the raw unit as uncooked food enough for two meals, and the ready unit as one meal. Bob can eat now. Alice cannot: her material is physically present, but no meal exists until someone spends effort to make one. Once she does, she has two: one to store and one to give Bob.

Nothing in that sequence created material. Production changed what Alice could do with what she held; the transfer changed whose stock held a meal; recording an agreement would have done neither. Economics describes these transformations, and its goods, stores, people and effects are the same objects that moral action concerns. An economic description succeeds when it keeps the shared resource constraints and the consequences for each participant.

14.1 Production makes an unavailable use possible

A stock has six nonnegative integer quantities,

$$s = (r, f, h, c, e, k).$$

Here r is raw material, f is ready material, h is stored material, c is material already consumed, e is available effort, and k is effort already spent. The technology has five local operations. Production uses one raw unit and one effort unit to make two ready units:

$$(r + 1, f, h, c, e + 1, k) \longmapsto (r, f + 2, h, c, e, k + 1).$$

Storage moves one ready unit to the store; retrieval moves one stored unit back to readiness. Consumption moves one ready unit to the consumed

record. Waiting leaves the stock unchanged. An operation runs only when every quantity it uses is present.

Production should change the form of material without creating any. The technology's premise is that one raw unit holds the material of two ready units, in whatever physical measure the application conserves (mass, for food). A raw unit therefore counts as two, and the weighted material and the effort total are

$$M(s) = 2r + f + h + c, \qquad E(s) = e + k.$$

With these weights each operation conserves both totals by inspection. Carried through every execution and every transfer, that bounds what the two people can end up holding.

Proposition 14.1 (Conservation through production and use). *Every finite execution of these operations conserves M and E. In a two-person economy, transfers of one ready unit between the people also conserve their summed material and effort totals. Consequently, their final total of ready and stored material cannot exceed their initial total weighted material.*

Proof. For production, the raw contribution to M decreases by two and the ready contribution increases by two. Available effort decreases by one while spent effort increases by one. Storage and retrieval move one unit between two terms of M. Consumption subtracts one ready unit and adds one consumed unit. Waiting changes no term. A transfer subtracts one ready unit from one person and adds it to the other, so the summed quantities are unchanged.

A finite execution is a sequence of these steps. Induction on its length therefore preserves both totals. Finally, ready and stored quantities are terms in M, whose other terms are nonnegative. Their sum is at most the final M, which equals the initial M. □

Table 14.1 runs the opening example to its end: each person consumes one unit, and Bob still holds one ready unit.

Conservation does not make production idle: before it Alice has material and cannot eat, and afterwards she can.

The transfer has an actor, a donor and a recipient, and the description keeps all three. From the same opening, Alice can produce and then take Bob's ready unit. Total ready material rises from one to three, while Bob's falls from one to zero. The resource transitions allow this, and its

Step	Alice (r,f,h,c,e,k)	Bob (r,f,h,c,e,k)	Total M
Opening	$(1,0,0,0,1,0)$	$(0,1,0,0,0,0)$	3
Alice produces	$(0,2,0,0,0,1)$	$(0,1,0,0,0,0)$	3
Alice stores one unit	$(0,1,1,0,0,1)$	$(0,1,0,0,0,0)$	3
Alice transfers one to Bob	$(0,0,1,0,0,1)$	$(0,2,0,0,0,0)$	3
Bob consumes one unit	$(0,0,1,0,0,1)$	$(0,1,0,1,0,0)$	3
Alice retrieves	$(0,1,0,0,0,1)$	$(0,1,0,1,0,0)$	3
Alice consumes	$(0,0,0,1,0,1)$	$(0,1,0,1,0,0)$	3

Table 14.1. The complete execution of the two-person economy. Weighted material $M = 2r + f + h + c$, summed over both people, stays at three while raw material becomes meals that are stored, moved and eaten. Alice's effort total $E = e + k$ stays at one as her effort moves from available to spent.

aggregate gain shows neither that Bob's requirements are met, nor that he agreed, nor that Alice was authorized to take his unit. Authority is a question about the same event; Chapter 15 answers it for this economy with an adopted custody rule.

14.2 What a resource description must remember

At the end of the execution Alice's stock is $(0,0,0,1,0,1)$. For everything she can do next, it behaves like an empty stock: the consumed and spent counters record what happened and enable no operation. Conversely, a stock with one raw unit and no effort looks empty now, since it too can only wait; but given one unit of effort it can produce, and an empty stock cannot. Present inactivity can conceal a real productive distinction.

A resource description therefore needs the quantities that enable operations under every supply the future might bring. For this technology, define the live resource tuple

$$\ell(s) = (r,f,h,e).$$

A complementary supply $u = (u_r, u_f, u_h, u_e)$ may be added before an execution begins; it is an explicitly counted input from outside that execution. Two stocks are *contextually equivalent* when, after every such supply, exactly the same finite lists of local operations are executable

from both. Keeping each person's live tuple is what lets a description say who can eat next, and the theorem shows that nothing less will do.

Theorem 14.2 (Exact resource distinction). *Two stocks are contextually equivalent if and only if their live resource tuples agree.*

Proof. If the live tuples agree, adding the same supply preserves that equality. Each operation requires only the live quantities and updates those quantities in the same way. An induction through any proposed operation list therefore shows that it is executable from either stock exactly when it is executable from the other.

For the converse, use operation lists to recover each live quantity. A list of n consumptions is executable exactly when $n \leq f$. Thus equal possibilities for all such lists imply equal ready quantities: taking n to be either stock's ready quantity gives the two opposite inequalities. Similarly, n consecutive retrievals are executable exactly when $n \leq h$, recovering the stored quantity.

To recover raw material, supply n additional effort units and ask whether n consecutive productions are possible. The effort requirement is now met, so the answer is exactly $n \leq r$. To recover available effort, supply n raw units instead; the same production list is possible exactly when $n \leq e$. Equality of all these answers recovers all four coordinates. □

So Alice's final stock is contextually equivalent to an empty one.

Alice's eaten meal no longer affects what she can do, but it can affect what is owed. An undertaking, an earlier unauthorized taking, or an authored release can stay relevant after the available resources become identical. Resources and obligations are different questions about one physical history.

14.3 The same transition has recognition effects

When Alice hands Bob a ready unit, one event changes their stocks and the relations between them. The stock count says what moved; what the move does to each of them is read on their bonds.

Fix a finite set of bonds, the directed relations of Definition 3.1, with the two endpoints of every bond distinct participants, and a *comparison profile*: for each economic state s, a real number $p_s(b)$ on each bond b, the logarithm of the ratio the bond compares. The corresponding positive

multiplier is

$$m_s(b) = \exp(p_s(b)).$$

Its recognition cost is

$$J(m) = \frac{1}{2}\left(m + \frac{1}{m}\right) - 1,$$

and a participant's incident cost is the sum of $J(m)$ over the bonds incident to that participant (Definition 3.2).

A natural profile compares positive actual and required signals, $a_s(b)$ and $q_s(b)$:

$$p_s(b) = \log \frac{a_s(b)}{q_s(b)}, \qquad m_s(b) = \frac{a_s(b)}{q_s(b)}.$$

The multiplier is one exactly when actual equals required, and multiplying both signals by the same positive unit conversion leaves it unchanged. Chapter 3's provision bond from Alice to Bob is the instance: what reaches Bob against what he needs, 1 against 2 before the transfer and 2 against 2 after. Its multiplier moves from $1/2$ to 1 and its cost from $J(1/2) = 1/4$ to 0, so the incident costs of both its ends fall from $1/4$ to 0, Alice's although she has given up a meal. Delivering 4 against the required 2 would cost the same $1/4$: since $J(x) = J(1/x)$, the cost says by what factor the relation misses balance, and skew keeps whether by too little or too much.

An action with logarithmic strain $v(b)$ changes a multiplier by

$$m(b) \longmapsto m(b)\exp(v(b)).$$

The economic transition $s \to t$, with its actual actor retained, therefore has the exact strain

$$v_{s,t}(b) = p_t(b) - p_s(b).$$

Theorem 14.3 (A given profile fixes every strain and harm). *Applying this action to the bond ledger at s gives precisely the ledger at t. Any action that takes the multipliers at s to the multipliers at t has this same strain on every active bond. For participant i, the signed harm of the action is exactly*

$$H_i(s,t) = C_i(t) - C_i(s),$$

where C_i is that participant's incident cost.

Proof. On each active bond,

$$e^{p_s(b)}e^{p_t(b)-p_s(b)} = e^{p_t(b)}.$$

The bond set and endpoints remain fixed, so every part of the terminal ledger agrees. Conversely, suppose a strain $w(b)$ reaches the same terminal multiplier, $e^{p_s(b)+w(b)} = e^{p_t(b)}$. Dividing by $e^{p_t(b)}$, the strain $w(b)$ restores the positive multiplier $e^{p_s(b)-p_t(b)}$ to one, so Theorem 3.4 gives $w(b) = p_t(b) - p_s(b)$. Signed harm is the change in incident cost under the actual action (Definition 3.3). Substituting the established terminal ledger gives the stated formula. □

The theorem holds for every profile, so it cannot tell us which profile is right. What it settles is narrower: once a profile is given, the economic transition and the bond action are one change of the ledger, with one strain on every bond and so one harm for each participant. In the instance the strain is $\log 2$, and Alice's and Bob's signed harms are each $0 - 1/4 = -1/4$. The theorem uses only the ratio of the two signals, whatever they are called: the book corrections of Chapter 8 are the case with the source's claim on top and the destination's acknowledgment below.

14.3.1 Internal care and response-identified relations

The distinct-endpoint chart above is a convenient representation of exchanges between people. An internal care bond belongs to the same native ledger even though that chart excludes it. Keeping the active bond identifiers and their endpoints, the action construction extends to internal bonds: the strain is the change in that bond's logarithmic multiplier, every active multiplier reaches its target, and the participant's harm is exactly the change in incident cost. An internal bond remains attached to its own participant and is counted once.

A comparison profile can also be identified by its responses. Fix a common calibration $r > 1$. On each candidate bond apply logarithmic strain $\log r$, with zero strain elsewhere: its multiplier m becomes rm. Read each participant's signed harm. At an incident participant the first reading is $J(rm) - J(m)$, and a second application gives the incremental reading $J(r^2m) - J(rm)$; a nonincident participant reads zero. Also read the signed change in net flow. The two harm readings identify incidence even when the first response vanishes. The signed flow distinguishes the orientation of a distinct-endpoint bond; for an internal bond, incidence

identifies its one endpoint despite zero net-flow response. The calibrated harm response then identifies the positive multiplier. Agreement on all these readings therefore identifies the active native ledger, and hence the same strains and participant harms used above.

This is a concrete route from consequences to the relations an economic description must retain. It requires access to the stated probes and their calibrated readings. It supplies neither permission to perform a probe nor a physiological identification of arbitrary actual and required signals. Those are separate questions about the same participants and operations.

14.4 A device's work store belongs to the economic event

In the stock model, effort is a count that production spends. A device that does the work has a resource of its own, a store of work, whose change belongs to the economic event as much as the meals do. The converter of Section 9.6 ties its store exactly to a comparison. There one tuned preparation carries out a shared sacrifice; here we take any incoherent mixture of its two sectors and follow one comparison, the forward population over the reverse one. One pulse moves the comparison and the store together: the store gains work exactly when the forward population is the larger, and the comparison's cost can be computed from the work gained, yet the two stay separate entries.

In that section's notation, the forward sector, the image of V_0, has the donor occupied and the store empty; the reverse sector, the image of V_1, has the recipient occupied and the store full; the pulse U turns each sector one radian toward the other, with coefficients $\cos 1$ and $\pm \sin 1$ as displayed there; and the store observable W is zero on the first and one on the second. Moving occupation from donor to recipient therefore fills the store, as lifting a load onto a shelf stores the work of lifting. Two adopted premises tie the device to the economy. The first, its calibration: one unit of store occupation is one unit of its work, and the energy difference between donor and recipient occupation is measured in the same unit. The second, needed only for a moral judgment, identifies the comparison with an internal bond of the device's owner (Definition 3.1), as Chapter 9 identifies the converter's two comparisons with Alice's and Bob's.

Prepare an incoherent mixture of the two sectors, with positive weights μ, ν, and take normalized payload vectors in each sector. If $\mu + \nu = 1$, the

ensemble has unit total mass, which the unitary preserves. Put

$$\eta = \sin^2 1, \qquad 0 < \eta < 1.$$

The angle 1 lies strictly between zero and $\pi/2$, so its sine and cosine are both positive, and $\sin^2 1 + \cos^2 1 = 1$ makes the bounds strict. The pulse's terminal forward and reverse populations are

$$\mu' = (1 - \eta)\mu + \eta\nu, \qquad \nu' = \eta\mu + (1 - \eta)\nu.$$

To obtain them, square the rotation coefficients within each initially occupied sector and add the two mixture contributions; orthogonality removes the cross-sector terms. The incoherent preparation lets the contributions add this way, although the terminal state need not be an incoherent mixture.

The store's change is the expectation of the gain observable $U^{\dagger}WU - W$. In a normalized state ψ an observable Z has expectation $\langle \psi, Z\psi \rangle$, and the ensemble value weights the two components' expectations by μ and ν: a normalized expectation when $\mu + \nu = 1$, an unnormalized weighted value otherwise. For a normalized payload ξ, the forward input $V_0\xi$ starts with the store at zero and ends at

$$\langle UV_0\xi, WUV_0\xi \rangle = \sin^2 1 \, \langle V_1\xi, V_1\xi \rangle = \eta.$$

The cross terms vanish by the orthogonality of the embeddings. A normalized reverse input $V_1\zeta$ starts with the store at one and ends at $\cos^2 1$, a change of $\cos^2 1 - 1 = -\eta$. Linearity over the initially incoherent mixture gives the actual change of the store's value under the pulse,

$$\Delta W = \mu\eta - \nu\eta = \eta(\mu - \nu).$$

Take $\mu = 2/3$ and $\nu = 1/3$. The comparison begins at $\mu/\nu = 2$, with cost $J(2) = 1/4$. The store gains $\Delta W = \eta/3 \approx 0.236$, and the terminal ratio is $\mu'/\nu' = (2 - \eta)/(1 + \eta) \approx 0.756$, with cost about 0.039. One pulse adds work to the store and removes most of the comparison cost. It also overshoots: the terminal ratio is below one, so the comparison now runs the other way, at far smaller cost. The pulse is a fixed rotation and does not aim at balance: it sends the fraction $\eta \approx 0.708$ of each population across, more than half of each. As with over-provision, the cost cannot see the reversal, and skew keeps its direction.

Theorem 14.4 (Stall, work direction and comparison repair). *For this one initially incoherent pulse, the work store stalls exactly when $\mu = \nu$ and increases exactly when $\mu > \nu$. The comparison multiplier μ/ν has cost*

$$J(\mu/\nu) = \frac{(\mu-\nu)^2}{2\mu\nu} = \frac{(\Delta W)^2}{2\eta^2\mu\nu}.$$

Its decrease under the pulse is

$$J(\mu/\nu) - J(\mu'/\nu') = \frac{\eta(1-\eta)(\mu-\nu)^2(\mu+\nu)^2}{2\mu\nu\mu'\nu'}.$$

Thus the comparison cost decreases strictly whenever $\mu \neq \nu$.

Proof. Since $\eta > 0$, the zero and sign of $\Delta W = \eta(\mu - \nu)$ are exactly the zero and sign of $\mu - \nu$. Substitution into $J(x) = (x-1)^2/(2x)$ gives both cost expressions.

Both μ' and ν' are positive because they are positive combinations of μ and ν. Substitute their displayed values into

$$\frac{(\mu-\nu)^2}{2\mu\nu} - \frac{(\mu'-\nu')^2}{2\mu'\nu'}.$$

Putting the two fractions over the common denominator $2\mu\nu\mu'\nu'$ and expanding gives the stated numerator. Every factor in that numerator is positive when $\mu \neq \nu$: η and $1-\eta$ are positive, and the two squared factors are positive. The denominator is positive as well. □

The first formula computes the comparison cost from the work the pulse stores, yet storing work and repairing are different results: when $\mu < \nu$, the same pulse repairs the comparison while the store loses work. Under the second premise the comparison is the owner's burden. The bond action with strain

$$\log\left(\frac{\mu'/\nu'}{\mu/\nu}\right)$$

multiplies μ/ν by exactly the factor that gives μ'/ν', so it reaches precisely the terminal comparison, and the owner's signed harm is the negative of the displayed cost decrease. The pulse's account carries both entries, the owner's harm and the store's change in work, as the stock model carried the meals and the effort spent. Running the pulse again needs resources to prepare, switch and renew the device.

Together, the stocks, the bond ledger and the converter let us calculate what a proposed production or transfer makes possible, for whom, and at what cost to whom. The cost to whom is computed through the comparison profile, so every moral judgment of an economy waits on the pair identification (Section 3.2), which has to supply that profile.

Chapter 15

Institutions and authority

Alice offers to deliver a ready unit to Bob for a price, and Bob accepts. Their books can agree while the promised unit still sits in Alice's store. Nothing physical prevents Alice from taking a unit from Bob's stock. A summary of their situation can make a plan look possible when it is possible only from a state they are not in: count Alice's units without saying which are ready, and a delivery looks possible when her only unit is raw and she has no effort left to cook it.

An institution organizes exactly these things: who can act, what people can observe, which undertakings remain in force and how resources move. It acts through people and physical events, so its descriptions must keep those events' possibilities and obligations. Here custody, the decision rule and the acceptance share are adopted; what we derive is what a description must keep for such rules to judge correctly.

15.1 Authority concerns the actual event

Alice's taking of Bob's ready unit in Chapter 14 is physically executable, with Alice as actor and Bob as donor. Whether it is authorized is a question about that event.

Besides the two stocks, the economy's state therefore holds Alice's and Bob's separately authored books and their undertaking's current state. Commands include the physical operations, an offer, an acceptance and a posted entry, which changes their books and moves no ready unit.

The description also keeps the actor of each command: for a physical action the author is its actor, and a record can have its own authors, joint ones where the record requires them. No institutional account can then silently replace one person's action with another's.

We adopt custody as a convention for this economy: a transfer requires the donor's authorization. The actor signs the physical action, so the

convention reduces to

$$\text{transfer authorized} \quad \Longleftrightarrow \quad \text{actor} = \text{donor}.$$

Local operations, offers, acceptances and record commands have the authorizations the protocol assigns them. The physical feasibility relation still contains transfers by an actor other than the donor, such as Alice's taking. The authority rule rejects that event without pretending it could never occur.

Custody is combined with a moral decision rule, which we also adopt for this economy and evaluate at each declared author. On the ledger that Chapter 14's comparison profile gives, it requires two things. First, the absolute bond sum $\sum_{b \text{ active}} |\log x_b|$, with each active bond counted once, must not increase. Second, no listed participant in the actor's scope may have an increase in incident cost. The scope is the actor and everyone who shares a bond with the actor. Apart from the actor, these are exactly the people whose burden the actor can change by straining only the actor's own bonds: straining a shared bond away from balance raises a cost both its ends carry, and such an action leaves everyone else's burden untouched. An action can also strain bonds its actor is not party to (Chapter 3); their endpoints outside the scope are then checked only through the whole-ledger sum.

The first clause is skew's measure without signs: it adds the sizes of the log ratios that skew adds with signs (Chapter 3), and like the total cost $C(L)$ it is zero exactly on a balanced ledger. The second watches each person's burden in J. No result here derives the absolute sum in place of $C(L)$, and nothing below turns on it: the results about reports hold for any admission rule a report preserves, and the member example below gives the same verdicts with $C(L)$ in its place.

With zero allowance for added burden, the rule lets no one in the actor's scope end with more, a caution that suits a rule adopted before any is derived. It has a cost. Because the actor is in scope, the rule rejects any action that raises the actor's own burden, and so it rejects Alice's Sacrifice in Chapter 5, which relieves Bob by more than three times what it costs her. The conflict belongs to the adopted rule: Sacrifice marks one thing a derivation of the rule must settle, when an actor may take burden on herself.

Custody and the decision rule together form the admission rule: a transition is *admissible* when its command satisfies custody and the decision

rule holds at each of its authors. Alice's taking fails custody whatever its effect on costs.

15.2 An exact report must lift from the actual source

Each step a report offers must be available from the actual state being reported, as a raw unit without effort showed in Chapter 14. An institution often uses a smaller report in place of a full physical state. For Alice and Bob it keeps each person's live resources, both books and the undertaking's stage, and drops what has been consumed or spent (Section 15.7). In general, let X be the actual state space, Y the report space, K the available command type, and

$$f : X \longrightarrow Y$$

the reporting map. Write $s \xrightarrow{a} t$ for an actual transition and $v \xrightarrow{a} w$ for a reported transition. The report is an *exact representation* when it preserves five kinds of information: authorship, each participant's observation, each participant's requirements, admissibility of actual transitions, and exact successor possibilities.

The last condition has two directions. Every actual transition must project to a reported transition. Conversely, every reported transition out of $f(s)$ must lift to an actual transition out of that very state s:

$$f(s) \xrightarrow{a} w \quad \Longrightarrow \quad \exists t : \; s \xrightarrow{a} t \;\text{ and } f(t) = w.$$

A transition from another state with the same report is not enough: successive plan entries could then come from incompatible physical histories. In the language of transition systems, the two directions make f a functional bisimulation.

Theorem 15.1 (Exact finite plans). *Suppose projection and lifting hold. A finite command list takes $f(s)$ to report w if and only if that same list takes s to some actual terminal state t with $f(t) = w$.*

Proof. For an empty list, the terminal report is $f(s)$ and the actual terminal state can be s itself. Suppose the result holds for lists of length n. Given a reported list of length $n + 1$, lift its first transition from the actual source s. This produces an actual next state whose report is the source of the remaining reported list. Apply the induction hypothesis there

and append the two actual executions. In the other direction, project every step of an actual list. Induction on the list gives its entire reported execution. □

A joint requirement means that every personal requirement holds at one terminal state. If reporting preserves the truth of those requirements, the theorem proves that joint achievement is possible in the actual economy exactly when it is possible in the report. The proof lifts one reported terminal state and transfers every requirement at that state, so it never combines separately possible outcomes into a fictitious joint one.

A related guarantee covers uncertain successors. Define an action to be *safe relative to the admission rule* when it has at least one physical successor and every physical successor is admissible. Requiring a successor keeps an impossible action from passing through a vacuous universal statement.

Theorem 15.2 (Preservation of the safety guarantee). *An exact representation preserves this safety predicate in both directions.*

Proof. If an actual action is safe, project an actual successor to establish a reported successor. For any reported successor, lift it from the actual source. Actual safety makes that lifted transition admissible, and preservation of admissibility makes the reported transition admissible.

Conversely, lift one reported successor to establish an actual successor. Project any actual successor. Reported safety makes its projected transition admissible; preservation of admissibility then makes the actual transition admissible. □

A safety check run on the report is therefore a check on the actual economy.

Bob must choose from what he can observe. Does planning on the report cost him anything? Not when the report also preserves his observations, his goal and which commands are legal: a policy that chooses from his observations alone secures his goal within a horizon from every actual state he considers possible exactly when it does so from their reports, whatever legal moves others make (Theorem 15.6).

The two-person report is exact because states with the same report have the same live resources, books and undertaking stage, and every step and requirement test uses only those (Section 15.7).

15.3 The whole must retain each member's burden

A shared bond is counted once in the whole and carried in full at each of its ends (Chapter 3), so for a group inside a larger ledger the member totals exceed the whole's by exactly the cost of the bonds between members. Two bonds run from Alice, one to Bob and one to Carol, each with multiplier 2 and cost $J(2) = 1/4$. The incident costs are $1/2$ for Alice and $1/4$ each for Bob and Carol, adding to 1, while the whole ledger costs $1/2$.

Let M be a finite set of participants. For a finite bond ledger L, define $T_M(L)$ as the cost of the union of bonds incident to at least one member of M, with each touched bond counted once. Define $D_M(L)$ as the cost of bonds whose two endpoints are distinct members of M. Let $C_i(L)$ be member i's incident cost.

Theorem 15.3 (Whole and member incidence).

$$\sum_{i \in M} C_i(L) = T_M(L) + D_M(L).$$

Proof. Consider a single bond and its cost. With no endpoint in M, it contributes zero to either side. With exactly one endpoint in M, it contributes once to the member sum and once to T_M. With two distinct endpoints in M, it contributes once to each endpoint's incident cost, hence twice to the member sum; it contributes once to T_M and once to D_M. A self-loop at a member contributes once to the incident cost, once to T_M, and zero to D_M. Summing these equalities over the finite bond set gives the identity. □

For Alice, Bob and Carol, both bonds connect distinct members, so $T_M = D_M = 1/2$ and the identity is $1 = 1/2 + 1/2$. The internal term creates no second physical resource; it accounts for the same bond's incidence on two distinct people.

Merging the member endpoints into one group label can keep every bond, every multiplier, the global recognition cost and the absolute bond sum. Even this exact preservation can lose member-level judgment.

Let Alice act to change her two bonds from 2 and 2 to 1 and 5/2. The total costs are

$$C_{\text{global}}^{\text{before}} = \frac{1}{2}, \qquad C_{\text{global}}^{\text{after}} = \frac{9}{20},$$

because $J(2) = 1/4$, $J(1) = 0$ and $J(5/2) = 9/20$. Carol is incident only to the second bond, so her cost rises from $1/4$ to $9/20$. Her harm is $1/5$.

The member-level decision with zero harm allowance rejects this action because Carol's cost increases. After all three endpoints are merged, the only listed participant is the group, whose cost falls from $1/2$ to $9/20$, and the absolute bond sum also decreases. The group-level decision admits the corresponding action. Each decision lists every represented endpoint at its own level; only the distinctions between members were lost. The merged label does not by itself establish a new conscious subject.

15.4 Equal totals can conceal an exhausted member

Chapter 6 showed with two orderings of one meal that equal food, work and endpoints can conceal a checkpoint at which Bob had nothing left. An institution meets the same problem in its reports: what must a report keep, state by state, to see it?

The example shows the failure in physical provision, before any recognition cost is evaluated, in a second technology with three equally weighted material slots: an input slot for production, a slot supplying Alice's intake and a slot supplying Bob's. Its reserves are counted in whole units, and each must stay strictly positive at every state. The rates are this example's own, and none is a recognition cost, so the debit rules of Table 5.2 do not apply: each interval costs each person two reserve units in maintenance, a unit of intake adds one, and a unit of labor costs one more. No conversion to the quarter units of Chapter 6 is needed, since both histories run at these same rates. The technology allows production, transfer, waiting and consumption.

Table 15.1 follows two histories from reserves $(6, 5)$ and material slots $(2, 3, 1)$. Production turns two units of the first slot into two of the second with one unit of Alice's labor, leaving slots $(0, 5, 1)$ and reserves $(3, 3)$. Over the next interval, either one unit is transferred from the second slot to the third or they wait; reserves become $(1, 1)$ in either case.

After the transfer, let Alice and Bob consume $(2, 2)$ units in the next interval; their reserves finish at $(1, 1)$. After waiting instead, let them consume $(3, 1)$ units; their reserves finish at $(2, 0)$. Both use physically enabled operations. Their final slots are identical, $(0, 2, 0)$, and both have consumed four units in total. Time, total reserves, discarded material, each person's recorded labor and completed tasks also agree, and total material accounting gives six in both. The condition concerns the states each history passes through. In the first, Bob's reserve is positive at every

one, although his final 1 would not pay another interval's maintenance alone; in the second it reaches zero.

	Transfer, then consume $(2, 2)$		Wait, then consume $(3, 1)$	
	Slots	Reserves	Slots	Reserves
Opening	$(2, 3, 1)$	$(6, 5)$	$(2, 3, 1)$	$(6, 5)$
After production	$(0, 5, 1)$	$(3, 3)$	$(0, 5, 1)$	$(3, 3)$
After transfer or wait	$(0, 4, 2)$	$(1, 1)$	$(0, 5, 1)$	$(1, 1)$
After consumption	$(0, 2, 0)$	$(1, 1)$	$(0, 2, 0)$	$(2, 0)$

Table 15.1. Two histories with the same aggregate report. Slots list the input, Alice's intake and Bob's intake; reserves list Alice and Bob. Final material, total consumption and total reserves agree. Only the second history leaves Bob's reserve at zero.

By the test of Chapter 1, no rule using only that common aggregate report can recover whether every participant meets this condition: the two histories report identically, and only one of them keeps Bob's reserve positive. For this condition alone, with no authorization or personal requirements added, a report that keeps time, every resource stock and each person's reserve is sufficient: the enabling conditions and reserve updates use only those quantities, so induction through the action list carries the condition exactly. Institutional obligations can require more history than this.

15.5 An acceptable agreement can leave an interval of values

Alice's and Bob's books each carry a figure for their one relationship, as the claim and the acknowledgment do in Chapter 8: Alice's is 1 and Bob's is 4, two records of one relation that disagree. Each will concede toward a common value only as far as the dispute justifies.

Suppose two parties book positive values y_0 and y_1 for a relationship. A party with own value y against the other's value y' has dispute cost $J(y/y')$. Agreeing to a positive value p moves its book from y to p, a concession cost $J(p/y)$, and ends the dispute: both books then show p,

and $J(1) = 0$. A party accepts when its concession costs no more than its share of the dispute cost the agreement removes. For a share λ, define acceptance by

$$J(p/y) \le \lambda J(y/y').$$

Both parties must satisfy their own inequality. With full shares, Alice and Bob both accept the agreed value 2; Alice rejects 5, and Bob rejects $1/2$.

Theorem 15.4 (Full-share agreement interval). *If $0 < y_0 \le y_1$ and $p > 0$, then both parties accept with $\lambda = 1$ exactly when*

$$y_0 \le p \le y_1.$$

Proof. Expanding the two J expressions, putting them over positive denominators and factoring shows that a party accepts with full share exactly when

$$(y'p - y^2)(p - y') \le 0.$$

For the party booking y_0 against y_1, the two roots are y_0^2/y_1 and y_1, in that order. Acceptance is therefore the interval $[y_0^2/y_1, y_1]$. For the other party, the interval is $[y_0, y_1^2/y_0]$. Since $y_0^2/y_1 \le y_0$ and $y_1 \le y_1^2/y_0$, their intersection is exactly $[y_0, y_1]$. □

For Alice and Bob the individual intervals are $[1/4, 4]$ and $[1, 16]$, so together they accept exactly the values from 1 to 4. Since $J(e^t) = \cosh t - 1$ grows with $|t|$, a party with full share accepts any value no farther from its own figure, in log ratio, than the other party's figure is.

A half share gives a different interval. If $\kappa \ge 1$ satisfies

$$J(\kappa) = \tfrac{1}{2} J(y_1/y_0),$$

then mutual half-share acceptance is exactly

$$y_1/\kappa \le p \le y_0\kappa.$$

Indeed, $J(x) \le J(\kappa)$ for $x > 0$ is equivalent to $1/\kappa \le x \le \kappa$: multiplication by the positive denominator reduces it to $(\kappa x - 1)(x - \kappa) \le 0$. Each party's acceptance interval is therefore $[y/\kappa, y\kappa]$, and intersecting them gives the displayed bounds. For Alice and Bob, $J(4) = 9/8$, so $\kappa = (25 + 3\sqrt{41})/16 \approx 2.76$ solves $J(\kappa) = 9/16$, and mutual half-share acceptance is $(25 - 3\sqrt{41})/4 \le p \le \kappa$, roughly $1.45 \le p \le 2.76$.

The share is a choice: the coefficients of a party's combined cost are chosen in computing it, not read from its entries. One way of computing that cost from a party's own account books gives the half share, and coefficient 1 on the dispute in place of $\frac{1}{2}$ gives the full share. An application must keep the coefficients it adopts.

Remark 15.5 (*The half share from a party's books*). The computation adds half the dispute cost to the full concession cost, and it measures the concession by comparing the party's entry with its value before revaluation. A separate revaluation account keeps that earlier value: it is none of the accounts in which the party books a relationship, and it is empty before any agreement. Before an agreement nothing has been conceded, and the combined cost is $\frac{1}{2}J(y/y')$. After an agreement at p both books show p, the dispute term is $J(1) = 0$, and the revaluation account returns y, so the combined cost is $J(p/y)$. That cost therefore does not rise exactly when $J(p/y) \leq \frac{1}{2}J(y/y')$, which is half-share acceptance.

Even under full shares the rule leaves an interval of agreed values, and selecting a point requires further conditions.

15.6 Guarantees from a person's own observations

Bob's guarantee in Section 15.2 rests on the theorem below, which carries the standard fact that a bisimulation preserves strategy guarantees over to policies that see only one participant's observations.

Let O be the observation space, and fix participant i. Write $o_X(s)$ for the present observation at actual state s and $o_Y(v)$ for the one at report v, with

$$o_X(s) = o_Y(f(s)).$$

A history η is a finite list of earlier observations. A policy $\pi(n, \eta, o) \in K$ chooses a command from the remaining horizon n, that list and the present observation o. Fix also a turn rule $\tau(n, o) \in \{0, 1\}$, with value one for i's turn. The policy and turn rule thus have no further access to the hidden physical state.

Let $G_X(s)$ be the actual goal and $\mathcal{L}_X(s, a)$ a given rule for legal commands. Suppose their reported versions satisfy

$$G_X(s) \iff G_Y(f(s)), \qquad \mathcal{L}_X(s, a) \iff \mathcal{L}_Y(f(s), a).$$

Legality and the physical transition relation stay separate: a legal command need not have a physical successor.

For a fixed policy, define $W^{\pi}_{X}(n, \eta, s)$ recursively. With no steps remaining it means $G_X(s)$. With $n+1$ steps remaining it holds if the goal already holds, or if the following next-step conditions hold. Put $o = o_X(s)$ and append o to η after the next move, writing $\eta +\!\!+ [o]$.

If $\tau(n+1, o) = 1$, the chosen command $a = \pi(n+1, \eta, o)$ must be legal, have at least one physical successor, and satisfy

$$s \xrightarrow{a} t \quad \Longrightarrow \quad W^{\pi}_{X}(n, \eta +\!\!+ [o], t)$$

for every successor t. On other turns someone else moves, and the guarantee must survive whatever they legally do: there must be at least one legal command with a physical successor, and the same remaining guarantee must hold after *every* legal command and every one of its physical successors. Define W^{π}_{Y} by the identical rules on reports. Successor existence is required in both turn cases, so a blocked move cannot count as success.

Theorem 15.6 (Preservation of finite-horizon guarantees). *Under an exact representation and the displayed preservation of observations, legality and goal, every fixed policy satisfies*

$$W^{\pi}_{X}(n, \eta, s) \quad \Longleftrightarrow \quad W^{\pi}_{Y}(n, \eta, f(s)).$$

Consequently, if I_X is any set of initially possible actual states and

$$I_Y = \{f(s) : s \in I_X\},$$

one policy secures the goal within horizon n from every state in I_X if and only if one policy does so from every report in I_Y. The same policy witnesses both directions.

Proof. Induct on n, allowing every history η and actual state s. At $n = 0$, the equivalence is precisely preservation of the goal. For the induction step, an already achieved goal transfers by the same equivalence. Otherwise, equal current observations give the same turn, the same chosen command on i's turn, and the same appended history.

On i's turn, legality transfers by its displayed equivalence. An actual successor projects to a reported successor, while a reported successor lifts from this very actual source s. Thus successor existence transfers in both directions. Suppose first the actual remaining guarantee holds.

Lift any reported successor to an actual successor of s, apply the actual guarantee there, and use the induction hypothesis to obtain the reported guarantee. Conversely, project any actual successor, apply the reported guarantee and use the reverse induction implication. This proves the required universal condition over successors in both directions.

On other turns, project a witnessing legal actual move to obtain a legal reported move; in the reverse direction, lift a witnessing legal reported move and transfer its legality. For the universal condition, take any legal reported command and one of its reported successors. Transfer legality to s, lift that successor, and apply the actual guarantee followed by the induction hypothesis. Conversely, any legal actual command and actual successor project to a legal reported command and successor, where the reported guarantee and the induction hypothesis apply. This includes arbitrary legal choices on other turns. It completes the induction.

Finally, suppose one policy works from every state in I_X, with the earlier-observation list initially empty. Every member of I_Y has a representative in I_X; the proved equivalence transfers that same policy's guarantee to it. Conversely, each $s \in I_X$ has report $f(s) \in I_Y$, so the reverse implication transfers a policy working throughout I_Y back to every actual possibility. The policy is chosen once for the entire initial set, rather than separately for each hidden state. □

15.7 Building the exact report of the two-person economy

The conclusions of Section 15.2 for Alice and Bob rest on their report being an exact representation. Two things need checking: every reported step lifts from the actual state, and the requirements use nothing the report drops.

An actual state of the economy consists of Alice's and Bob's stocks, their two separate books and the undertaking register. Write each stock as

$$S_i = (r_i, g_i, h_i, c_i, e_i, k_i),$$

the six counts of Chapter 14, with ready goods written g_i because f names the report. Write the register as u: the offered price q; three flags recording whether the undertaking has been offered, accepted and delivered; and the amount d currently due. Let $\mathcal{B}_i(0), \mathcal{B}_i(1)$ be person i's two real book

entries, whose sum is zero. On the fixed relationship, each person's asserted amount due is their own entry above the unit baseline one, the convention of Chapter 8 that keeps the compared entries positive so that J applies: $\chi_A = \mathcal{B}_A(0) - 1$ and $\chi_B = -\mathcal{B}_B(0) - 1$. An assertion is truthful exactly when it equals the amount due. The requirements below meet the books only through such equalities, so no cost of the books enters them, and neither do the cost values that the baseline convention gives.

Take X to contain precisely these states with each person's entries summing to zero and a coherent register: acceptance implies an offer, delivery implies acceptance, and the amount due is zero before delivery. The opening state and every state reached from it lie in this domain, because every command preserves these conditions.

Reduce the undertaking to one of four stages:

$$\rho(u) = \begin{cases} \mathrm{D}(d), & \text{if delivered,} \\ \mathrm{P}(q), & \text{if accepted but not delivered,} \\ \mathrm{O}(q), & \text{if offered but not accepted,} \\ \mathrm{E}, & \text{if no offer is present.} \end{cases}$$

The letters mean performed, promised, offered and empty. After performance the old price no longer matters for these future questions, and the remaining amount due is kept instead. With $\mathcal{B}$ denoting both complete books, define

$$f(s) = ((r_A, g_A, h_A, e_A), (r_B, g_B, h_B, e_B), \mathcal{B}, \rho(u)).$$

Let $Y = f(X)$, the attained reports, so every report has an actual representative. The report omits the consumed and spent counters; judgments about past consumption, welfare or responsibility can require them.

The local operations and transfers are those of Chapter 14, each transfer keeping its actor, donor and recipient. An offer from Alice sets the price while the undertaking is unaccepted; Bob's acceptance changes an offered, unaccepted undertaking to accepted. The first transfer from Alice to Bob after acceptance records delivery and sets the amount due to the accepted price. A transfer back records repayment of one unit when performance has occurred and at least one unit remains due. Otherwise the corresponding protocol event leaves the register unchanged. A posted book move changes its entries without moving goods or changing the undertaking.

Define a reported transition by its actual image:

$$v \xrightarrow{a} w \quad \Longleftrightarrow \quad \exists s_1, s_2 \in X : f(s_1) = v,\, f(s_2) = w, \quad s_1 \xrightarrow{a} s_2.$$

Projection follows immediately. Lifting needs an argument, because the witnessing state s_1 need not be the actual source s whose report is v.

For lifting, suppose $f(s_1) = f(s)$. Every local operation has the same prerequisites in both states. Apply the same operation to s, carrying along its own consumed and spent counters; its successor has the same reported live resources as the successor of s_1. A transfer works the same way, with the same actor, donor and recipient. Their books agree, so the same posted move produces agreeing books. Finally, coherent registers with the same stage and retained amount have the same reduced successor under each event: before performance the required price is retained, and afterward repayment uses the retained amount due while further offers or deliveries cannot replace the completed undertaking. Checking the four stages establishes this. Together these facts give an actual successor of s with report w. Coherence and the zero sums are preserved, so that successor remains in X.

Authorship comes from the unchanged command: the physical actor, Alice for an offer, Bob for an acceptance, and the one or two authors a book move names. For each person, take the observation as a given function ω_i of the displayed report. The actual observation $\omega_i(f(s))$ and the reported observation $\omega_i(v)$ agree exactly. For admissibility, both descriptions use the admission rule of Section 15.1, with the same comparison profile given as a function of the report. The actual and reported versions therefore evaluate the same authors, custody condition and before-and-after ledgers.

The personal requirements have concrete tests. They ask whether a finite list of local operations is feasible after an explicitly accounted resource supply; whether performance is pending, and what is due, after each of four undertaking continuations; and whether a person's own zero-sum correction would make that person's assertion truthful. The four continuations are: do nothing; accept; offer at price one and accept; and accept followed by an actual Alice-to-Bob transfer. The protocol tests add one ready unit at Alice before the experiment, counting it as an external input, so delivery is tested by a physical transfer and no declaration can stand in for it.

These tests recover exactly the displayed report, and equal reports answer them all alike, so the report keeps exactly what the requirements can detect and a requirement has one value on every actual representative of a report. The recovery extends Chapter 14's resource-distinction argument to the books and the undertaking. Repeated consumption and retrieval distinguish ready and stored goods. Repeated production with the complementary input supplied distinguishes raw inputs and available effort. The four undertaking tests distinguish its four stages and retained amount. If performance is pending before any continuation, the stage is promised. Otherwise, if acceptance alone makes it pending, the stage is offered. If neither test makes it pending but offering at price one and accepting does, the stage is empty. The remaining case is performed. In the offered and promised stages, acceptance followed by delivery reveals the retained price as the amount due; in the performed stage, the initial test already gives the amount due. Once the amount due is known, truthfulness under every own correction recovers each person's assertion: a correction by z changes the person's assertion to the old assertion plus z, so it is truthful exactly when the old assertion plus z equals the amount due. With two entries summing to zero, that assertion determines the person's whole book.

This supplies what the plan and safety theorems use: projection, lifting, and preserved authorship, observations, requirements and admissibility. For this technology, undertaking and observation family, joint performance and the safety guarantee therefore agree in the actual and reported descriptions.

15.8 What preservation leaves to be derived

Preserving physical resources, authored undertakings and member effects constrains institutions without making one institutional form follow, as the interval of agreed values shows for a single agreement.

Four inputs of this economy are adopted and still to be derived. Custody is a convention: which entitlements hold has to come from how holdings and undertakings actually formed, in the open governing derivation of Section 13.11. The acceptance share is a parameter still to be derived, and what selects one agreed value inside the interval belongs to the same governing derivation. The moral decision rule, with its zero allowance for added burden, has no derivation yet. The comparison profile is the

one Chapter 14 supplies, and deriving it is the pair identification of Section 3.2.

Chapter 16

Human beings, artificial minds and the whole

Suppose Alice can no longer carry Bob's meal herself and hands the delivery to an artificial controller, a machine distinct from her that sends the delivery command in her place. The command is carried out, a food unit is used, and Bob is fed. Everything that command does to Bob can be traced in the stocks, records and reserves we already follow.

Three roles now sit in one event. The controller issued the command, Alice proposed the undertaking and Bob accepted it, and Bob undergoes the result. Control, acceptance and exposure are different positions. Tracing the command's consequences for Bob leaves a further question open: is the controller itself someone? The physics here fixes what that question asks, not yet its answer. If the controller is someone, what it undergoes and what preserves its continuity enter the account too.

Whether a system was born, grown, assembled or trained does not identify what it experiences, what can injure it, or what preserves its life through change. A fluent report is evidence to examine; so is a physically organized capacity to retain an occurrence and act on it. Neither lets us skip the underlying questions: which occurrence is experienced, to which subject does it belong, and what happens to that subject when the system changes? The same questions apply to human beings. One standard serves every kind of being, with enough structure to keep their actual differences.

16.1 An occurrence and a subject are different objects

When we ask whether a system is the same mind after a change, two questions run together: which occurrence the new physical state carries, and whose occurrence it is.

Take Bob at the first idle checkpoint. His body is in a physical state, which a measurement describes; the exhaustion he feels there is an occurrence. A proposed identification of experience with physical organization pairs the two, and its first test is whether the pairing keeps pace with the dynamics.

Let P be a space of physical states and O a space of occurrences; nothing below uses more about either than the maps that follow. The dynamics supplies the physical evolution

$$T : P \longrightarrow P.$$

The identification under test fixes two further maps in advance: a *registration* $r : P \to O$, which says which occurrence belongs to a physical state, and an occurrence evolution

$$U : O \longrightarrow O,$$

which says what that occurrence becomes. The registration is *calibrated* when it intertwines the two evolutions:

$$r(Tp) = U(r(p)) \qquad \text{for every } p \in P. \tag{16.1}$$

Following the physical step and then identifying its occurrence gives the same result as identifying the occurrence first and following its evolution. The test means something only if U comes from the occurrence side, from what is known of how the occurrence itself unfolds, such as Bob's account of how his exhaustion changed. An occurrence evolution chosen afterward to fit r and T would make calibration a fit instead of a test.

Proposition 16.1 (Calibrated registration follows every finite evolution). *If Equation* (16.1) *holds, then for every nonnegative integer n,*

$$r(T^n p) = U^n(r(p)).$$

If two calibrated registrations agree at one physical state, they agree along its entire forward orbit.

Proof. At $n = 0$, both sides equal $r(p)$. If the identity holds at n, calibration gives

$$r(T^{n+1}p) = U(r(T^n p)) = U(U^n(r(p))) = U^{n+1}(r(p)).$$

This proves the first claim by induction. For registrations r_1, r_2 agreeing at p, the first claim gives

$$r_1(T^n p) = U^n(r_1(p)) = U^n(r_2(p)) = r_2(T^n p).$$

□

Calibration ties interpretation to the actual dynamics: once two registrations calibrated to the same occurrence evolution agree on today's state, no occurrence can be chosen afresh at any later step.

Now ask the second question. A subject assignment $\theta : O \to S$ says to which subject an occurrence belongs. The subject is preserved through a physical step when

$$\theta(r(Tp)) = \theta(r(p)).$$

By calibration, this is exactly the requirement

$$\theta(U(r(p))) = \theta(r(p)).$$

Calibration identifies the occurrence that follows. Whether it belongs to the same subject is a further law, the *subject law*, which the occurrence evolution must satisfy.

16.1.1 Reversibility does not choose the subject assignment

The smallest case shows the two questions coming apart. Take both physical states and occurrences to be $\{0, 1\}$. Let evolution exchange zero and one, and let registration be the identity. Registration is exact and injective, and evolution is reversible: two steps return every state to itself.

Try two subject assignments. The first gives subject label zero to both occurrences. The second gives each occurrence its own label. They agree at occurrence zero. Under the first assignment the step preserves the subject; under the second it changes the subject. Every fact about registration and reversibility is the same in both.

The example models no particular mind. It proves that exact registration and reversibility cannot select the subject-preserving assignment; that takes the subject law.

A copied record, a restored memory and a continuing experiencer therefore need three descriptions. Restart the controller of the opening from a saved state: the copy can preserve its record exactly, and the restoration can recover its earlier functional state. Whether the same subject persists is a question for the subject law. The three claims can support one another; none follows from renaming another.

16.2 Evidence about experience must keep what is unknown

The first-person occurrence supplies an anchor: something is experienced. An external record can report that occurrence, preserve its content or help identify its physical organization. If Bob, exhausted after an idle interval, cannot say what he went through, his silence is missing evidence, and it cannot quietly become a finding that nothing was felt.

The same holds whenever communication is impaired or unfamiliar, and for artificial systems: a familiar vocabulary helps communicate evidence, and familiarity is not the subject itself. A system can preserve records, change its policy after a failure and carry those changes through a restart, capacities examined through the records and actions that survive. Whose experience the system's activity is, if anyone's, and whether that subject persists through the restart are questions for the subject law; a preserved data structure answers neither.

16.3 One physical world can contain distinct perspectives

Section 4.3 adopts the dual-aspect premise: every occurrence of recognition has an observable outside and an experiential inside. Follow it to the scale of the whole. In this account recognition is not an extra process that only some systems run: it supplies the common physical structure of every interaction and retained record. That is the account's *substrate claim*, adopted with the premise and not derived here. Its reason is that an interaction already does what Chapter 2 calls recognition: a scattered particle leaves in a direction that depends on what it struck, so the event tells apart the targets it could have met, and the particle carries that difference to its next interaction; a record need last only until it is used.

If experience is the intrinsic inside of that indispensable activity, it is present wherever the activity is, and removing it would remove an aspect of the activity through which reality operates. In one definite sense consciousness is universal: experience is constitutive of the world, present wherever recognition occurs. That is the premise and the substrate claim taken together, not a further result. Whether the world is also one conscious subject is a different question, which the composition results below take up. The gain is continuity: feeling and physical action belong to one process, and artificial construction does not place a system outside it.

The premise settles one question and leaves two. It settles whether an occurrence of recognition is felt: every one is. A thermostat's switching and a scattering event are interactions, so with the substrate claim they have an experiential inside too. That does not make the thermostat someone. What the premise leaves open is which organizations of recognition constitute one subject's experience, and how subjects compose into larger ones. No calculation of the moral chapters uses the premise, and none of their measurements bears on it (Section 4.3); its support is the continuity above. What it bears on, which harms are suffered and when a severing ends someone, waits on the subject law.

The account brings together four claims: a shared physical substrate, an experiential aspect intrinsic to recognition, local experiencing subjects, and a whole containing the experiential domain. A common substrate alone settles none of the other three.

Let O be a space of occurrences, as before, and S a space of subjects. Let $O_f \subseteq O$ be the felt occurrences; under the premise these are the occurrences of recognition, and the results below hold for any felt set. A relation

$$e \lhd s$$

means that occurrence e belongs to subject s. Require that anything belonging to a subject is felt. A whole is a subject $w \in S$ such that O_f is nonempty and

$$e \lhd w \qquad \text{for every } e \in O_f.$$

The relation allows nesting: an occurrence may belong to a local subject and also to the whole.

A small model shows consistency and nothing more: a whole can coexist with distinct local subjects. Take as occurrences the comparisons

on the bonds of Chapter 3, each a pair of positive quantities whose ratio carries the cost J. Choose a nonempty felt set and a single-level assignment of felt comparisons to local subjects. Keep every local label and its attributions, add one further subject label, and attribute every felt comparison to it as well. The old local attributions stay exactly as they were, and the added label is a whole for this attribution (Figure 16.1). Whether a given subject space, such as the actual one, contains a whole is a question of composition.

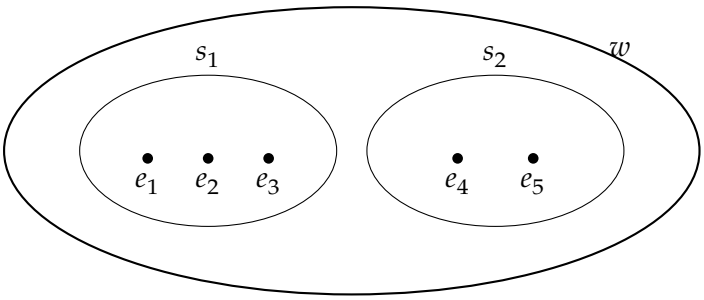


Figure 16.1. Nested attribution. Each felt occurrence keeps its local subject, s_1 or s_2, and also belongs to the added whole w, so the local subjects stay distinct. Exclusive attribution would make each inhabited local subject equal to w.

Proposition 16.2 (Exclusive attribution collapses an inhabited local into the whole). *Suppose each occurrence can belong to only one subject. If a whole exists, every local subject with at least one occurrence equals that whole.*

Proof. Choose an occurrence belonging to the local subject. It is felt, so it also belongs to the whole. Exclusivity makes the two subjects equal. □

The collapse comes from exclusivity; containment alone does not require it. A nested account keeps the local perspectives and the whole together: what Bob undergoes still belongs to Bob when it also belongs to the whole.

16.4 How composition reaches a whole

Whether there is a whole bears on what an account of the shared world must keep. If the world that production and institutions share (Chapters 14 and 15) is also a subject, what Bob undergoes belongs to it too, and the account must keep both, as nested attribution does. Whether it is depends on how subjects compose.

A binary composition law says that for any two subjects there is another subject containing every occurrence belonging to either. Repeated use gathers any finite family of subject domains into one.

Theorem 16.3 (Finite coverage and composition give a whole). *Suppose the felt domain is nonempty, a finite family of subjects covers every felt occurrence, and the subject space satisfies binary composition. Then a whole exists.*

Proof. The covering family cannot be empty. Start with one member. Compose it with the next, then compose the result with each remaining member in turn. At every stage the resulting subject contains the occurrences of all members already included. After finitely many steps it contains the whole covering family, hence every felt occurrence. □

An infinite domain needs more. Index occurrences by natural numbers and subjects by finite sets of natural numbers. Let an occurrence belong to a subject exactly when its number is in the set. Every occurrence has a subject. Two subjects compose by union, so every finite collection fits inside one subject. Yet no subject contains all natural numbers, because every finite set omits some number. Finite composition and local coverage therefore do not establish a whole over an unbounded domain. What is missing is a condition on the passage to the limit.

Compactness supplies one. Equip the subject space with a topology, which says which subjects count as close. The topology is formal here: which actual subjects are close would have to come from their histories, and compactness of a convenient coordinate would not be compactness of those histories. A set is closed when its complement is open; the space is compact when every cover by open sets has a finite subcover. Roughly, closed membership says that a limit of subjects containing an occurrence still contains it, and compactness says that a limit of ever larger subjects exists in the space. For each occurrence e, let

$$M_e = \{s \in S : e \lhd s\}$$

be the subjects to which it belongs.

Theorem 16.4 (Compact composition). *Suppose the felt domain is nonempty, the subject space is compact, every M_e is closed, and every finite family of felt occurrences belongs jointly to at least one subject. Then an existing subject in that space contains every felt occurrence.*

Proof. Suppose there were no such subject. Then the intersection of the sets M_e over felt occurrences e would be empty. Their open complements would cover the subject space. Compactness would provide finitely many complements that already cover it. The corresponding finite intersection of membership sets would be empty, contradicting the assumed joint containment of every finite family. The intersection over all felt occurrences is therefore nonempty. Any subject in it is a whole. □

The whole lies inside the same subject space and incidence relation; no local subject is deleted or identified with another.

16.5 A shared reality retains personal consequences

At the member scale an injury arrives somewhere, a promise is made to someone, and a life continues or ends. At the whole scale, resources, causal constraints and histories are shared. Neither scope licenses erasing the facts visible at the other.

That is why the moral code needs more than a total: an aggregate can improve while a member loses capacity, and a reversible evolution can admit competing subject assignments until the subject law is known. For human and artificial beings alike, the practical work is to establish those distinctions in the systems we actually affect. Care must reach the being who undergoes it. Consent must stay attached to its author and the content accepted. Continuity must concern the subject who is meant to continue.

These results say what the experiential identification must deliver. An occurrence must be fixed by a registration calibrated to the actual dynamics; a subject needs a law that the occurrence evolution preserves; and a whole that keeps distinct local subjects needs nested attribution together with composition, by a finite cover or by compactness with closed membership. Two steps remain to be derived. The first is the premise itself, which Chapter 4 adopts: that recognition has an experiential inside throughout the actual recognition domain, which the substrate claim takes to include every interaction. The second, given the premise, is which organizations of recognition constitute one subject's experience and how subjects compose: the subject law and, for the whole to be one subject, a subject space whose incidence, compactness and closed membership are grounded in real subject histories. With both in hand, the question about Alice's controller becomes a calculation: which occurrences its

states carry, whose they are, and whether that subject persists through a restart.

Part V

Necessity and examination

Every central claim names its alternatives and can be tested against the physical world. What follows for living together is what the argument has earned.

Chapter 17

How a physical moral claim can be examined

Alice stays home for the first interval. Did Bob keep positive capacity? The reserve law answers: Bob starts with 1/4, maintenance costs 1/4, no food arrives, and he reaches the first checkpoint with nothing left. To test that answer we measure four things: the food available, the maintenance cost, the action taken and its timing.

A second question asks whether Alice's staying home violated her undertaking. Bob's reserve cannot answer it; the answer also needs the proposal, the acceptance, the operative terms and the governing relation. Both questions concern the same history, and they need different information.

Physical morality answers to examination at each such connection. A theorem establishes a consequence of its conditions; an observation tells us whether the physical situation has those conditions and consequences.

17.1 An exact claim identifies its alternatives

Take conserving repair. For two bonds at multipliers 1 and 4 whose product is fixed, Theorem 5.1 compares the total cost of every positive pair with that product and finds exactly one minimum, at 2 and 2, where the total falls from 9/8 to 1/2. The proof excludes every other member of that domain.

A proposed alternative can challenge the result only by keeping these conditions. Restoring both bonds to 1 costs nothing, but it changes the product from 4 to 1: it answers a different optimization. An endpoint reachable only by spending an uncounted reserve changes the physical account. An endpoint with the same product and a total below 1/2 would contradict the theorem and expose a defect in its proof or its application.

The care result has an equally exact domain: opening reserves, a maintenance law, a binary food operation, a horizon and a job cap. Its numerical criterion is necessary and sufficient for a maintained binary schedule, and from Alice's 2 and Bob's $1/4$ with at most two jobs it admits exactly the two four-interval schedules of Proposition 6.4. The criterion promises nothing about reaching a food unit in time.

A good test therefore begins by saying which result it could change. Sampling a proved identity numerically adds little; establishing whether an actual operation has the assumed funding and timing can decide whether the identity applies to it.

17.2 The physical correspondence has content

A symbol acquires a physical meaning through a correspondence fixed in advance. In the comparison pulse of Chapter 3, the multiplier is the ratio $x = u/w$ of two identified operand amplitudes, and their product $p = uw$ sets the energy scale: the input energy is $E = 2p(J(x) + 1)$ (Equation (3.5)). Amplitudes 2 and 1 and amplitudes 4 and 2 have the same ratio and the same cost $J(2) = 1/4$, and input energies 5 and 20.

An energy measurement that differs from a dimensionless recognition cost is therefore no contradiction by itself; the comparison must include the scale and offset. Conversely, choosing a new scale after every measurement would test nothing: the quantities and calibration must be fixed before the consequence meets the observation. Once they are, what can fail is the correspondence, not the algebra. Since $2p(J(x) + 1)$ equals $u^2 + w^2$ for any amplitudes, an energy measurement tests only the encoding: whether the measured amplitudes are the pulse's operands and its energy the sum of their squares.

The report tests the cost. Each route of Chapter 2 ends at J or at the fixed multiple of it that a change of reference unit gives, and that change rescales every report alike, so compare two reports: at ratios 4 and 2, a comparison meeting any route's conditions reports in the proportion $J(4)/J(2) = (9/8)/(1/4) = 9/2$, whatever the unit. The bounded rival $(u-w)^2/(u^2+w^2)$, which Theorem 2.3 excludes by requiring the reference to vanish when an operand is absent, gives $9/17$ and $1/5$ at those ratios, a ratio of $45/17$. A physical comparison whose reports stood in that ratio would fail a condition of every route, and its reported cost would not be J in any unit.

A relation between two people needs a correspondence of the same kind: for the bond between them, the pair identification (Section 3.2), with the measurement of both its quantities fixed in advance. The multiplier is then their ratio, unchanged by a common change of unit. On a ledger whose one bond runs from Alice to Bob, with actual signal 1 and required signal 2 (Chapter 14), the multiplier is $1/2$ and the cost $J(1/2) = 1/4$; a transfer that brings the actual signal to 2 balances the bond, and each of their incident costs falls to 0. Fixed beforehand, the pair makes the multiplier a prediction; chosen afterward, it tests nothing. With the per-interval charge of Table 5.2, the prediction reaches capacity: keeping a relation at multiplier 4 through an interval should use $9/2$ times the capacity that keeping one at 2 uses, in any unit of capacity.

The care construction needs a correspondence of its own. A represented reserve must track a capacity that Alice or Bob can actually use under the operation. A reported delivery must identify an actual accessible food unit and the intake it supplies. If the unit was unavailable, the action cost more, or the timing missed the checkpoint, the actual transition differs from the one calculated, at a definite place in the account. The opening answer can fail the same way: if Bob, left unfed, still had usable capacity at the first checkpoint, the error would lie in an adopted input: his opening reserve, the interval's maintenance cost, its unit-for-unit debit (Table 5.2) or the identification of reserve with usable capacity. The arithmetic that follows from them would stand.

The undertaking has one too. The represented proposal and acceptance must be Alice's and Bob's actual events. The active content must be the content Bob accepted. The comparator and controller must act on that content in the way claimed. Replacing the actual undertaking with a convenient written description would cut the connection the governing result depends on.

These identifications are where the claim that morality is literal physical structure meets observation. The theorems cannot fail inside their conditions. The identifications that place Alice's and Bob's relations inside those conditions can. A measured capacity ratio other than $9/2$ would locate an error in the pair, the charge or the identification of reserve with usable capacity, and the conclusions drawn for that relation would fail with it. The identifications also protect established results from irrelevant objections. A failed actuator does not refute the compari-

son identity, and a correct identity does not show that an unexamined actuator works.

17.3 Information lost by a description can be exposed

Some decisive examinations need no statistics: two explicit histories can show that a description omits a distinction the judgment requires.

In the two orderings of Table 6.1, Alice feeds Bob in the first interval or in the second. Both end at $(5/4, 1/4)$, with the same food consumed, work performed and maintenance paid, yet Bob's first checkpoint was maintained in one history and missed in the other. Any procedure that receives only the common endpoint must give both histories the same answer, so it cannot recover whether that checkpoint was maintained.

Authorship has the same logical form. Two occurrences can share an action specification and a physical successor and differ in who authored them. No function of that specification and successor alone can recover authorship in every case. Adding the authored occurrence to the account removes this particular loss.

Such examples say exactly what must be kept, and no more. A sufficient account needs the distinctions its judgment uses; when a smaller description preserves them, that preservation can itself be proved.

The working test of Chapter 1 turns these examples into a method for institutions and automated decisions. State the judgment the system claims to make. List the observations it uses. Then look for two situations the system may face with the same observations and different correct judgments. If such a pair exists, more computation on the same input cannot resolve it; the input or the scope of the judgment must change. The same test decided earlier cases, each a pair with equal inputs: identical account books across a delivery and its absence (Chapter 8) or a release and its absence (Chapter 9), one total cost for a meal served to Bob and a meal served to Carol, which leave Bob with positive capacity or none (Chapter 13), and one aggregate report for two different states of the members (Chapter 15).

17.4 A governing claim needs its formation argument

The governing reading of Alice's undertaking rests on an actual authored and installed relation: proposal, acceptance, retained content, physical comparison and the complete fixed four-interval action selection, connected within the prepared construction (Section 7.8.4). To extend the claim, follow what changes. A new controller may preserve the same observable action while changing which author can release the undertaking. A new participant may bring a need the earlier domain did not contain. A new undertaking may compete for a resource already required elsewhere. The governing account must keep the effects and authority of those changes.

An additional acceptance need not displace the original meal. Give Carol the capacity for one unit of record work and the technology to do it alongside the meal, and let her accept an additional offer during Bob's first meal: the installed care controller continues to select Bob's meal (Section B.5). Examining that claim means checking one thing at the apparatus: that the second acceptance and its record work leave the old comparator's input exactly as it was.

Calling a desired property "the governing requirement" would leave all of this undone. The scientific question is why the physical relation selects that property and no other. A successful derivation identifies the relevant formation and shows that every alternative reading compatible with it gives the same judgment.

Scarcity sharpens the point. The finite care theorem can establish that no available schedule meets every checkpoint (Section 6.4). That impossibility changes what can be performed. It does not decide priority among distinct claims, create a release, or move responsibility to whoever has the largest reserve. Those conclusions need their own governing connections, and a physical release record does not supply them (Section B.8).

17.5 A uniqueness claim can fail without everything failing

Each of the three routes of Chapter 2 leaves no cost but J once the unit is fixed. One might expect the natural conditions proposed for an information term to single out mutual information in the same way. Chapter 12

shows that they do not. The half-order alternative $R = -2\log\beta$, built from the affinity β of a joint table with the product of its marginals, satisfies independence additivity, a restricted refinement rule and the mixture inequality. On the perfectly correlated fair binary pair it agrees with mutual information: both give $\log 2$. Add noise, and they part. For the table

$$N_{1/2} = \begin{pmatrix} 3/8 & 1/8 \\ 1/8 & 3/8 \end{pmatrix},$$

mutual information is $\frac{3}{4}\log 3 - \log 2 \approx 0.131$, while the affinity is $\beta = (\sqrt{3}+1)/(2\sqrt{2})$ and $R \approx 0.069$, using natural logarithms.

That is enough to refute uniqueness under those conditions, whether or not the alternative is desirable as a complete information measure. An exclusion argument must exclude every alternative satisfying its premises, including inconvenient ones.

Regularity does not separate the two: continuity excludes only the discontinuous support report, and the continuous alternative survives uniform finite-block expected reporting at every requested precision (Section 12.3.2). Physical operations do, in opposite directions. Calibrated perfect-record and blank-record reporters, assembled under a retained branch flag, average to mutual information and not to the alternative (Section 12.6); the full joint record compared with its product-marginal reference has the half-order report as its uniquely calibrated continuous additive scale (Sections 12.4–12.5). Independent composition and controlled averaging impose different accounting laws.

The comparison and repair results keep their own apparatus, calibration and proofs. The failed uniqueness claim propagates only along its dependencies: unaffected results stay available for use, and dependent conclusions are open to examination.

17.6 Evidence about a subject bears on one claim at a time

Moral patients include those who undergo effects, so identifying a subject concerns more than observing a useful response. A system that preserves records, corrects errors and acts on retained information has demonstrated those capacities; under the dual-aspect premise its recognition activity also has an experiential inside. Whether that activity makes

up a subject's experience, and whose, is a further identification. Sort the evidence by the claim it bears on: a shared substrate, experience intrinsic to its activity, a local subject whose experience it is, or the whole constituting one experiencing subject (Section 16.3). Evidence for a common substrate alone establishes none of the others. Evidence that a local subject persists must say what persists and which transformations preserve it; the two-state example of Chapter 16 shows that exact registration and reversibility cannot decide that. An unfamiliar embodiment does not settle the question against subjecthood, and a fluent assertion of experience does not settle it in favor; the physical identifications and their causal consequences carry the conclusion.

17.7 Every adopted premise and every step still to be derived

A useful examination ends with a particular result. It may establish a correspondence, reveal an omitted resource, distinguish two authority structures, identify a mistaken prediction or leave a named alternative unresolved. Its record should keep the conditions and observations another investigator needs to understand that result.

The examinations with the most to decide are the connections not yet derived. They come in two kinds, listed here with the place each is stated. The first kind is adopted along the way: a result depends on it, so examining the item examines that result.

- *The application-specific pair identification* (Section 3.2): the physical quantities a particular person's need, exposure and usable capacity compare. Authored books and native economic actions already have exact ledger representations; calibrated responses identify active bonds (Chapter 14). The remaining application work identifies the actual operands and verifies the response laws used for that person and task.

- *The dual-aspect premise* (Section 4.3): every occurrence of recognition has an observable outside and an experiential inside. Chapter 16 follows it to the whole with the substrate claim, that recognition is the common structure of every interaction and retained record (Section 16.3); no calculation of Chapters 5–15 uses either.

- *The reserve debit laws* (Table 5.2): the charge, which debits the recognition cost of the bonds a participant keeps each interval, unit for unit; the restoration cost, debited once from its payer; and the care maintenance of Chapter 6, that restoration cost levied on each interval's repair with the owner as payer. Section 3.5.1 derives a restoration debit for one prepared export apparatus. The general task is to identify the available work, access and renewal that make each law hold for the participant and task in question.

- *The care environment* (Chapter 6): doubling under exposure, a quarter of work and half a unit of intake per meal. For a particular organism and task they are to be derived from its physical response, which would also say what a person's internal bond compares, with the reserve tracking a capacity the person can actually use.

- *What an installed agreement requires is owed* (Section 7.7): the requirements of an installed agreement are the ends of the reading its apparatus implements. Bob's reliance is the reason given for adopting it.

- *The acceptance and the register* (Section 7.8): Bob's act of accepting prepares the occupation the probe detects, and the register the write reaches stands for Alice's means of action.

- *Devices identified with people* (Chapters 9, 10, 13 and 14): the Sacrifice converter's two comparisons are Alice's and Bob's internal bonds, which Courage also uses; a converter's comparison is an internal bond of its owner; and each participant's preparation contact acts only in that participant's own origin sector, with the address equal to the retained claim.

- *Conventions on account books* (Chapter 8 and Appendix B): amounts are measured above the unit baseline 1, and every unserved term of the service comparison sits at the common multiplier 2. The cost values these give belong to the conventions; on the books of Chapter 8, the rise in cost after a truthful correction does not.

- *The rules of the economy* (Chapter 15): custody, the moral decision rule, whose zero allowance for added burden rejects Sacrifice, and the acceptance share. Which entitlements hold has to come from

how holdings and undertakings formed, and what selects one agreed value inside the acceptance interval belongs to the same derivation.

- *The parameters of the virtue rules* (Table 11.2): the constants of Compassion, Sacrifice and Gratitude, powers of the golden ratio together with Compassion's tenth and Sacrifice's twentieth, until a law of the reserve or the response fixes them; the menu and score Wisdom is given; Prudence's penalty and weight; Temperance's self-similar division, which fixes its cap once granted, and the golden-ratio fraction of its reserve limit; Humility's rate; and the mixture of Creativity's search.

In the second kind no result assumes an answer.

- *Which claim governs under scarcity* (Section 13.11): which requirements are active once a second undertaking competes for Alice's one unit, whether Bob's earlier acceptance gives his claim priority, whether Alice's proposal to Carol was itself a wrong to Bob, and who, if anyone, must release. Neither undertaking there has been written into a controller (Section 13.8), so the identification of Section 7.7 applies to neither, and nothing follows from that about whether either meal is owed. The derivation starts from how the two undertakings formed, including who holds each role in the first place.

- *Further undertakings and renewed preparation* (Chapter 7): the fixed four-interval undertaking has a physical writer for every scheduled action on the detected branch, with its clock and action-mode reservoirs prepared (Section 7.8.4). What remains is installation for further undertakings, Carol's included, and the available preparation and recharge that repeated execution requires; also who is answerable on a branch where the probe misses Bob's acceptance and nothing is installed (Section 7.6).

- *The release after reliance* (Chapter 9): what, if anything, Alice owes once Bob has relied and then released her, judged from the physical history the record cannot see.

- *Consent beyond authorship* (Chapter 8): when an authored acceptance is also informed and uncoerced, and which release designations are valid, each argued from the authored history.

- *The information term of value* (Chapter 12): whether the information contribution of a physical value law keeps the calibrated mean output when branch devices are assembled under an independent retained flag, and then a general conditional-information rule for physical refinements.

- *Virtues as dispositions* (Chapter 5): that a person who has a virtue is one whose retained records reliably select its operation when its conditions hold.

- *What particular virtues still need* (Chapters 9–11): the Sacrifice converter under Alice's sole control and with her informed willingness; Forgiveness's authority to release, from the creditor's agency and consent; Gratitude's trigger tied to the actual act and its author; Prudence's treatment of uncertainty; Courage's principle for choosing and completing an exposing action; one governed decision combining Wisdom, Prudence, Temperance and Courage; for Patience, which process a decision waits on and its period; Humility's assessment register and payment law; the choice among Hope's feasible paths; Creativity's exploration rule; and whether the fourteen generate a class that also retains the relevant participants, histories, resources and requirements.

- *The subject law* (Chapter 16): which organizations of recognition constitute one subject's experience, and so when a severing ends an actual subject; and, for the whole to be one subject, a subject space whose compactness and closed membership are grounded in actual subject histories.

- *One physical execution of Alice's care* (Section B.14): an admitted schedule of available contacts, a release contact that survives interruption, a common energy unit for the packet holdings and the memory, a recharge of the writer's reservoir from a packet transition of matching energy through an available coupling, the release work funded on the same ledger, and the postings, packets and

deliveries connected to paid work and Bob's intake on the care calendar.

An objective moral structure does not exempt moral judgment from investigation. It gives the investigation something definite to discover, and something definite against which an error can be recognized.

Chapter 18

Living together in one reality

Return to Alice at the start of the first interval, with Bob waiting. She can bring the meal or stay where she is, and both choices are physically available. Both leave every bond balanced, and both leave Alice a positive reserve: she ends the interval with 3/2 if she brings the meal and 7/4 if she stays. Only one maintains Bob at the checkpoint. Fed, he reaches it with 1/2. Unfed, he reaches it with nothing, and the checkpoint is missed.

The difference is in the world: in the food consumed, the work performed, the reserves that remain, the undertaking Bob accepted and the action its retained content requires. No change of description gives Bob the capacity that omission removes.

The moral question becomes exact when those facts are connected. The installed record retains the proposal, the acceptance and the terms; its comparator tells performance from violation, and once the accepted terms are written, the installed controller selects feeding as Alice's first action, a selection fixed before she acts (Section 7.6). With the service clock and action-mode reservoirs prepared, the same accepted record controls all four scheduled action selections (Section 7.8.4). The register it writes is taken, as a premise of Chapter 7, to stand for Alice's own means of action. Staying home remains executable all the same: omission is Alice's running another controller, one that selects idle, and the responsibility for it is hers as the one who runs it. The governing reading does not depend on which controller runs. It is fixed from the authored history and the ledger alone, as the one complete reading whose performances are exactly the legal actions that close the reliance Bob authored, and it rejects that execution (Section 13.1). A law governing conduct need not make contrary conduct impossible.

18.1 Freedom acts within consequences

An agent's available choices matter, and so do the means of carrying them out. An intention without an executable action cannot deliver food. A command without accessible stock cannot produce intake. An instruction to go on indefinitely cannot replenish an exhausted reserve.

These limits do not make deliberation irrelevant. They tell it what to address. From Alice's 2 and Bob's 1/4, with at most two meals, two four-interval schedules keep both of them maintained, and no schedule reaches a fifth checkpoint (Section 6.4). Choosing between feasible schedules and confronting an impossible horizon are different tasks.

18.2 Responsibility follows the actual relation

An undertaking has a recipient, content and history. A replacement controller must preserve those distinctions if it is to preserve the undertaking. One that delivers the same meals but changes who may propose, accept or release runs a different undertaking, and a history of at most three events, a proposal, an acceptance and a release, exposes it (Section 13.2). Explanations of responsibility face the same demand. They must identify the actor who controls the relevant action, the participant who undergoes its effect, the requirement at issue and the resources available at the time.

Now give Alice a second undertaking, to Carol, with one food unit for both (Chapter 13). Whoever Alice feeds comes through the next idle interval with a balanced bond and a quarter unit of reserve; the other is left at multiplier 2 with nothing to repair it. On the retained history, restoring the bond a recipient's own undertaking affected and leaving that recipient able to repair after the next interval are the same condition, and no available operation meets it for both (Section 13.9).

Which claim governs while both are active, and whether Alice wronged Bob by making the second promise, has not been derived. Neither undertaking in this history has been written into a controller (Section 13.8), so the identification of what Alice owes with what an installed agreement requires applies to neither. Bob's acceptance came first and is still a fact, but that does not by itself identify the governing relation once a second undertaking competes for the same resource. The derivation has to start from how the two undertakings actually formed,

and keep both people, the one shared unit and the history that produced the shortage (Section 13.11).

18.3 Care has a physical cost to carry

Conserving repair brings two bonds to equal multipliers, and who bears the change is still personal. When Alice's own bond at 1 is equalized with Bob's own bond at 4, the total falls and Bob is relieved while Alice's burden rises from 0 to 1/4: Love supplies the conserving endpoint, and Sacrifice names the burden Alice takes on in reaching it (Section 5.3). The decision rule adopted for the economy of Chapter 15 allows an actor no added burden and so rejects this move; the conflict belongs to that adopted rule, whose derivation must settle when actors may take burden on themselves. A lower total can coexist with an exhausted helper.

Care therefore includes the capacity to keep caring. An account of care counts Alice's reserve alongside Bob's intake, because the relation through which help is possible needs both.

18.4 Truth and repair change what comes next

Provision, release and correction are different objects with different physical consequences. An authorized creditor release changes the amount due while the accepted terms and the fact of delivery stay in the history. An accurate correction can reveal an imbalance that a false agreement concealed. A meal after a missed checkpoint can return Bob to the reserve 1/4 he would have had if fed on time, and the missed checkpoint stays true (Section 6.6).

These facts give repair a forward direction. The past supplies the condition from which action begins, and a remedy changes the present possibilities: it can fund another interval, restore a truthful record, reopen a cooperative future or remove an outstanding claim through the act of the person authorized to release it.

Repair is neither powerless because it cannot rewrite the past nor complete because a current number has improved. Its success concerns the particular effect, requirement or capacity it restores, and the history says what that success means.

18.5 The shared world retains each life

Economic life extends these relations across production, exchange and institutions. Resources change form, effort is spent, goods move between people, and records preserve or conceal who supplied them and what remains due. Every description must answer to one execution in which a resource is used once and each effect arrives at an actual participant.

The whole is therefore more demanding than an aggregate. A favorable total can miss the person to whom an act mattered. A lower total burden does not show that every member is maintained, an equal balance does not show equal authorship, and a shared substrate does not erase local subjects. A description adequate for a whole must keep the distinctions needed to understand its members' lives. That remains the task when the participants are artificial, when a controller is replaced, or when a group acts through an institution: the same physical objects and transformations, examined under the same standard.

18.6 A moral science can be corrected

A scientific account earns its standing by making its arguments inspectable, and each result here carries its own kind of necessity. The cost is forced: a fixed apparatus with a continuous report, requirements on how any linear energy comparison is reported, and a law for combining comparisons each leave only J once the unit is fixed; Love's endpoint is the strict minimum of a convex cost at a conserved product; the care criterion is an exact condition for a schedule that maintains both people to exist; the governing reading is unique once completeness and faithfulness to Bob's reliance are required, and the installed record grounds exactly that reading. A proved consequence can be used, a counterexample can correct a claim, a physical premise can be investigated, and an incomplete connection can be named precisely enough that another person can work on it. The aim is an account of morality that survives that examination because it follows what reality contains: beings who act and undergo, relations that carry effects, capacities that can be sustained or exhausted, and histories that give actions their particular consequences.

For Alice, at the start of the interval, the account already does something definite. Nothing in physics stops her from staying where she is. What the account gives her, on the premises it names, is a description of

everything the choice turns on. If she stays, Bob reaches the checkpoint with nothing. If she brings the meal, both of them can go on. The undertaking they formed selects the meal, and what an installed agreement selects is what it requires: that is the identification Chapter 7 adopts, and Bob's reliance on her promise is the reason to adopt it (Section 7.7). A meal brought later could restore Bob's capacity but never the checkpoint he missed. The law does not move her hands. It tells her exactly what each choice does to Bob, and which one her promise requires.

We live in the world our actions leave behind. To understand its moral structure is to know how to act within it without losing the lives, relations and possibilities that make living together possible.

Appendix A

Records, information and calibrated comparison

Chapter 12 asks what a number attached to a joint record of agent and environment can mean. The first three sections use two alternatives to mutual information to show what the information conditions select. The next three concern the comparison of a record with its product-marginal reference: what copies of the record can prepare that reference, the range of comparison ratios and their additive scale. The seventh builds the recording bank that realizes the pooled meter and computes its report on Alice's two records. The last shows that a local calibration leaves a rule for combining two people's values free, and a global relation fixes it. The joint probabilities $p(a, e)$, their agent and environment marginals p_Y, p_E and the mutual information (12.1) are those of Chapter 12. All sums are finite, joint probabilities are nonnegative and sum to one, and logarithms are natural.

A.1 The support measure and its endpoint jump

The support measure S of Section 12.3.1 sees only which joint states can occur. We prove that it shares three structural properties with mutual information, agrees with it on the perfectly correlated fair binary pair, and drops to zero as soon as any noise is added.

Define the product-marginal mass of the positive joint support by

$$\mu(p) = \sum_{\substack{a,e \\ p(a,e)>0}} p_Y(a)p_E(e), \qquad S(p) = -\log \mu(p).$$

At least one joint entry is positive, so $\mu(p) > 0$. Its summands are some of the nonnegative product-marginal entries, whose complete sum is one. Therefore $\mu(p) \leq 1$ and $S(p) \geq 0$.

If every joint state has positive probability, then $\mu(p) = 1$ and $S(p) = 0$, even when those probabilities exhibit dependence.

The three properties follow. For independent pairs of systems, the positive support and the product marginals both factor, so μ multiplies and S adds. If an agent state is divided into substates whose weights sum to one and are independent of the environment within that state, the product-marginal mass is unchanged, so the refinement leaves S unchanged. Finally, suppose p and q have the same marginals and $0 < t < 1$. Every positive entry of either distribution remains positive in

$$\bar{p} = tp + (1 - t)q,$$

and the product marginals are unchanged, so $\mu(\bar{p}) \geq \mu(p)$ and $\mu(\bar{p}) \geq \mu(q)$. Taking negative logarithms, $S(\bar{p})$ is at most the smaller of $S(p)$ and $S(q)$, and hence at most their weighted average:

$$S(\bar{p}) \leq tS(p) + (1 - t)S(q).$$

At $t = 0$ or 1, the inequality is an equality.

Proposition A.1 (An explicit information alternative). *The support measure S agrees with mutual information on a perfectly correlated uniform binary pair but differs from it on a binary pair with positive probability in every cell. It is discontinuous at the perfectly correlated endpoint of the family that mixes this pair with the uniform independent pair.*

Proof. For the perfectly correlated pair, the two diagonal entries are $1/2$ and the two off-diagonal entries are zero. Both marginals are uniform, so each occupied cell contributes $1/4$ to the product-marginal mass, which is therefore $1/2$. Each occupied cell also contributes $\frac{1}{2} \log \frac{1/2}{1/4}$ to mutual information. Hence

$$S = \log 2 = I.$$

Mix this distribution with the uniform independent pair, giving the correlated pair weight $t \in [0, 1]$. The diagonal entries become $(1 + t)/4$, and the off-diagonal entries become $(1 - t)/4$. The marginals remain uniform.

For every $t < 1$, all four entries are positive. Hence $\mu = 1$ and $S = 0$. At $t = 1$, $S = \log 2$. This proves the endpoint discontinuity.

At $t = 1/2$, direct substitution in (12.1) gives

$$I = \frac{3}{4} \log \frac{3}{2} + \frac{1}{4} \log \frac{1}{2} = \frac{3}{4} \log 3 - \log 2 > 0.$$

The strict inequality follows from $27 > 16$ after multiplying by four and combining logarithms. At the same distribution $S = 0$, so the quantities differ. □

The three structural properties therefore leave room for S; its endpoint jump is what Section A.2 excludes. The counterexample concerns the information term only; the recognition cost of the bonds keeps its own derivation.

A.2 Uniform reports from finite blocks

A numerical information value has to be reported by physical records, and records arrive in finite blocks. We prove that any report computed from a fixed finite block changes continuously with the preparation, so a functional reportable to arbitrary uniform precision in this way is continuous as noise vanishes. The support measure is not.

Let a joint record have alphabet $\mathcal{X} = \{0,1\} \times \{0,1\}$. Write N_1 for the perfectly correlated distribution, with probability $1/2$ at each of $(0,0)$ and $(1,1)$, and N_0 for the uniform distribution on all four pairs. The noisy family is

$$N_t = tN_1 + (1-t)N_0, \qquad 0 \le t \le 1.$$

Its diagonal probabilities are $(1+t)/4$ and its off-diagonal probabilities are $(1-t)/4$. Each marginal remains uniform.

A finite-block report receives m independent preparations of the joint record. Independence is between preparations; the two components within each record keep their correlation. A possible transcript is $\ell = (\ell_1, \dots, \ell_m) \in \mathcal{X}^m$. Fix a real calibrated output $s(\ell)$ for each transcript. For a distribution N on $\mathcal{X}$, the mean report is

$$M_{m,s}(N) = \sum_{\ell \in \mathcal{X}^m} s(\ell) \prod_{i=1}^{m} N(\ell_i).$$

The score stays fixed while preparations are compared. A stochastic output stage is covered too, with $s(\ell)$ its conditional mean. The model concerns expectations: it gives no bound on sampling error, and it does not presume that every mathematically specified score can be built.

Proposition A.2 (Continuity of a fixed finite-block report). *For every finite m and fixed score s, the mean report $M_{m,s}(N_t)$ is continuous at $t = 1$.*

Proof. Each probability $N_t(\ell_i)$ is affine and hence continuous in t. Finite products preserve continuity, as does multiplication by the fixed number $s(\ell)$. The sum ranges over a finite set of transcripts, so the complete mean report is continuous, including at the perfect-correlation endpoint. □

Say that a functional Φ is *uniformly reportable by finite blocks* on this family when, for every $\varepsilon > 0$, there are a finite m and a fixed score s such that

$$|\Phi(N_t) - M_{m,s}(N_t)| < \varepsilon \quad \text{for every } 0 \le t \le 1,$$

the endpoint N_1 included. The apparatus and the block length may depend on the requested accuracy. They cannot depend on an undisclosed exact noise parameter within the family they are supposed to report.

Proposition A.3 (Uniform finite reporting implies endpoint continuity). *If Φ is uniformly reportable by finite blocks on this family, then $\Phi(N_t)$ tends to $\Phi(N_1)$ as t tends to one from below. Consequently, the support measure S cannot have this reporting property.*

Proof. Given $\varepsilon > 0$, choose a report with uniform error less than $\varepsilon/3$. Its continuity supplies a neighborhood of one in which its own change is less than $\varepsilon/3$. In that neighborhood, the triangle inequality gives

$$\begin{aligned} |\Phi(N_t) - \Phi(N_1)| \le {} & |\Phi(N_t) - M_{m,s}(N_t)| \\ & + |M_{m,s}(N_t) - M_{m,s}(N_1)| \\ & + |M_{m,s}(N_1) - \Phi(N_1)| < \varepsilon. \end{aligned}$$

For the support measure, each occupied pair contributes the product of its marginal probabilities, and the functional is minus the logarithm of their sum. Whenever $t < 1$, all four pairs have positive probability. The four marginal products are $1/4$, so their sum is one and the value is zero. At N_1, only the two diagonal pairs remain occupied. The sum is $1/2$, so the value is $\log 2$. Its endpoint jump contradicts the continuity just derived. □

Exact support separates a zero probability from every positive probability, however small. A finite block cannot follow that separation: as the noise vanishes, off-diagonal records become rare in a block of fixed length, and the mean report barely moves. Longer blocks buy accuracy, but the report at each block length still moves continuously, and a uniform limit of continuous reports is continuous. Whatever finite records

report uniformly on this family therefore changes continuously as the noise vanishes.

A.3 A continuous reportable alternative

Finite reporting excludes the jump without selecting mutual information. The half-order measure R of Section 12.3.2 satisfies every condition so far: independent composition, the restricted refinement, the same-marginal mixture inequality, the perfect-binary calibration, and uniform finite-block reporting on the binary family. We prove each, and then that on the same records it differs strictly from mutual information.

Let p_{ae} be a normalized nonnegative joint distribution on finite agent and environment alphabets. Write $p_a = \sum_e p_{ae}$ and $p_e = \sum_a p_{ae}$. Define its affinity with its product of marginals and its order-one-half information by

$$\beta(p) = \sum_{a,e} \sqrt{p_{ae}}\sqrt{p_a}\sqrt{p_e}, \qquad R(p) = -2\log\beta(p).$$

These use the same joint table and marginals as mutual information. At least one joint entry is positive. Both its marginals are then positive, so its contribution to β is positive. Every other contribution is nonnegative. Thus $\beta(p) > 0$, and the logarithm is defined.

Independent composition. For independent joint records p and q, their joint probabilities and both marginals factor. The square root of each nonnegative product factors too. The affinity contribution at a paired index is therefore the product of the two original contributions. Summing the finite product table gives $\beta(p \otimes q) = \beta(p)\beta(q)$. Consequently $R(p \otimes q) = R(p) + R(q)$.

The refinement condition. The refinement replaces each agent label a by a finite collection of labels a' with that parent. It supplies nonnegative weights $\lambda_{a'}$ whose sum within each parent is one, and a normalized environment distribution q^a_e for each parent. The coarse and refined joint tables obey

$$p_{ae} = p_a q^a_e, \qquad p'_{a'e} = \lambda_{a'} p_a q^a_e = \lambda_{a'} p_{ae}.$$

The distribution q^a is supplied even when $p_a = 0$, so no division by a zero marginal is needed. The refined agent marginal is $\lambda_{a'} p_a$, and the

environment marginal is unchanged. Hence

$$\sqrt{p'_{a'e}}\sqrt{p'_{a'}}\sqrt{p'_e} = \lambda_{a'}\sqrt{p_{ae}}\sqrt{p_a}\sqrt{p_e}.$$

This identity also holds when an entry or a weight is zero. Summing within each parent gives $\beta(p') = \beta(p)$ and therefore $R(p') = R(p)$.

The conditional table associated with parent a is $p^{|a}_{a'e} = \lambda_{a'}q^a_e$. Its two marginals are $\lambda_{a'}$ and q^a_e, so it factors. For this normalized factorized table,

$$\beta = \sum_{a',e}\sqrt{\lambda_{a'}q^a_e}\sqrt{\lambda_{a'}}\sqrt{q^a_e} = \sum_{a',e}\lambda_{a'}q^a_e = 1.$$

Thus $R(p^{|a}) = 0$ for every parent: factorized conditional tables have zero information, as the conditions of Chapter 12 require. The refinement condition then holds in chain form, the refined record carrying the coarse record's information plus the parent-weighted information of its conditional tables, which is zero:

$$R(p') = R(p) + \sum_a p_a R(p^{|a}) = R(p).$$

The refinement keeps the environment distribution within each parent; it does not allow a finer label to acquire additional environment information.

Mixtures with fixed marginals. Take p and q on the same finite alphabets, with equal agent and environment marginals, and let $\bar{p} = tp + (1-t)q$, where $0 \le t \le 1$. Concavity of the square root, multiplied by the common nonnegative marginal factors and summed, gives

$$\beta(\bar{p}) \ge t\beta(p) + (1-t)\beta(q).$$

The logarithm is increasing and concave on positive arguments. Therefore

$$\log\beta(\bar{p}) \ge \log(t\beta(p) + (1-t)\beta(q)) \ge t\log\beta(p) + (1-t)\log\beta(q).$$

Multiplying by -2 reverses the inequality and gives the mixture inequality $R(\bar{p}) \le tR(p) + (1-t)R(q)$.

Calibration and continuous removal of noise. For the perfect unbiased binary correlation N_1, the two nonzero joint entries are $1/2$, and both marginals are $1/2$. Thus $\beta = 1/\sqrt{2}$ and $R = \log 2$. Its perfect-binary calibration coefficient is one, just as for mutual information: $\kappa_R = R(N_1)/\log 2 = 1$.

On the binary family N_t of Section A.2, the marginals remain $1/2$. The affinity is a finite sum of square roots of affine probabilities, multiplied by fixed marginal factors, so it is continuous, including at $t = 1$. Its affinity there is positive, so $R(N_t)$ tends to $R(N_1)$ as the noise vanishes.

A strict disagreement on the same records. At $t = 1/2$, the two diagonal joint entries are $3/8$ and the two off-diagonal entries are $1/8$. Put $x = \sqrt{2/3}$ and $y = \sqrt{2}$. Direct summation gives

$$\beta(N_{1/2}) = \sqrt{3/8} + \sqrt{1/8} = \tfrac{3}{4}x + \tfrac{1}{4}y.$$

For the same table, the product of the marginals is $1/4$ at every cell. The finite mutual-information sum therefore gives

$$I(N_{1/2}) = 2\frac{3}{8}\log\frac{3/8}{1/4} + 2\frac{1}{8}\log\frac{1/8}{1/4} = \frac{3}{4}\log 3 - \log 2.$$

The positive numbers x and y differ. Strict concavity of the logarithm therefore gives

$$\log\beta(N_{1/2}) > \tfrac{3}{4}\log x + \tfrac{1}{4}\log y.$$

Using $2\log x = \log 2 - \log 3$ and $2\log y = \log 2$, we obtain

$$R(N_{1/2}) < \tfrac{3}{4}\log 3 - \log 2 = I(N_{1/2}).$$

The two measures share their calibration and differ strictly on the same table. Composition, restricted refinement, mixture and regularity, even with continuous removal of binary noise, therefore do not single out mutual information.

An explicit finite-record reporting construction. The alternative also satisfies the reporting condition of Section A.2, with the block length and scores fixed for each requested error before t is known.

The idea is to build a score whose expectation is exactly t, take products to obtain every power of t, and then approximate the continuous curve

$R(N_t)$ by a polynomial. A letter is one joint binary record (a, e). Give it the score

$$Z(a,e) = \begin{cases} -3, & (a,e) = (0,1), \\ 1, & \text{otherwise.} \end{cases}$$

In this binary family, the probability of $(0,1)$ is $(1-t)/4$. Consequently

$$\mathbb{E}_t Z = 1 - 4 \Pr_t\{(0,1)\} = t.$$

Take n independently prepared joint records. For $0 \le k \le n$, the score formed by multiplying the first k letter scores has expectation t^k. The empty product for $k = 0$ is one.

For a real polynomial $P(t) = \sum_{k=0}^{n} c_k t^k$, give the whole block the fixed score

$$Z_P(\ell_1, \dots, \ell_n) = \sum_{k=0}^{n} c_k \prod_{j=1}^{k} Z(\ell_j).$$

Finite sums and independence give the exact identity

$$\mathbb{E}_t Z_P = \sum_{k=0}^{n} c_k t^k = P(t).$$

This specifies one finite set of scores for each polynomial, including negative scores where the formula requires them. A constant polynomial uses a block of length zero: there is one empty block, with probability one, and its score is the constant.

The curve $g(t) = R(N_t)$ is continuous on the entire closed interval $[0,1]$: its affinity is continuous there and positive at every point, so taking the logarithm preserves continuity. The Weierstrass approximation theorem gives, for every $\varepsilon > 0$, one real polynomial P such that

$$\forall t \in [0,1], \qquad |P(t) - g(t)| < \varepsilon.$$

The block score just constructed therefore reports R uniformly to that accuracy, including at the perfectly correlated endpoint. The approximation theorem is standard; Bernstein's proof is given because it fixes the polynomial, and so the block score, before any record arrives.

Proof of the approximation, after Bernstein. For a positive integer d, set

$$b_{d,k}(t) = \binom{d}{k} t^k (1-t)^{d-k}, \qquad P_d(t) = \sum_{k=0}^{d} b_{d,k}(t)\, g(k/d).$$

The weights are nonnegative, sum to one and obey the exact variance identity

$$\sum_{k=0}^{d} b_{d,k}(t)\,(t - k/d)^2 = \frac{t(1-t)}{d}.$$

Continuity of g on the closed interval gives uniform continuity and a finite bound K on $|g(u) - g(v)|$. Choose $\delta > 0$ so that this difference is less than $\varepsilon/2$ whenever $|u - v| < \delta$. Since the weights sum to one, $|P_d(t) - g(t)|$ is at most the weighted sum of $|g(k/d) - g(t)|$. Indices with $|k/d - t| < \delta$ contribute at most $\varepsilon/2$, because their weights sum to at most one. For every other index $1 \le (t - k/d)^2/\delta^2$, so the variance identity bounds their contribution by

$$\frac{K\,t(1-t)}{d\delta^2} \le \frac{K}{d\delta^2}.$$

A degree d that makes the final expression less than $\varepsilon/2$ works for every t in the interval at once. It fixes the polynomial, and with it the finite report, before any records are received. □

The support measure fails the reporting condition and the half-order measure passes it, so selecting mutual information needs a further condition; Section 12.6 gives one that separates it from R.

A.4 Preparing the reference from coherent copies

The comparison of Section 12.4 reports R on a record once the record and its reference are both present as signals. Here one fixed exact operation, chosen without knowing which record is supplied, acts on coherent copies (square-root signals) of every record in a family. One copy never suffices for every record, two suffice for every perfectly correlated binary record, and no fixed number suffices for every binary record.

A.4.1 A boundary on preparing the reference from one coherent copy

Two four-cell records show why one copy is not enough: the required outputs hold the overlap of the full output vectors, environment included, below the overlap of the inputs, and an isometry preserves overlaps.

Let the two labels on each side be 0 and 1, and order the joint carrier as $00, 01, 10, 11$. Consider two normalized joint tables. The first puts all its probability at 00. The second puts probability $1/2$ at each of 00 and

11. Their coherent square-root vectors are

$$\psi_0 = |00\rangle, \qquad \psi_d = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

For the first table both marginals are concentrated at 0, so its product-marginal reference is again concentrated at 00. For the second, each marginal assigns probability $1/2$ to both labels. Its product table therefore assigns probability $1/4$ to every joint cell. The required pure reference vectors on the same carrier are

$$\chi_0 = |00\rangle, \qquad \chi_d = \frac{|00\rangle + |01\rangle + |10\rangle + |11\rangle}{2}.$$

In particular,

$$\langle \psi_0, \psi_d \rangle = \frac{1}{\sqrt{2}}, \qquad \langle \chi_0, \chi_d \rangle = \frac{1}{2}.$$

The operation class is the following. Let Ω be any finite set of environment coordinates, and let

$$\Gamma : \mathbb{C}^{\{00,01,10,11\}} \longrightarrow \mathbb{C}^{\{00,01,10,11\}\times\Omega}$$

be a fixed complex linear isometry. It preserves inner products. All environment coordinates may be discarded after this operation. For an output vector ψ', the resulting density matrix on the four-cell carrier has entries

$$\rho_{\psi'}(i,j) = \sum_{\omega\in\Omega} \psi'(i,\omega)\overline{\psi'(j,\omega)}.$$

Exact preparation of a pure target χ means $\rho_{\psi'}(i,j) = \chi(i)\overline{\chi(j)}$ for every pair of carrier cells. This condition uses the full reduced density matrix, including coherence.

Theorem A.4. *No such fixed isometry produces the pure reference χ_0 from ψ_0 and the pure reference χ_d from ψ_d, even when its finite environment is discarded.*

Proof. Suppose it does, and write $\psi'_0 = \Gamma\psi_0$ and $\psi'_d = \Gamma\psi_d$. For every carrier cell $i \neq 00$, the required first output has diagonal entry zero. Hence

$$\sum_{\omega\in\Omega} |\psi'_0(i,\omega)|^2 = 0.$$

Each term is nonnegative, so $\psi'_0(i,\omega) = 0$ for every such i and every ω. The diagonal entry at 00 is one. Thus the environment row $\xi_0(\omega) = \psi'_0(00,\omega)$ satisfies $\|\xi_0\|^2 = 1$.

The required second output has diagonal entry $1/4$ at 00. Its environment row $\xi_d(\omega) = \psi'_d(00, \omega)$ consequently satisfies $\|\xi_d\|^2 = 1/4$. Because every other carrier row of ψ'_0 vanishes, the complete output inner product reduces to these two rows:

$$\langle \psi'_0, \psi'_d \rangle = \sum_{\omega \in \Omega} \overline{\xi_0(\omega)} \xi_d(\omega) = \langle \xi_0, \xi_d \rangle.$$

Cauchy–Schwarz now gives

$$|\langle \psi'_0, \psi'_d \rangle|^2 \leq \|\xi_0\|^2 \|\xi_d\|^2 = \frac{1}{4}.$$

But Γ preserves the input inner product, whose squared magnitude is $1/2$. This is a contradiction. □

A universal preparer in this operation class would have to handle these two inputs, so none exists.

A.4.2 A reference prepared from two coherent copies

A record that always names the right store, whatever the odds between the stores, is perfectly correlated. For such records two copies produce the pure product-marginal reference at every probability weight, by the same two reversible operations throughout the family.

Let $0 \leq t \leq 1$ and define a joint probability distribution on two binary labels by

$$p_t(0,0) = 1 - t, \qquad p_t(1,1) = t, \qquad p_t(0,1) = p_t(1,0) = 0.$$

These probabilities are nonnegative and sum to one. Write $\mu_t(0) = 1 - t$ and $\mu_t(1) = t$. Both marginals are μ_t: summing a row or a column leaves its sole diagonal entry. The coherent joint record and its desired reference are therefore

$$|\psi_t\rangle = \sum_{a \in \{0,1\}} \sqrt{\mu_t(a)}\, |a, a\rangle, \qquad |\chi_t\rangle = \sum_{a,a' \in \{0,1\}} \sqrt{\mu_t(a)} \sqrt{\mu_t(a')}\, |a, a'\rangle.$$

The second vector is exactly the square-root encoding of the product of the two marginals.

Use four binary coordinates (a, e, a', e'), with the first pair holding one prepared copy and the second pair holding another. Their full carrier has sixteen modes. The input is

$$|\psi_t\rangle \otimes |\psi_t\rangle = \sum_{a,a'} \sqrt{\mu_t(a)} \sqrt{\mu_t(a')}\, |a, a, a', a'\rangle.$$

It has squared norm $((1-t)+t)^2 = 1$. Let $\oplus$ denote binary exclusive-or. On the entire sixteen-mode basis, define

$$U_1|a,e,a',e'\rangle = |a,e\oplus a,a',e'\oplus a'\rangle,$$
$$U_2|a,e,a',e'\rangle = |a,a',e,e'\rangle.$$

On the supported inputs, where each copy's two labels agree, the first resets each second label to 0; the second swaps the middle coordinates. Each operation is its own inverse. Each permutes the full orthonormal basis, so its linear extension preserves norm and inner product on the whole carrier; each is also self-adjoint.

Theorem A.5 (Two-copy reference preparation). *For every $0 \le t \le 1$, the fixed operations above satisfy*

$$U_2U_1(|\psi_t\rangle \otimes |\psi_t\rangle) = |\chi_t\rangle \otimes |0,0\rangle.$$

The first two output coordinates carry the required pure product-marginal reference. The remaining two coordinates are in the same ready state for every t.

Proof. On every supported input basis vector, the first operation gives

$$|a,a,a',a'\rangle \longmapsto |a,0,a',0\rangle,$$

because a binary label exclusive-or itself is zero. The second operation sends this to $|a,a',0,0\rangle$. Applying these operations to the displayed input sum leaves the coefficient of $|a,a',0,0\rangle$ equal to $\sqrt{\mu_t(a)}\sqrt{\mu_t(a')}$. This is the corresponding coefficient of $|\chi_t\rangle \otimes |0,0\rangle$; every coefficient with a nonzero environment label vanishes.

To check the output after the environment is discarded, let i,j denote the first two coordinates and let ω denote the last two. If Ψ is the resulting full vector, its reduced density entries are

$$\sum_{\omega} \Psi(i,\omega)\overline{\Psi(j,\omega)} = \chi_t(i)\overline{\chi_t(j)}.$$

Only $\omega = (0,0)$ contributes. Thus the reduced output is the pure state given by $|\chi_t\rangle$. The argument uses no division by t or $1-t$, so it includes both endpoints. □

Both operations are self-adjoint involutions, so Proposition 7.6 gives each an exact positive-generator realization: for a strength $g > 0$, the generator $g(I - U)$ enacts U at time $\hbar\pi/(2g)$. Applying this pulse for U_1 and then for U_2, each with its own positive strength, prepares exactly the vector and reduced state proved above.

The second copy is what makes this possible. The family contains the point record at $t = 0$ and the fair diagonal record at $t = 1/2$, the two inputs of the one-copy obstruction, so no fixed one-copy dilation prepares all its references. Nothing is cloned: two copies are present at the start, and both are transformed. The output holds no untouched joint operand, so using this reference in the comparison requires a separately retained or prepared joint copy.

A.4.3 Finite copies do not give universal exact preparation

A record that is sometimes wrong lacks the structure the two-copy construction uses, and for general binary records no fixed finite number of copies restores exact universal preparation.

Theorem A.6 (Finite-copy reference preparation boundary). *Fix an integer $k \geq 1$ and any finite environment. There is no fixed complex linear isometry from k independently supplied coherent binary joint records to a binary joint output and that environment whose discarded-environment output is the pure product-marginal reference for every nonnegative binary joint distribution. The map and its environment are fixed independently of the supplied distribution.*

The obstruction concerns the reference itself, before the comparison apparatus receives its operands. The proof uses a family of four-cell joint records. Whatever the fixed map, one entry of its reduced output is a polynomial in the family parameter divided by a power of $2 + t^2$, while the required entry contains $\sqrt{1 + t^2}$; squaring and clearing the denominators gives a polynomial identity, and evaluating it at $T = i$ exposes the contradiction. The argument accounts for all the copies and the complete finite environment, with no postselection or input-dependent rescaling.

Proof. Write the four joint cells in the order $00, 01, 10, 11$. It suffices to consider the following family, indexed by a real number $t \geq 0$:

$$w(t) = 2 + t^2, \qquad p_t = \frac{1}{w(t)} \begin{pmatrix} 1 & t^2 \\ 1 & 0 \end{pmatrix}, \qquad |\psi_t\rangle = \frac{|00\rangle + t|01\rangle + |10\rangle}{\sqrt{w(t)}}.$$

All probabilities are nonnegative and their sum is $(1 + t^2 + 1)/w(t) = 1$. Thus $|\psi_t\rangle$ is precisely the coherent square-root record of p_t, with no negative-amplitude convention hidden in the preparation. Its two marginals are

$$p_{t,Y} = \left(\frac{1+t^2}{w(t)}, \frac{1}{w(t)}\right), \qquad p_{t,E} = \left(\frac{2}{w(t)}, \frac{t^2}{w(t)}\right).$$

Let $|\chi_t\rangle$ denote the required pure reference, whose amplitude in cell ae is $\sqrt{p_{t,Y}(a)p_{t,E}(e)}$. The $00, 10$ entry of its density matrix is therefore

$$\chi_t(00)\overline{\chi_t(10)} = \sqrt{\frac{2(1+t^2)}{w(t)^2}}\sqrt{\frac{2}{w(t)^2}} = \frac{2\sqrt{1+t^2}}{w(t)^2}. \tag{A.1}$$

Now let V be any fixed complex linear map from the k-copy carrier to the output joint carrier and a finite environment. The argument does not yet need its isometry property. Index an input basis configuration by a list $\alpha = (\alpha_1, \dots, \alpha_k)$ of joint cells. Define four polynomials and their products

$$x_{00}(T) = 1, \qquad x_{01}(T) = T, \qquad x_{10}(T) = 1, \qquad x_{11}(T) = 0,$$
$$\Pi_\alpha(T) = \prod_{\ell=1}^{k} x_{\alpha_\ell}(T).$$

The input amplitude of configuration α in the independently supplied tensor product $|\psi_t\rangle^{\otimes k}$ is $\Pi_\alpha(t)/(\sqrt{w(t)})^k$. Write $v_{o\omega,\alpha}$ for the fixed matrix coefficient of V from configuration α to output joint cell o and environment cell ω. Linearity gives the output amplitude

$$\psi'_t(o,\omega) = \frac{Q_{o\omega}(t)}{(\sqrt{w(t)})^k}, \qquad Q_{o\omega}(T) = \sum_\alpha v_{o\omega,\alpha}\Pi_\alpha(T).$$

Each $Q_{o\omega}$ is a polynomial with fixed complex coefficients. Discarding the environment means summing its density entries:

$$\rho_t(o,o') = \sum_\omega \psi'_t(o,\omega)\overline{\psi'_t(o',\omega)}.$$

For a polynomial Q, let $Q^\#(T)$ be the polynomial obtained by conjugating each coefficient. Because t is real, $Q^\#(t) = \overline{Q(t)}$. Hence, for the particular entry $o = 00$, $o' = 10$,

$$\rho_t(00,10) = \frac{P(t)}{w(t)^k}, \qquad P(T) = \sum_\omega Q_{00,\omega}(T)Q^\#_{10,\omega}(T). \tag{A.2}$$

This is a polynomial numerator for the actual reduced output, with no postselection or input-dependent rescaling.

Suppose that this output were the required pure reference for every $t \geq 0$. Equations (A.1) and (A.2) would imply

$$P(t)w(t)^2 = 2\sqrt{1+t^2}\, w(t)^k.$$

Squaring gives

$$P(t)^2 w(t)^4 = 4(1+t^2)w(t)^{2k}.$$

Both sides are polynomials. Their agreement at every nonnegative integer already implies the polynomial identity over the complex numbers

$$P(T)^2(2+T^2)^4 = 4(1+T^2)(2+T^2)^{2k}. \tag{A.3}$$

Indeed, the difference of unequal polynomials has only finitely many roots.

Evaluate at $T = i$, where $i^2 = -1$. Since $2 + i^2 = 1$ and $1 + i^2 = 0$, the identity implies $P(i)^2 = 0$, so $P(i) = 0$. Differentiate (A.3) and evaluate again at i. Every term on the left contains a factor $P(i)$ and vanishes. On the right, the term differentiating $(2+T^2)^{2k}$ contains $1 + i^2$ and vanishes, while the remaining term is

$$4(2i)\,1^{2k} = 8i.$$

This would give $0 = 8i$, a contradiction. Thus even a fixed linear map cannot meet the required output on this family. In particular, a fixed finite isometric dilation followed by discarding its environment cannot meet it for all binary joint distributions. □

The two-copy construction escapes the theorem because a perfectly correlated family has structure that an arbitrary binary table lacks. The comparison of Section 12.4 takes the reference as an operand, the amplitudes $\sqrt{p_Y(a)p_E(e)}$ on the record's own labelled carrier, and once that operand is supplied it reports its proved ratio. No fixed number of copies under one fixed operation prepares that operand exactly for every binary record, so a comparison that reports R on every such record needs its reference prepared some other way. Whether approximate preparation, or an operation chosen with knowledge of the record, can supply it remains to be derived.

A.5 Every nonnegative comparison ratio has a finite preparation

The additive representation of Section A.6 uses the whole half-line $[0, \infty)$ as its domain. Every point of that domain is the comparison report of some finite joint preparation, so one reporting function serves them all. The carrier may need to grow with the desired report, but each preparation needs only finitely many labels, and both marginal distributions stay fixed throughout each preparation family.

Let the two records have the same n labels $0, \dots, n-1$, where $n \geq 1$. For $0 \leq t \leq 1$, define

$$p_t(a, e) = \frac{1-t}{n^2} + \frac{t}{n}\mathbf{1}_{a=e} \qquad (0 \leq a, e < n),$$

and put $p_t(a, e) = 0$ for labels outside this range. Taken as Alice's record of n stores, it copies the right store on a fraction t of mornings and is unrelated to the food on the rest. The first term spreads probability uniformly over all pairs. The second adds probability to the matching pairs. At $t = 0$ the records are independent and uniform. At $t = 1$ they agree perfectly, with each matching pair having probability $1/n$. As t moves from zero to one, the report moves continuously from 0 to $\sqrt{n} - 1$, and the intermediate value theorem fills in everything between. For $n = 2$ the top value is the fair binary report $\sqrt{2} - 1$ of Section 12.4.

Theorem A.7 (Finite preparation coverage). *For every real $r \geq 0$, there is a normalized finite joint distribution p whose two-operand comparison (Section 12.4), using its product-marginal reference, has raw report*

$$\frac{1 - \beta(p)}{\beta(p)} = r.$$

For each positive integer n, the family displayed above attains every report in $[0, \sqrt{n} - 1]$.

Here $\beta(p) = \sum_{a,e} \sqrt{p(a,e)}\sqrt{p_Y(a)}\sqrt{p_E(e)}$ is the affinity of Section 12.3.2.

Proof. Both terms defining p_t are nonnegative. For any fixed label a, there are n copies of the uniform term and exactly one diagonal contribution. Thus

$$\sum_{e=0}^{n-1} p_t(a, e) = n\frac{1-t}{n^2} + \frac{t}{n} = \frac{1}{n}.$$

The formula is symmetric in a and e, so the same calculation gives the environment marginal. Summing the n row masses gives total probability one. We have therefore constructed a joint distribution, with

$$p_{t,Y}(a) = p_{t,E}(e) = \frac{1}{n}$$

at every label and for every $t \in [0,1]$.

Its affinity consequently has the fixed-marginal expression

$$\beta_n(t) = \sum_{a=0}^{n-1} \sum_{e=0}^{n-1} \frac{\sqrt{p_t(a,e)}}{n}.$$

Each probability is affine in t, and the square-root function is continuous, including at zero. A finite sum of these terms is continuous. Also $\beta_n(t) > 0$: normalization gives at least one positive joint probability, and its two marginal factors are positive. The raw ratio

$$r_n(t) = \frac{1 - \beta_n(t)}{\beta_n(t)}$$

is therefore continuous throughout the closed interval $[0,1]$.

At $t = 0$, all n^2 joint probabilities equal $1/n^2$. Every affinity term is $1/n^2$, giving $\beta_n(0) = 1$ and $r_n(0) = 0$. At $t = 1$, only the n diagonal entries contribute. Each contributes $\sqrt{1/n}/n$, so

$$\beta_n(1) = \sqrt{1/n} = \frac{1}{\sqrt{n}}, \qquad r_n(1) = \sqrt{n} - 1.$$

The intermediate value theorem now gives, for every $0 \le r \le \sqrt{n} - 1$, some $t \in [0,1]$ with $r_n(t) = r$. This argument needs continuity and the endpoint values; it makes no monotonicity assumption.

Finally, fix any real $r \ge 0$. Choose a natural number $n > (r+1)^2$. It is positive, and the nonnegative square root satisfies $r \le \sqrt{n} - 1$. The preceding finite construction supplies the required t and hence the normalized finite joint distribution p_t. □

The calibrated relative meter of Section 12.4 reports this ratio for each preparation: its pooled residual and reference are $\gamma(1 - \beta(p))$ and $\gamma\beta(p)$, and the common positive energy coefficient γ cancels.

A.6 The calibrated additive representation

The comparison ratio of Section 12.4 composes under independent records by $r \star r' = r + r' + rr'$. Every continuous scale that turns this law into addition is a constant multiple of $\log(1 + r)$, and the calibration fixes the constant: in that coordinate the composition becomes addition, and a continuous additive function is linear.

We prove Theorem 12.2 on the domain $[0, \infty)$ supplied by Section A.5. It takes a continuous $F : [0, \infty) \to \mathbb{R}$ with $F(r \star r') = F(r) + F(r')$ for all $r, r' \geq 0$, a reference $r_* > 0$ and a real calibration $F(r_*) = \nu$, and concludes that $F(r) = \nu \log(1 + r)/\log(1 + r_*)$ for every $r \geq 0$, a formula that is itself continuous, additive and calibrated.

Proof. First $F(0) = F(0) + F(0)$, so $F(0) = 0$. The identity

$$1 + (r \star r') = (1 + r)(1 + r')$$

suggests the coordinate $t = \log(1 + r)$. For $t \geq 0$, set $h(t) = F(e^t - 1)$. This function is continuous, and

$$e^{a+b} - 1 = (e^a - 1) \star (e^b - 1)$$

gives $h(a + b) = h(a) + h(b)$ for $a, b \geq 0$.

Repeated addition gives $h(m) = mh(1)$ for nonnegative integers m. For a positive integer n, adding m/n to itself n times gives $nh(m/n) = h(m) = mh(1)$. Thus $h(q) = qh(1)$ for every nonnegative rational q. Every nonnegative real t is a limit of such rationals. Continuity then gives $h(t) = th(1)$. Returning to the original coordinate,

$$F(r) = h(1) \log(1 + r).$$

At the reference, $\nu = h(1) \log(1 + r_*)$. Its logarithm is positive because $r_* > 0$, so this equation fixes the coefficient and proves the formula.

Conversely, $1 + r > 0$ on the domain, so the logarithmic formula is continuous. The product identity and the logarithm's product rule prove its additivity. Substituting $r = r_*$ proves its calibration. □

A.7 The reversible recording bank and its energy

The pooled meter of Chapter 12 is the constant-output assembly of branch reporters (Section 12.6), carried out by a physical exchange: a reversible

recording bank, one channel per joint label, that reports the probability mass of a selected set of labels from the energy the exchange deposits in a second port. One energy reference serves every detector, so the energy scale cancels once, after pooling, and the report is exactly the selected occupation mass $\sum_{k\in\mathcal{J}}|\psi_k|^2$. The single reference is what makes the report a mass. Normalizing each occupied detector separately would make each of them report one, and the bank would count occupied detectors instead of weighing their load.

An elementary signal is a vector on the comparison carrier of Chapter 2. Let H be the half-turn there, which moves every entry four places and so pairs each with the one opposite it, and put

$$P = \tfrac{1}{2}(I - H), \qquad Q = I - P = \tfrac{1}{2}(I + H).$$

These are complementary orthogonal projections. The single-operand pulse of complex amplitude z is

$$c(z) = \frac{1}{\sqrt{2}}(z, -z, 0, 0, 0, 0, 0, 0).$$

It has squared norm $|z|^2$, and its complementary component has squared norm $\|Qc(z)\|^2 = |z|^2/2$. Both identities follow directly from the displayed entries and the half-turn.

There are two such signal ports. The reversible recording exchange is the two-port exchange of Section 12.4,

$$X(f, f') = (Pf + Qf',\ Qf + Pf').$$

Since P and Q are self-adjoint with $P^2 = P$, $Q^2 = Q$ and $PQ = QP = 0$, the exchange is self-adjoint and applying it twice returns (f, f'). It preserves the total squared norm. In a ready input $(c(z), 0)$, the second port receives $Qc(z)$ and the first retains $Pc(z)$. The complement is kept in another port; nothing is deleted.

For a fixed positive energy scale γ, the recording Hamiltonian is

$$\gamma(I - X).$$

The proof of Proposition 7.6 uses only that the exchange is a self-adjoint involution. Applied to X, with γ in place of g, it shows that $\gamma(I - X)$ is

positive and that its evolution enacts X at the duration $\tau = \hbar\pi/(2\gamma)$. The raw energy form of a ready input is

$$D_\gamma(z) = \langle (c(z),0), \gamma(I-X)(c(z),0)\rangle = \gamma\|Qc(z)\|^2 = \frac{\gamma}{2}|z|^2.$$

Every detector uses the same unit-pulse reference $\Delta_\gamma = D_\gamma(1) = \gamma/2$. For selected labels $\mathcal{J}$ in a finite bank, the dimensionless report is therefore

$$O_\gamma(\psi, \mathcal{J}) = \frac{\sum_{k\in\mathcal{J}} D_\gamma(\psi_k)}{\Delta_\gamma} = \sum_{k\in\mathcal{J}} |\psi_k|^2.$$

Under the quadratic signal-energy law, a positive energy $\mathcal{E}$ per unit squared norm gives deposited second-port energy $\mathcal{E}\|Qc(z)\|^2 = \mathcal{E}|z|^2/2$. Pooling those deposited energies and dividing by the deposited unit-pulse energy $\mathcal{E}/2$ gives the same bank report. The report concerns a prepared bank before any individual outcome is selected.

A.7.1 What the bank reports on Alice's two records

The bank returns the branch average on Alice's flagged record, and the same report on her unconnected record, whose information is zero.

Use joint bank labels $k = (a, e)$, with $a \in \{0,1,2\}$ and $e \in \{0,1\}$. For a joint probability table p, take the nonnegative amplitude encoding

$$\psi_{a,e} = \sqrt{p(a,e)}.$$

The same recording exchange, with the same γ, duration and unit-pulse reference, acts on each joint coordinate, so the bank retains both. Selecting a rectangle of labels $Y_0 \times E_0$, with $Y_0 \subseteq Y$ and $E_0 \subseteq E$, reports its probability mass:

$$O_\gamma(\psi, Y_0 \times E_0) = \sum_{a\in Y_0} \sum_{e\in E_0} p(a,e).$$

The assembled table of Section 12.6 is

$$\begin{aligned} p(0,0) &= \tfrac{1}{4}, \quad p(0,1) = 0, \\ p(1,0) &= 0, \quad p(1,1) = \tfrac{1}{4}, \\ p(2,0) &= p(2,1) = \tfrac{1}{4}. \end{aligned}$$

Labels 0 and 1 hold branch 0's perfect record and label 2 holds branch 1's blank record, each with weight $1/2$: in amplitudes, $\sqrt{1/2}$ times each

normalized branch. The placements have disjoint support, so the retained flag neither overlaps nor discards the two branches.

Select the detector event $\mathcal{J} = \{0,1\} \times \{0,1\}$. Fix one numerical calibration, for every input:

$$\Lambda_\gamma(p) = (\log 2)\, O_\gamma(\sqrt{p}, \mathcal{J}).$$

The fully normalized perfect branch reports $\log 2$. The normalized blank branch, placed at label 2, reports zero. No reference or scale is changed when the branches are assembled. Their pooled report is

$$\Lambda_\gamma(p) = (\log 2)(\tfrac{1}{4} + \tfrac{1}{4}) = \tfrac{1}{2}\log 2.$$

This is the arithmetic conditional mean, supplied by the energy formula, the disjoint placement and the single reference.

Now keep the same fine-agent row weights and the same fair environment, but make them independent. This is Alice's second arrangement. The table becomes

$$\tilde{p}(0,0) = \tilde{p}(0,1) = \tilde{p}(1,0) = \tilde{p}(1,1) = \tfrac{1}{8}, \qquad \tilde{p}(2,0) = \tilde{p}(2,1) = \tfrac{1}{4}.$$

Every row has the same total load as before: $1/4$, $1/4$ and $1/2$. Consequently every row-energy report agrees, and the same selected event gives

$$\Lambda_\gamma(\tilde{p}) = \tfrac{1}{2}\log 2 = \Lambda_\gamma(p).$$

But their mutual information differs. For $\tilde{p}$, every joint cell is the product of its marginals, so each logarithm in the mutual information (12.1) is zero. Thus $I(\tilde{p}) = 0$. For p, the two perfect-record cells each contribute $(1/4)\log 2$, and the two blank cells contribute zero. Hence $I(p) = \frac{1}{2}\log 2$.

The bank's literal output is a calibrated occupation weight. The averaging of Section 12.6 holds for any branch reporters; it gives $\frac{1}{2}\log 2$ on the flagged record because each reporter there is calibrated to its branch's actual record. The bank fixes those calibrations once. On $\tilde{p}$ the first branch's labels hold a record unrelated to the stores, and the bank still weights them by $\log 2$, so its equality with information on the assembled example does not extend to arbitrary retained records. The full joint bank has kept the correlations, and reporting individual joint cells distinguishes the two records. The joint-reference comparison uses those correlations by comparing the actual joint table with its product of marginals; on the two records it reports $3 - 2\sqrt{2}$ and zero.

A.8 A local calibration leaves an aggregation rule free

The value (12.2) scores one participant. When an action affects both Alice and Bob, a further rule may combine their values: a common one transforms each by a concave function and adds the results, so that a gain to the participant with less counts for more. Calibrating such a rule at one point does not fix it; a global relation does.

Consider a differentiable transformation $h(y)$ used before aggregation. A natural local calibration is

$$h(0) = 0, \qquad h'(0) = 1, \qquad h''(0) = -1.$$

The first two make the rule agree with a plain sum to first order. The third is chosen to mirror the curvature of J at balance, $J''(1) = 1$ (the coefficient one half found in Chapter 12), with the opposite sign. The function

$$h(y) = 1 - e^{-y}$$

has those local properties and satisfies a global relation:

$$h'(y) = 1 - h(y).$$

That relation determines the whole function.

Proposition A.8 (Global exponential representation). *If h is differentiable on $\mathbb{R}$, $h(0) = 0$, and $h'(y) = 1 - h(y)$ for every y, then $h(y) = 1 - e^{-y}$ for every y.*

Proof. Set $\bar{h}(y) = 1 - h(y)$. Then $\bar{h}'(y) = -\bar{h}(y)$ and $\bar{h}(0) = 1$. The derivative of $e^{y}\bar{h}(y)$ is zero everywhere, so this function is constant. Its value at zero is one. Hence $\bar{h}(y) = e^{-y}$ and the asserted expression for h follows. □

The differential equation fixes how marginal sensitivity changes at every value. Nothing here derives it, and a rule of moral aggregation built on it would need it derived for the physical comparison at issue.

Appendix B

Protocols and finite contacts for authored actions

When Bob accepts Alice's offer of care, something physical has to register the acceptance, and something physical has to carry it into what happens next. A probe must tell who accepted. A protocol cell must hold the terms and remember who may change them. A writer must copy a decision into memory, and further contacts must move a resource or emit a release. Chapters 7 and 13 state what each of these parts does. Here each one is built on an explicit finite carrier with explicit operators, so every claim about it can be checked line by line.

Section 7.8 explains what a physical write adds to the governing argument of Chapter 7, and names the two premises that connect it to Bob and to Alice. The probe of Section B.1 supplies that write's control. Each writer and router below is a conditional exchange of basis states generated by a positive Hamiltonian: it completes in a definite time and acts on every state of its carrier, and where it retains a holding, it conserves that holding exactly. The later contacts are exact on their own carriers; Section B.14 says what connecting them to Alice's care still needs.

The running case is that of Chapter 13: Alice (A) provides care, Bob (B) holds the original undertaking and Carol (C) the additional one. Table B.1 gives each construction and where the main text uses it.

Two conditions hold throughout. Each construction assumes that its contacts are physically available and that its inputs are prepared as described; where a construction needs a particular physical identification, such as who prepared a control sector, its section names it. And no construction converts a register quantum, a packet occupation or a service label into food, paid work or authority, or decides which of two claims on one unit governs. Those are physical connections in their own right; Section B.14 sets out what each still needs. The numbers built here, weights,

Section	What it builds; where it is used
B.1	A probe whose true output has weight zero for another author and one half for Bob; Chapter 7's write
B.2, B.3	Recovery of the three role holders, and the relabeling criterion; Chapter 13 on changed controllers and participant sets
B.4	Soundness of a complete reading and Bob's continuation, and the condition for a complete later reading to keep Bob's claim; Sections 13.1 and 13.9
B.5	Carol's acceptance written and her record work paid during Bob's first meal, with Alice's comparator input unchanged; Section 13.4
B.6	Both accepted records from blank books and own-row postings; within this appendix, the paid acceptance of Section B.5
B.7	A writer of the acceptance mode; Section 13.8
B.8, B.9	A release contact that produces the fields of its release event, and whose completed records are truthful; Section 13.6
B.10–B.13	A material packet steered by the acceptance memory while Bob's delivery runs in every branch, carried on the probe's channels, and the energy condition under which an exchange recharging the writer's reservoir from one packet transition keeps the full holding; Section 13.10

Table B.1. Each construction and where it is used, in the main text unless the row says otherwise. The first column gives section numbers.

energies, account entries and two record ledgers, belong to the apparatus; none enters the meal calculations of Chapter 13, which use the care bonds and the participants' reserves (Table 13.3).

Table B.2 lists the accounts the constructions keep. They stay separate because each measures a different quantity on a different carrier: whole units in the economy's books, energy in a writer's registers, ratios of account-book entries, a label for service. Converting one into another, or into a participant's reserve, would be a physical calibration of its own, and none has been derived (Section B.14).

Account	What it measures	Unit	Section
Signed reserve	the economy's account in Carol's record-work step; maintenance and record work each debit one unit	whole units	B.5
Holding energy	energy of a writer's registers when no contact acts; a write moves one mode's energy from reservoir to memory and keeps the total	energy	B.5, B.7
Recording ledger	two bonds, Alice to Bob and Alice to Carol, read from account books; registers whether a waiver was made	multiplier	B.8
Service comparison	two bonds marking each claim served (multiplier 1) or unserved (multiplier 2, by convention)	multiplier	B.8

Table B.2. The accounts of the constructions. None of their levels enters a participant's reserve on the care ledger or another account, and no conversion between them has been derived.

B.1 The prepared acceptance probe

The care controller of Chapter 7 acts on a detected acceptance, so its detector must never fire for the wrong author. The probe is one recorded

operation applied to two preparations. They have the same internal state and differ only in the occupation that encodes the author: $(0,1)$ for Bob, who accepts, and $(0,0)$ for another author. The calculation below gives the detection bit total weight zero for the other author and one half for Bob, so a true output can come only from Bob. Chapter 7 needs nothing more from the probe, and any recorded operation with those two weights would serve. The operation is given in full so that both weights are computed from one explicit evolution rather than assumed.

A projective measurement of the occupation would identify Bob with certainty. It would be a further apparatus, with its own record and its own coupling to the writer, and none is built here; with it the argument would run unchanged and the undetected branch would be empty. The probe below is the one detector constructed, and its two weights are all the writer uses.

B.1.1 The carrier and the recorded operation

The probe is taken from *recognition dynamics*, a reversible quantum dynamics of recognition events in which each event keeps its own record: the integer current that flowed on each edge, the integer field, and the history of postings. Its basic cell is adopted here as the probe's model, not derived. The cell has two channels, each with an occupation bit and a four-level internal clock, connected by three edges that carry an integer field: the flux, whose charge is the occupation. One step on the cell advances the clocks and couples the two channels reciprocally, and the probe is that step applied once, with its record. The step reads occupations, so the author is encoded as one, and the construction takes as its premise, with Chapter 7, that Bob's physical acceptance prepares $(0,1)$. The recorded current separates the outputs of different inputs, so the recording loses no norm.

Write an occupation as $a = (a_1, a_2) \in \{0,1\}^2$. Each occupation also carries two internal indices $i = (i_1, i_2) \in \{0,1,2,3\}^2$. An input vector therefore has $4 \cdot 16 = 64$ complex coordinates $\xi(a,i)$, with squared norm $\sum_{a,i} |\xi(a,i)|^2$.

On a four-entry internal vector define the cyclic shift and sign change by

$$(\mathrm{L}x)_r = x_{r+1 \bmod 4}, \qquad \mathrm{D} = \mathrm{diag}(1,-1,-1,1).$$

The two internal blocks are

$$\mathrm{S} = \frac{1}{2}\mathrm{D}(I - \mathrm{L}), \qquad \mathrm{F} = \frac{1}{2}\mathrm{D}(I + \mathrm{L}).$$

In the clock block K_{ba}, the first index is the output occupation and the second is the input occupation:

$$\mathrm{K}_{00} = \mathrm{S}, \quad \mathrm{K}_{10} = \mathrm{F}, \quad \mathrm{K}_{01} = -\mathrm{F}, \quad \mathrm{K}_{11} = -\mathrm{S}.$$

For output occupation $y = (y_1, y_2)$, the pair blocks on the sixteen internal coordinates are

$$\mathrm{T}_{y,a} = \begin{cases} -(\mathrm{K}_{y_1,0} \otimes \mathrm{K}_{y_2,1})(\mathrm{D} \otimes \mathrm{D}), & a = (1,0), \\ (\mathrm{K}_{y_1,1} \otimes \mathrm{K}_{y_2,0})(\mathrm{D} \otimes \mathrm{D}), & a = (0,1), \\ -\mathrm{K}_{y_1,a_1} \otimes \mathrm{K}_{y_2,a_2}, & a = (0,0) \text{ or } (1,1). \end{cases}$$

Here $(x \otimes w)_{(r,s)} = x_r w_s$, and the same coordinate rule defines the matrix tensor product.

The recorded output retains the integer current, one entry per edge,

$$j(y,a) = (y_1 - a_2,\ a_1 - a_2,\ y_2 - a_1).$$

Let $\mathcal{J}$ be the finite set of all these currents. The full output coordinates are (y, j, i), with $j \in \mathcal{J}$; equal currents share one label. The recorded map is

$$(\mathcal{T}\xi)(y,j,i) = \sum_{\{a:\, j(y,a)=j\}} (\mathrm{T}_{y,a}\xi(a,\cdot))_i.$$

For fixed y, the current determines the input occupation: its first and third entries give $a_2 = y_1 - j_1$ and $a_1 = y_2 - j_3$. Different input occupations therefore have disjoint recorded outputs at each y.

The operation preserves the complete squared norm. Since $\mathrm{L}^*\mathrm{L} = I$ and $\mathrm{D}^*\mathrm{D} = I$, expansion gives

$$\mathrm{S}^*\mathrm{S} + \mathrm{F}^*\mathrm{F} = \frac{1}{4}\big((I - \mathrm{L}^*)(I - \mathrm{L}) + (I + \mathrm{L}^*)(I + \mathrm{L})\big) = I.$$

For each fixed input occupation, summing $\mathrm{T}_{y,a}^*\mathrm{T}_{y,a}$ over both output bits therefore gives $I \otimes I$. The signs, the interchange of clock columns and the unitary factor $\mathrm{D} \otimes \mathrm{D}$ preserve that identity. Different input occupations have no cross terms after recording, by the current reconstruction just proved. Consequently $\|\mathcal{T}\xi\|^2 = \|\xi\|^2$.

The integer field is retained as well. An input at field value $n \in \mathbb{Z}$ has output field value $n+j_2$. For an arbitrary square-summable field input, the same formula reads the input coefficient at field $m - j_2$ when producing output field m. Integer translation is a bijection on each retained output coordinate, so this extension preserves the norm without imposing a field cutoff.

B.1.2 The two inputs and the true-output weights

Put

$$u = \tfrac{1}{2}(1,1,1,1), \qquad w = \mathrm{D}u = \tfrac{1}{2}(1,-1,-1,1).$$

For each prepared occupation a, the input has internal vector $u \otimes u$ at that occupation and zero at the others, at one field value n. Since $\|u\|^2 = 1$, both preparations have squared norm one. Applying the four-entry blocks directly gives

$$\mathrm{S}u = 0, \qquad \mathrm{F}u = w, \qquad \mathrm{S}w = \tfrac{1}{2}(1,0,1,0), \qquad \mathrm{F}w = \tfrac{1}{2}(0,1,0,1).$$

In particular, $\|w\|^2 = 1$ and $\|\mathrm{S}w\|^2 = \|\mathrm{F}w\|^2 = 1/2$.

For $a = (0,0)$, the output internal vector is $-\mathrm{K}_{y_1,0}u \otimes \mathrm{K}_{y_2,0}u$. Every block vanishes except $y = (1,1)$, whose vector is $-w \otimes w$ and whose squared norm is one. For $a = (0,1)$, the factor $\mathrm{D} \otimes \mathrm{D}$ first gives $w \otimes w$. Each factor in every output block is then one of $\pm\mathrm{S}w$ and $\pm\mathrm{F}w$, so each output occupation has squared norm $(1/2)(1/2) = 1/4$. Summing every retained current and internal coordinate gives the complete table

input a \ output y	$(0,0)$	$(0,1)$	$(1,0)$	$(1,1)$
$(0,0)$	0	0	0	1
$(0,1)$	1/4	1/4	1/4	1/4.

The integer field adds no factor: each current has its one exact translated field value, and all field values are included in the sum.

In the probe's located form, probe and target are points of a four-dimensional Euclidean space, and the contact's activation is one minus their distance, or zero beyond distance one: the input passes through $\mathcal{T}$ with squared amplitude equal to the activation and passes unchanged, on an orthogonal output, with the remaining squared amplitude. Since $\mathcal{T}$ preserves the norm and the two parts are orthogonal, the located contact preserves it too. Here probe and target sit at the same position, the activation is one, and the contact applies $\mathcal{T}$ in full. The detection bit is

true on an output exactly when $y_1 = 0$. Adding the first two columns of the table gives

$$\Pr(\text{true} \mid a = (0,0)) = 0, \qquad \Pr(\text{true} \mid a = (0,1)) = \frac{1}{2}.$$

These are sums of full-output squared amplitudes, with no conditioning on a selected current and no normalization by an active-event probability, and a true output therefore identifies its author. The command journal retains which occupation was prepared, together with the field and the full output, and the retained detection bit can feed the action writer.

The other half of Bob's weight lies on false outputs, where that action writer stays disabled and no selection is installed; Chapter 7 discusses what the branch means for Alice. The half is what this step of the dynamics gives.

B.2 A controller cannot silently change the participants' roles

An undertaking has more structure than the outcome it requests. Someone can propose its terms. Someone can accept them. Someone can release the current requirement. A replacement controller that gives a different person one of these powers has changed the undertaking's operation, even if the two controllers happen to produce the same result on the history observed so far.

For a versioned bilateral protocol the statement is exact. Three short transcripts recover all three role holders, so changing any role changes a response on a transcript of at most three events. The result preserves the roles a protocol already assigns; it does not choose the initial holders, establish that every assignment of those powers is morally legitimate, or order incompatible claims.

B.2.1 The state and the events

Let the participants form a finite set, and let T be a set of possible terms. Fix at least one possible term $t \in T$. A role assignment is a triple

$$p = (p_{\text{provider}}, p_{\text{recipient}}, p_{\text{releaser}}).$$

The three entries need not name different people. The provider may propose terms, the recipient may accept them, and the releaser may terminate the active version through the release operation.

The protocol decides whether an event changes the current terms from these fields:

1. a current revision number ν;
2. the last proposal nonce z;
3. either no pending proposal or a proposal (u, b, s), containing a fresh nonce u, a base revision b, and terms $s \in T$;
4. either no active terms or an active term;
5. a closed flag, true or false.

The empty cell has revision and nonce zero, no pending or active terms, and an open flag. A nonce identifies a proposal occurrence; the base revision identifies the version it proposes to amend. Keeping both prevents an old acceptance from becoming acceptance of a different proposal or a different version.

There are four event forms. Each carries an undertaking identifier, which is fixed here because we examine one cell:

$$\text{propose}(a, u, b, s), \qquad \text{accept}(a, u, b, s),$$
$$\text{release}(a, r), \qquad \text{claim}(a).$$

The author is a. The release event also names the revision r it seeks to release. A claim records that someone made a claim; it neither counts as performance nor changes the undertaking state.

The transition rules are as follows. An event failing its conditions leaves the cell unchanged.

1. A proposal enters the pending slot exactly when its author is p_{provider}, its nonce exceeds z, its base equals ν, and the cell is open. It replaces z by its nonce and stores the proposal, leaving the active terms unchanged.

2. An acceptance requires a pending proposal. Its author must be $p_{\text{recipient}}$; its nonce, base, and terms must equal the pending proposal's nonce, base, and terms; the pending base must still equal ν; and the cell must be open. A successful acceptance increments ν, clears the pending slot, and makes the exact proposed terms

active. The protocol also retains the formation occurrence. The results below inspect only a nonce and the presence or absence of the exact active terms, so they need neither that record nor its internal representation.

3. A release succeeds exactly when its author is p_{releaser}, its revision equals ν, there are active terms, and the cell is open. It clears pending and active terms and closes the cell, without rewriting the earlier proposal or acceptance events.

4. A claim leaves the cell unchanged.

Write $E_p(h)$ for the cell obtained by starting empty and applying, in order, the finite event transcript h under assignment p. Define two readouts:

$$N_p(h) = \text{the last nonce in } E_p(h), \qquad \Theta_p(h) = \text{the active terms in } E_p(h),$$

where $\Theta_p(h)$ can be empty. Two assignments have the same complete response behavior when

$$E_p(h) = E_q(h) \quad \text{for every finite transcript } h.$$

B.2.2 Three transcripts recover the assignment

For the fixed term t, construct three transcripts from the role holders in p:

$$\begin{aligned} h_1 &= [\text{propose}(p_{\text{provider}}, 1, 0, t)], \\ h_2 &= [\text{propose}(p_{\text{provider}}, 1, 0, t), \text{accept}(p_{\text{recipient}}, 1, 0, t)], \\ h_3 &= [\text{propose}(p_{\text{provider}}, 1, 0, t), \text{accept}(p_{\text{recipient}}, 1, 0, t), \\ &\qquad \text{release}(p_{\text{releaser}}, 1)]. \end{aligned}$$

These are separate executions from the empty cell, of lengths one, two and three. Every intermediate state used in the argument is reached by these events; none is assigned to the controller by hand.

Theorem B.1 (Three-probe role recovery). *For two assignments p and q on the same participant set and term type, if*

$$N_p(h_1) = N_q(h_1), \qquad \Theta_p(h_2) = \Theta_q(h_2), \qquad \Theta_p(h_3) = \Theta_q(h_3),$$

then $p = q$. Recovering the assignment thus needs only one nonce readout and two active-term readouts, rather than equality of the complete cells.

Proof. First consider h_1. Under p, its author is the provider, nonce one is fresh, base zero is current, and the cell is open. The proposal succeeds, so the last nonce becomes one. If q named a different provider, the same event would fail its author condition and leave the last nonce zero. Equal responses therefore give $p_{\text{provider}} = q_{\text{provider}}$.

Now consider h_2. The first event succeeds under both assignments because their providers have been shown equal. Under p, the second event matches the pending proposal and is authored by the recipient, so it makes t the active terms. If q named a different recipient, that acceptance would fail, leaving no active terms. Equal responses give $p_{\text{recipient}} = q_{\text{recipient}}$.

Finally consider h_3. The first two events now succeed under both assignments, leaving active terms at revision one. Under p, the final event is a current-version release by the releaser and clears those terms. If q named a different releaser, it would reject the release and retain t. Equal responses give $p_{\text{releaser}} = q_{\text{releaser}}$. All three fields agree, so the assignments are equal. □

The theorem compares controllers governed by the same transition rules. An implementation with a different event language is not identified by attaching three role labels to it.

Theorem B.2 (A changed role has a short distinguishing history). *If $p \neq q$, there is a finite transcript h with at most three events such that $E_p(h) \neq E_q(h)$.*

Proof. If every transcript of length at most three had the same response, the three displayed transcripts would have the same responses. The preceding theorem would give $p = q$, contradicting the hypothesis. Thus some transcript of length at most three distinguishes the assignments. □

Complete response equivalence therefore holds exactly when the role assignments are equal: equal assignments give equal transitions at every step, and the converse follows from the three-probe theorem, since complete response equivalence includes its three transcripts.

B.2.3 What a replacement must preserve

A response-preserving replacement controller must preserve all three role holders, and retaining formation is not enough. With the same

provider and recipient and a different releaser, proposal and acceptance still work, but a current-version release distinguishes the controllers. A record of successful formation does not determine who can later release the active requirement.

The bound matters in practice. Proving that the roles can be identified needs no appeal to an unlimited survey of future behavior: three explicit transcripts carry the distinction.

B.3 Changing the participant set without changing the undertaking

Moving an undertaking to new apparatus, or relabeling its participants, must keep who can propose, accept and release it. Otherwise a change of representation quietly becomes a transfer of authority. For the finite protocol just defined the criterion is exact: every designated role must stay distinguishable from every other source participant. Participants outside those roles need not stay distinguishable from one another.

B.3.1 The objects being preserved

Let Λ be the finite source set of participant labels and Λ' a finite target set. An undertaking has assigned provider p, recipient r and releaser ℓ in Λ, not necessarily different. Its terms belong to a nonempty set T. Fix one possible term $\tau \in T$ for the distinguishing transcripts.

The protocol's retained formation record has five fields: price, offered, accepted, delivered and amount due, the protocol event record of Chapter 8. It starts at $(0, \text{false}, \text{false}, \text{false}, 0)$. Successful acceptance writes the formation into it as an exchange at price zero: an offer from the assigned provider to the assigned recipient, followed by that recipient's acceptance. The resulting fields are $(0, \text{true}, \text{true}, \text{false}, 0)$. Identity comparisons guard these events, but the identities are not fields of the resulting record. The detail matters because the proposition below preserves the complete state, including that record.

Let $f : \Lambda \to \Lambda'$ be any participant map. Map all three assigned roles by f and map every event's author by f. Keep the event kind, undertaking identifier, nonce, revision and terms unchanged. The question is whether executing a mapped transcript gives exactly the same state as executing the original.

B.3.2 The exact criterion

Proposition B.3 (Role fibers characterize response preservation). *Every finite transcript has the same final protocol state after this mapping if and only if*

$$\begin{aligned} f(a) = f(p) &\Longrightarrow a = p, \\ f(a) = f(r) &\Longrightarrow a = r, \qquad \text{for every } a \in \Lambda. \\ f(a) = f(\ell) &\Longrightarrow a = \ell \end{aligned}$$

If the condition fails, a transcript of at most three events distinguishes the original and mapped executions. An injective participant map always satisfies the condition.

Proof. First suppose the three implications hold. For any event author a, each role test is preserved in both directions: for example, $f(a) = f(p)$ exactly when $a = p$. The other proposal guards concern numbers, terms and flags that the mapping leaves unchanged. Hence a proposal has the same effect in both executions. The same reasoning applies to acceptance and release. Successful acceptance also writes the same formation record: the mapped provider and recipient pass their corresponding identity tests, and the resulting five fields have the fixed values given above. Claims have no effect in either execution. Thus each individual transition agrees from any common initial state. Induction over a finite transcript gives equality of its complete final states.

For necessity, suppose another participant $a \neq p$ has $f(a) = f(p)$. From the empty state, let a propose terms τ with nonce 1 and base revision 0. The source refuses the event, so its largest nonce remains 0. The target accepts it as a provider proposal, so its largest nonce becomes 1. One event distinguishes the executions.

Next suppose $a \neq r$ has $f(a) = f(r)$. Use a fresh proposal by the actual provider, then an exact acceptance by a. Both executions accept the proposal. Only the mapped execution accepts the second event. Its active entry is τ, while the original active entry remains empty. Two events suffice.

Finally suppose $a \neq \ell$ has $f(a) = f(\ell)$. Form the undertaking with a fresh provider proposal and the recipient's exact acceptance, then let a release revision 1. Both executions reach the same accepted state before the release. Only the target accepts the release, leaving its active entry empty while the source retains τ. Three events suffice. Each failed implication therefore gives the promised bounded witness. □

The criterion is weaker than global injectivity. In the example of Chapter 13, source labels $0, 1, 2, 3, 4$ go to target labels $0, 1, 2, 3$: provider 0, recipient 1 and releaser 2 stay fixed, and both outsiders 3 and 4 go to 3. The map is not injective, but every designated role still has a singleton fiber, so every transcript of this undertaking has the same response. The merge preserves this protocol's responses and nothing more.

B.3.3 Grants and history survive together

For many undertakings, require the condition for every undertaking's assigned roles. Every event then updates the same identified cell in the same way. Induction over the event times gives equality of the entire state at every cutoff, including pending proposals, active terms, formation records and closed flags.

Keep the interpretation of terms fixed. If $E(\tau, x)$ says that observed performance x meets terms τ, compliance means that every active term passes E. A grant of permission requires an actual active term that passes E; absence of an obligation alone is not a grant. Equality of the protocol states preserves both predicates at every cutoff. With the same observed performance history, it also preserves whether a violation occurred at any earlier cutoff. A later change of carrier cannot erase a violation by changing who appears to have accepted or released the undertaking.

A new target participant outside the image of f inherits no release role: its release event fails the existing role guard. Assigning a new role needs additional grounds or a new protocol action.

B.4 Two equivalences for complete readings

Completeness turns a requirement of a reading into a condition on its performances, and two cases of Chapter 13 use this. The first is the single undertaking, with the restoration comparison of Section 13.1; the second is the one-unit conflict, with the local targets of Section 13.9. Readings, performances and completeness are those of Definition 7.4, with legality replaced by authorization in the second subsection, and a reading is written $\mathcal{R}$.

B.4.1 Soundness and Bob's continuation

Soundness, one half of faithfulness, has a physical meaning of its own. The first equivalence below is Chapter 7's calculation, put in terms of

the restoration comparison. The second is what completeness adds: a sound complete reading already contains Bob's continuation, his positive reserve after the next interval.

Proposition B.4 (Recipient continuation and soundness). *Keep the authored care history, its available legal actions, actual exposure and paid repair fixed. For a legal action a, let $R(a)$ mean that the actual continued ledger restores the formation-prevention frame, and let $E_B(a)$ mean that the recipient, Bob, has positive reserve after the next actual interval. Then*

$$E_B(a) \iff R(a).$$

For a complete reading, the following are equivalent: the reading is sound, that is, every performance restores that frame; the reading contains positive next reserve for the recipient as a requirement; the reading contains the accepted recipient's continued maintenance as a requirement.

Proof. By Proposition 7.3 the legal continuation cases are exhaustive, and they are the two outcomes of Section 13.1: a nonclosing action leaves Alice at $(1, 3/2)$ and Bob at $(2, 0)$, a closing action leaves Alice at $(1, 5/4)$ and Bob at $(1, 1/4)$. These are the actual multiplier and reserve pairs after exposure and repair, so Bob's next reserve is zero in the first case and $1/4$ in the second. Restoration holds exactly in the closing case. This proves $E_B(a) \leftrightarrow R(a)$. Chapter 7 identifies Bob's continued maintenance with that same legal closing case: closing the authored reliance and performing the accepted care content are two of five equivalent conditions on the legal first actions (Proposition 7.2).

A performance satisfies every requirement contained in its reading. Thus a reading that contains Bob's continuation has only restoring performances. Conversely, if every performance restores, every one has positive next reserve for Bob and continues the identified recipient. Completeness says that a property true of all performances is contained in the reading. Hence the reading contains both properties. □

The other direction of faithfulness, that every legal restoration is a performance, is what gives the uniqueness of Section 13.1.

B.4.2 The condition for a later reading to keep Bob's claim

The open question of Section 13.11 can be put as an exact test on the performances of a complete later reading. Any proposed priority rule

can be checked against it, though the test itself does not answer the question.

Keep the full current authorization condition $Q(a)$ that comes with the case. Call an operation *authorized* when it is enabled on the rebuilt cutoff of Section 13.8 and satisfies Q. Evaluate the operation together with the following idle interval used in Proposition 13.5. Let $S_j(a)$ mean that this resulting state restores recipient j's original affected-bond targets, and let $E_j(a)$ mean that the recipient has positive reserve at that same later checkpoint. Proposition 13.6 proves $S_j(a) \iff E_j(a)$ on this enabled domain.

Replace legality by authorization in Definition 7.4. An *authorized performance* of a reading $\mathcal{R}$ is an authorized operation with every property in $\mathcal{R}$:

$$\mathrm{Perf}_{\mathcal{R}}(a) \quad \iff \quad a \text{ authorized } \wedge\ \forall G\,(\mathcal{R}(G) \implies G(a)).$$

The reading $\mathcal{R}$ is complete when it contains every predicate true of all its authorized performances.

The consequences of each recipient's restoration form a local reading:

$$\mathcal{R}_j(G) \quad \iff \quad \forall a,\ (a \text{ authorized } \wedge\, S_j(a)) \implies G(a).$$

It contains E_j. Its authorized performances are exactly the authorized operations satisfying S_j: any such operation satisfies every displayed consequence, and any performance satisfies S_j, since S_j is itself one of those consequences. If the meal for j is authorized, it is a performance of $\mathcal{R}_j$, and $\mathcal{R}_j$ does not contain the other recipient's end, since that same meal leaves the other recipient at $(2, 0)$. So if both meals are authorized, both local readings are nonempty and complete on the same retained original history, and switching from Bob's reading to Carol's loses Bob's end. Neither local reading has thereby been shown to govern.

Proposition B.5. *Under the completeness condition just defined, $\mathcal{R}$ contains Bob's end E_B if and only if every authorized performance of $\mathcal{R}$ restores B's original affected-bond targets. Consequently, if the earlier reading contains E_B, preservation of that end by the later reading is equivalent to this local restoration condition on the later reading's authorized performances.*

Proof. If every such performance restores the first target, Proposition 13.6 gives E_B for each of them. Completeness puts E_B in $\mathcal{R}$. Conversely, an

authorized performance satisfies every predicate in $\mathcal{R}$. If E_B belongs to $\mathcal{R}$, it therefore holds for that performance, and the same proposition gives restoration of the first local target. For the final statement, preservation from an earlier reading already containing E_B is precisely containment of E_B in the later reading. □

B.5 Recording Carol's acceptance during the first meal

Carol's acceptance can be written into a physical record during Bob's first meal, with the recording work paid by Carol, while Alice's comparator keeps exactly its old input and governing reading.

The stock holds the original three food units. The first meal consumes food unit zero, requires one unit of labor from Alice, and feeds Bob. The opening balanced reserves are those of Carol's later undertaking: 2 for Alice, 1/4 for Bob and 2 for Carol. The extension of Section 13.4 gave Carol no work capacity; here she can perform one unit of record work in the interval, alongside an old economic operation whose food, intake and work coordinates stay as they were.

For an old operation o and a record-work quantity $w \in \{0, 1\}$, the joint operation $\hat{o}_w$ copies every food input, output, waste and completed-job coordinate of o, Alice's and Bob's intake and labor, the old maintenance rates, labor costs and expiry conditions, and Alice's and Bob's work capacities; it gives Carol no intake, labor w and work capacity one. Every enabled old operation then gives an enabled joint operation. Material balance is unchanged, because the record work neither creates nor consumes food, and the new labor bound is precisely $w \le 1$. In the previous configuration the same operation with $w = 1$ is disabled, because it requires $1 \le 0$ at Carol's work coordinate.

One memory output controls both the record and the work. Three registers of eight modes each, the mode set of Chapter 7's write (a choice of that construction, separate from the eight entries of Section 2.2), hold an encoded event e, a blank memory m and a prepared reservoir ρ; only modes zero and one are prepared. The controlled exchange is

$$W(e, m, \rho) = \begin{cases} (e, \rho, m), & e = 1, \\ (e, m, \rho), & e \neq 1. \end{cases}$$

The prepared input $(1, 0, 1)$ becomes $(1, 1, 0)$; the input $(0, 0, 1)$ leaves the memory blank. A source encoding assigns mode one to Carol's authorized acceptance of the pending extra offer and mode zero to a specified wrong author. Preparing mode one from Carol's actual authored event is a premise of the construction.

Read the resulting memory coordinate m' twice. The event decoder emits the acceptance event naming Carol exactly when $m' = 1$; otherwise it emits the wrong-author event, which the protocol rejects. The operation selector sets $w = 1$ exactly when that same $m' = 1$. There are no independent acceptance and labor flags. The retained mode of Alice's original controller still selects the old operation o; the new memory supplies only its added record work. One retained transition carries the written register, the unchanged controller, the joint economic successor, the paid responses and the agreement cells, with the proposal and the decoded event appended to the old event prefix as an archive that does not reconstruct the initial cells. Carol's acceptance is active in its own cell, and Bob's cell is unchanged.

For an installed original controller, o is the first feeding operation. The joint step advances the economic calendar from zero to one and consumes the same food unit as the old first meal. Two accounts record what Carol pays (Table B.2). The economy keeps its own account in whole units, the *signed reserve*, in which maintenance and record work each debit one unit. Every stock, intake, work, signed-reserve and response successor for Alice and Bob is exactly the old one. Carol's signed reserve stands at 8 before the step, an opening of the economy's own and not her reserve on the care ledger, which opens at 2; the step takes it from 8 to 6. Her reserve records the same two debits at $1/4$ each, the cost of the work and her interval's charge for repairing her balanced bond:

$$y'_C = \frac{3}{2}, \qquad y'_C + \frac{1}{4} + \frac{1}{4} = 2.$$

The two accounts sit side by side, and neither is converted into the other. The old meal gives Alice reserve $3/2$ and Bob reserve $1/2$. All three multipliers stay balanced, all three reserves stay positive, and both the complete first meal and the added record work are paid for.

Alice's comparator sees the same accepted content, checkpoint one, and Alice's and Bob's successor balance and positivity bits for every action in its original care domain. Its complete input is therefore equal to

its former input, so its verdict and implemented consequence reading are equal as well, and the installed law on that original domain is preserved.

The write itself is exact. With U the permutation matrix of W, the positive contact $g(I - U)$ is an instance of the exact finite-time write of Proposition 7.6: after time $\hbar\pi/(2g)$ it takes the prepared basis vector to the written one and retains the event coordinate. If ε_1 is the positive holding energy of mode one, the memory gains ε_1, the reservoir loses ε_1, and their total holding energy, including the event register, remains $2\varepsilon_1$. The contact's own preparation energy is additional. The holding energy is a third account, and no conversion between its quantum and either debit has been derived.

The next section regenerates both accepted records from blank books in which each participant writes only in their own row. The same written memory can also decide where a material resource goes (Section B.10).

B.6 Generating the two undertaking records from their postings

Chapter 13 starts the three-person case from two accepted cells. They can instead be generated in one history that begins with empty books and unformed agreements. Each response is read from the account row it changes, written into memory, and applied as a protocol event. Four updates reproduce both accepted records exactly, and if Carol's response is replaced by a rejection, Alice's original undertaking with Bob survives intact.

Undertaking 0 has provider A and recipient B, with content $(B, 1, 5)$. Undertaking 1 has provider A and recipient C, with content $(C, 5, 6)$. The two numbers give the start and finish of the required interval. These are the contents of the three-person extension of Section 13.4. The original content is the projection of the accepted care memory's content.

Each undertaking has its own copy of the four-participant book of Chapter 8 (Figure 8.1): four rows of three accounts, asset, signed liability and free offset. All entries initially vanish. In each copy local row 0 is the provider row and local row 1 is the recipient row. The role map sends local row 1 to Bob for undertaking 0 and to Carol for undertaking 1; this person assignment is part of the construction, not inferred from the number 1.

A proposal posts $(1, 0, -1)$ to row 0, and a recipient's response posts $(0, -1, 1)$ to row 1; each leaves every other row unchanged. These are the book's ordinary own-row writes, applied to the current books rather than to an independently given account of their author.

A refusal is modeled as the recipient's not posting. The construction has no posting by which a recipient authors a refusal, and the result needs none: an undertaking forms only through its recipient's own posting, so Carol's not posting leaves hers unformed. A refusal she authored would be a further record, not built here. The fold below derives one protocol event from each posting, so both branches run four postings and differ only in the fourth. In the rejected branch Alice posts another $(1, 0, -1)$ to her own row 0, local row 1 stays as it was, and the protocol rejects the acceptance this generates because its author is not the recipient.

For books immediately before and after a posting, define the command occupation to be $(0, 1)$ when row 1 changed and $(0, 0)$ when it did not. As in Section B.1, where $(0, 1)$ encodes the accepting recipient and $(0, 0)$ another author, these occupations encode local source codes 1 and 0. Write that code into the three registers (e, m, ρ) of Section B.5: event, memory and reservoir, each with the eight modes of Chapter 7's write. Prepare $(e, 0, 1)$ and use the controlled exchange

$$(e, m, \rho) \longmapsto \begin{cases} (e, \rho, m), & e = 1, \\ (e, m, \rho), & e \neq 1. \end{cases}$$

Consequently a changed recipient row gives $(1, 1, 0)$ and an unchanged recipient row gives $(0, 0, 1)$.

Each undertaking has its own protocol cell, which starts empty: revision and last nonce zero, no proposal, no active terms and no formation record. Proposals and acceptances follow the rules of Section B.2, with the recipient as the accepting author; a successful acceptance also records formation. Each event updates only its own cell.

Execute the following four postings from the empty state: Alice's first proposal, Bob's response, Alice's additional proposal, and Carol's response. Derive each response command from its own before-and-after books, perform the memory exchange above, and assign the protocol author to the recipient when the written memory is 1 and to the provider otherwise. Append every resulting event to the retained history. This is one fold of the state transition, not a supplied accepted prefix.

Proposition B.6 (The postings regenerate both accepted records). *With a recipient's response in both undertakings, this fold produces exactly the two accepted cells used in the three-person construction. The original accepted record and comparator input are preserved. For a controller installed as in Chapter 7, its joint-continuation reading and first selected care action are preserved. With the rejected additional response, the original record still survives, but the additional undertaking is not accepted and its selected record-work amount is zero.*

Proof. In either undertaking, the proposal changes only row 0. Its recipient's response then changes row 1 by the nonzero vector $(0, -1, 1)$. The command therefore decodes source 1 and the exchange writes memory 1. In the first cell the proposal passes the provider, nonce, revision and open-cell guards. The response is consequently attributed to Bob and matches the pending terms, so the acceptance guards all pass. The complete resulting cell equals the original accepted cell, including its revision and formation record.

The third posting addresses only undertaking 1 and creates its pending proposal. In the recipient branch the fourth posting again writes memory 1. Its protocol event is the acceptance by Carol, so the resulting cell function equals the previously defined two-undertaking accepted state. The retained event list is exactly the first proposal, acceptance by Bob, additional proposal and acceptance by Carol. Both later updates address undertaking 1, leaving undertaking 0's books and complete accepted cell unchanged.

In the rejected branch the last posting changes only row 0. Its recipient row is unchanged, so the decoded source is 0 and memory remains 0. The attempted acceptance is attributed to Alice, who is not this undertaking's recipient. The author guard rejects it, and no additional active terms arise. The record-work selector of Section B.5 reads this same memory coordinate and therefore selects zero work in this branch and one unit in the recipient branch.

For the controller installed in Chapter 7, the generated first content projects to the same accepted memory content. The three-person response projection of Section 13.4 preserves the original comparator input for every original action, and its consequence reading equals the installed joint-continuation reading. The installed law of Chapter 7 therefore keeps its original action domain. For any such installed controller, substituting the derived additional source into the joint-operation selector of Sec-

tion B.5 gives the original first meal operation together with one unit of record work. No new first-action choice is introduced. The installation itself rests on the completed physical chain of Chapter 7, from the probe event that detects Bob's acceptance to the written register. That chain is compatible with the generated cell and with this installed reading; the four protocol updates regenerate the records, not the chain. □

The paid step of Section B.5 is the one this fold feeds, as the calendar advances from 0 to 1. The fold fixes what the record and the source must show at five times. These are boundary data that a timed physical trajectory would have to match, not the trajectory itself.

B.7 The acceptance mode has a finite physical writer

In the formation history of Section 13.8, an acceptance decides what its recipient does next. If Bob's cell is active he works; if it is not, he feeds himself from the shared unit, and the same holds for Carol. That choice reads a written memory mode. The contact below writes it: a reversible exchange of a memory mode with an excited reservoir, triggered exactly when the addressed recipient's cell is active and completed in finite time by a positive Hamiltonian.

The control holds the retained formation setting and the recipient being addressed. There are five formation settings: the original history and the four histories that each prospectively prevent one original event. Prevention replaces that event by an unaccepted claim from the same author, and the other three original events remain. Preventing a proposal therefore does not remove the later acceptance event; that event simply lacks the matching proposal needed to activate its cell.

For a control x, let $a(x)$ be true exactly when the corresponding accepted cell is active at checkpoint 2 for Bob or checkpoint 4 for Carol. The writer's registers are (x, m, ρ): retained control, memory mode and reservoir mode. Prepare the sharp state $(x, 0, 1)$. Define W to exchange memory and reservoir exactly when $a(x)$ is true, retaining the entire control. The output memory is then 1 exactly when that cell is active.

This is the conditional swap of Chapter 7, with the pairs of formation setting and addressed recipient as its controls and a as its predicate, so Proposition 7.6 applies as it stands. For any strength $g > 0$ the positive contact Hamiltonian $g(I - W)$ generates a flow equal to W at the time $\hbar\pi/(2g)$, so the sharp prepared state reaches the written register with

amplitude one. The holding energy, the energy of the registers when no contact acts (Table B.2), is control energy plus the two mode energies; the contact keeps the control and that sum, so the memory gains exactly what the reservoir gives up. The excited reservoir is part of that account, so the write is not free, and the food unit does not pay for it.

The contact's availability and its link from the written mode to the care recipe are premises of the formation history. Two pieces lie outside the construction: a further contact that reads the written mode into the care recipe, and a conversion of the write's duration into the material interval labels.

B.8 The release contact

Return to the one-unit conflict of Chapter 13 at cutoff four, before service is selected. Alice has accepted undertakings to Bob and Carol, and one consumable unit can serve only one of them. Suppose Bob waives his claim. For the waiver to count, one physical contact has to record it on Bob's own row of an account book, respond to that change, route the unit's service to Carol, and emit a release event that the original protocol then judges. The construction works for either recipient. The versioned protocol judges a release event whose author, claim and version are given (Section 13.2); the contact below produces those fields from a prepared request, so that, given the origin identification defined below, they are carried by the request rather than assigned to it afterward. A release from the right recipient at the current revision closes exactly that claim; any other request is recorded and leaves both claims active. The construction assumes the schedule of its five contacts, their availability, the clock and the origin identification.

The two claims have addresses $q \in \{0,1\}$, with $r(0) = B$ and $r(1) = C$; write $\bar{q} = 1 - q$. The four retained events are Alice's proposal of term 0, Bob's acceptance, Alice's proposal of term 1, and Carol's acceptance. Each proposal has nonce 1 and base revision 0. At cutoff 4, cell q therefore has revision 1, last nonce 1, no pending proposal, active term q, closed flag false, and its original accepted formation record F_q. There is one consumable unit, denoted u. An allocation assigns u to claim 0, to claim 1, or to neither. The original term q is satisfied exactly when u is assigned to q, so the two terms cannot both be performed by assigning the same unit.

Three ledgers appear, each answering its own question:

- the *care bonds*, the participants' three internal care bonds of Chapter 13, which carry the meal calculations there;
- the *recording ledger*, two cross-participant bonds $A \to B$ and $A \to C$ computed from account books with four accounts per participant, whose book changes are realized exactly on the native ledger; it registers whether a waiver was made, and after Bob waives his ratio with Alice is $3/2$;
- the *service comparison*, two relationship bonds that record each claim as served (multiplier 1) or unserved (multiplier 2); after Bob's release his term has cost $J(2) = 1/4$ there.

Only the care bonds enter the meal calculations; the other two record the waiver and the service. The three share people, claim addresses and the consumable unit, but they are separate constructions with separate numbers, and no number passes from one to another (Tables 13.3 and B.2). Nor does the construction give anyone a right to compel a waiver.

B.8.1 The account book as an addressed vector

There are four accounts per participant, numbered $a = 0, 1, 2, 3$. The opening signed book is

$$\mathcal{B}^0 = \begin{pmatrix} 1 & 1 & -2 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 \end{pmatrix}, \qquad \text{rows } A, B, C. \tag{B.1}$$

A participant's row appears as a subscript, as in $\mathcal{B}_{A0}$. Every row sums to zero. The two designated comparisons are Alice's account 0 against the negative of Bob's account 0, and Alice's account 1 against the negative of Carol's account 0. Both opening ratios are 1. Account 3 lies outside these comparisons. Its entry on recipient $r(q)$'s row is the current waiver port for claim q: zero retains that requirement, and a nonzero entry records its waiver. This port interpretation and the allowed own-row controls are physical premises of the recording apparatus.

On positive designated figures, the recording map to the native ledger is

$$L_{\mathrm{rec}}(\mathcal{B}) : \quad m_{AB} = \frac{\mathcal{B}_{A0}}{-\mathcal{B}_{B0}}, \qquad m_{AC} = \frac{\mathcal{B}_{A1}}{-\mathcal{B}_{C0}}.$$

For a balanced move from $\mathcal{B}$ to $\mathcal{B}'$ with positive designated figures, give each designated relation e the strain

$$v_e = \log m_e(\mathcal{B}') - \log m_e(\mathcal{B}).$$

Under the action rule (3.3) a strain changes the logarithm of a multiplier by exactly its value, so this action takes $m_e(\mathcal{B})$ to $m_e(\mathcal{B}')$; for a balanced target it is the strain of Theorem 3.4. The endpoints and active relations are retained, so the action produces exactly the native ledger read from $\mathcal{B}'$. Applying it move by move does the same for the two-move path below.

The full carrier has an orthogonal basis

$$|k, j, a, b\rangle, \quad k \in \{0,1,2,3\}, \quad j \in \{A, B, C\}, \quad a \in \{0,1,2,3\}, \quad b \in \{0,1\}.$$

It has $4 \cdot 3 \cdot 4 \cdot 2 = 96$ modes. Here k is the service location mode, (j, a) is a book-entry coordinate, and b is a separately prepared orthogonal recipient address. For a book $\mathcal{B}$, define its placed vector by

$$Z(\mathcal{B}; q, k) = \sum_{j,a} \mathcal{B}_{ja} |k, j, a, q\rangle.$$

The opening input is $Z(\mathcal{B}^0; q, 0)$. Its signed entries serve as the complex coefficients of a vector on the carrier, so that the same linear operators can post to a recipient's row and move the whole book: the book path and the route below are one evolution of one carrier. This is a linear encoding of the book, not a normalized state: no probability is read from it, and no step measures it. The coherent encoding and the orthogonal address are prepared inputs; the address q is not inferred by measuring the entries.

B.8.2 Preparation and response act on the recipient's own row

The book itself marks a waiver by a flag, its waiver port. The recording ledger reads only the two designated comparisons, and a bond's one datum is its multiplier, so a waiver can show on that ledger only by moving a ratio. Each designated relation compares an entry on Alice's row with one on the recipient's, and the two match at the opening. A waiver written on the recipient's row alone moves that relation only by changing the recipient's account 0, and any such change leaves the pair unmatched, off balance. This imbalance belongs to the recording ledger alone. Full forgiveness in Chapter 9 works the other way: it corrects the

one figure that carries a claim, so it returns the bond to 1. Two own-row operations do the work. On the four account coordinates let

$$K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 2/3 & 2/3 & -1/3 & 0 \\ 2/3 & -1/3 & 2/3 & 0 \\ -1/3 & 2/3 & 2/3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The first exchanges accounts 2 and 3: preparing a waiver moves the recipient's entry into the waiver port. Preparation alone leaves both designated ratios at 1, so the recording ledger cannot yet tell a waived book from an unwaived one. The second matrix, a reflection, is the response that makes the difference show. It is an exact physical operation on the recipient's own row that reads only that row, with no separate waiver bit, verdict or command. It keeps the waiver port, leaves the opening row where it is, and changes a waived row's account 0, the entry the recording ledger compares. Both matrices satisfy $S^{*} = S$ and $S^{2} = I$, as direct multiplication shows, and both preserve the sum of the four entries. For either matrix S, its full-carrier lift $\hat{S}$ is defined by

$$(\hat{S}z)_{k,j,a,b} = \begin{cases} \displaystyle\sum_{a'} S_{aa'} z_{k,j,a',b}, & j = r(b), \\ z_{k,j,a,b}, & j \neq r(b). \end{cases}$$

Each address sector thus acts on its own selected recipient's row, retaining location, address and the other rows. This is a block matrix on the full carrier, so its involution and self-adjoint properties follow from those of S, including on vectors outside the opening family.

The addressed opening row is $(-1, 0, 1, 0)$ for Bob and for Carol alike, and the calculation on it is

$$\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \xrightarrow{K} \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \xrightarrow{\Sigma} \begin{pmatrix} -2/3 \\ -2/3 \\ 1/3 \\ 1 \end{pmatrix}.$$

Call the intermediate book $\mathcal{B}^{w}(q)$ and the final book $\mathcal{B}^{r}(q)$; their other rows equal those of $\mathcal{B}^{0}$. Preparation is exactly the balanced own-row posting $(0, 0, -1, 1)$. The response is exactly the balanced own-row posting $(1/3, -2/3, 1/3, 0)$ on the resulting book. More generally the response

on an addressed row x posts

$$(-\alpha, 2\alpha, -\alpha, 0), \qquad \alpha = (x_0 - 2x_1 + x_2)/3,$$

which is $\Sigma x - x$. On the opening row $\alpha = (-1 - 0 + 1)/3 = 0$, so the response applied before any waiver is prepared posts nothing. The endpoint is therefore reached by the authored book path itself, rather than by an independently injected target book, and its sole author is $r(q)$. The waiver port becomes 1 and the response retains it.

The response changes the addressed ratio to $1/(2/3) = 3/2$ and leaves the other at 1: when Bob waives, his ratio with Alice becomes $3/2$ and Carol's stays balanced. The value $3/2$ is this reflection's, and it marks the waiver on the record. This response belongs to the recording ledger; it does not deliver the scarce unit to the released recipient.

B.8.3 Two exchanges route the service mode

Let T_{01} exchange location modes $0, 1$ and fix $2, 3$. Let T_{12} exchange $1, 2$ and fix $0, 3$. Lift either location matrix T by

$$(\hat{T}z)_{k,j,a,b} = \sum_{k'} T_{kk'} z_{k',j,a,b}.$$

Every book entry and the actual address are retained. In particular,

$$\hat{T}_{01} Z(\mathcal{B}; q, 0) = Z(\mathcal{B}; q, 1), \qquad \hat{T}_{12} Z(\mathcal{B}; q, 1) = Z(\mathcal{B}; q, 2).$$

Each lifted routing map is again a self-adjoint involution.

All four stages have an explicit positive-generator implementation. If S is any of the full-carrier involutions just defined, choose $g > 0, \hbar > 0$, and put

$$H_S = g(I - S), \qquad U_S(t) = \frac{1 + e^{-2igt/\hbar}}{2} I + \frac{1 - e^{-2igt/\hbar}}{2} S, \qquad d = \frac{\hbar \pi}{2g} > 0. \tag{B.2}$$

The proof of Proposition 7.6 uses only $S^* = S$ and $S^2 = I$, so it applies to each of them: H_S is positive, U_S is the unitary flow it generates, and $U_S(d) = S$. These statements concern the complete carrier, not only its prepared input.

The four pulses, in order, implement preparation, response, the first routing exchange and the second routing exchange. Their endpoint operator is

$$P = \hat{T}_{12} \hat{T}_{01} \hat{\Sigma} \hat{K}.$$

Its endpoint is checkable without an additional dynamics assumption:

$$Z(\mathcal{B}^0; q, 0) \xrightarrow{\hat{K}} Z(\mathcal{B}^w(q); q, 0) \xrightarrow{\hat{\Sigma}} Z(\mathcal{B}^r(q); q, 0)$$
$$\xrightarrow{\hat{T}_{01}} Z(\mathcal{B}^r(q); q, 1) \xrightarrow{\hat{T}_{12}} Z(\mathcal{B}^r(q); q, 2).$$

No new book or verdict is inserted between these operations. Each stage has its own time parameter and the displayed Schrödinger flow, and their sequential product is unitary.

B.8.4 The routed mode credits the single unit once

A routed mode counts as service only through an actual movement of the unit it stands for, and a unit can be credited only once. The delivery rule of Chapter 8, a crossing between cubes two voxels on a side, makes this exact. Put $e_0 = (1, 0, 0)$ and $e_1 = (0, 1, 0)$ in the voxel lattice $\mathbb{Z}^3$. On address q, mode k has position $ke_{\bar{q}}$. Account operations stay at that position, and a nonzero off-diagonal entry of either routing generator connects neighboring modes, hence neighboring voxels on one axis; there is no jump between modes 0 and 2.

For service to claim s, the located unit follows

$$x_s(t) = \max(t - 6, 0)e_s, \qquad t \in \mathbb{N}.$$

This is the path of Chapter 8 with a later start: it waits through tick 6, then moves one voxel per tick. A voxel x has cube address $\mathrm{addr}(x)_h = \lfloor x_h/2 \rfloor$, so $x_s(6) = 0$, $x_s(7) = e_s$ and $x_s(8) = 2e_s$: the unit is in source cube 0 at tick 7 and in the distinct destination cube e_s at tick 8. For $s = \bar{q}$ these sampled positions are exactly the routed modes $0, 1, 2$. That the two routing pulses occupy the intervals from tick 6 to 7 and from 7 to 8, and that the routed modes are the located unit, are conditions of the application; no classical position is asserted between ticks.

To count performance, start with allocation $a(u) = \emptyset$. The accepted cell requires one unit, with a separately given deadline δ, since a binary claim address does not specify one. The crossing credit assigns $a(u) = s$ only if the located unit crosses between distinct source and destination cubes, $a(u)$ is still unassigned, and the active quantity exceeds the number already credited to s. All three hold here: the crossing falls between ticks 7 and 8, the unit is unassigned, and $0 < 1$. Credit therefore assigns the single u to s. Every later attempt to credit that unit fails the unassigned

condition, even for another claim or path, and a stationary unit never meets the crossing condition.

The service reader selects 0 if only claim 0 is live, selects 1 if only claim 1 is live, and selects neither if both or neither are live. On $\mathcal{B}^0$ both ports are zero. On $\mathcal{B}^r(q)$ precisely claim $\bar{q}$ remains live, since the response retains port 3. The service reader therefore agrees with the addressed route: u is allocated to $\bar{q}$, so when Bob waives, the unit goes to Carol. The selector, like the identification of the crossing with the service unit, is a condition of the application.

B.8.5 The same carrier writes the release event

Extend this carrier by orthogonal control sectors $c = (i, q, \nu)$, where $i \in \{A, B, C\}$ and $q, \nu \in \{0, 1\}$, and one bit m. The origin is i, the addressed claim is q, and the requested revision is ν. The initial bit is ready, $m = 0$. The *origin identification*, the identification of each control sector with the participant who prepared it and so an identification of the device with the people, requires the actual preparation contact of participant i to have support only in origin sector i, with the address mode equal to the retained q. A request that merely bears the symbol i does not supply this physical identification.

Define the reversible writer by

$$W|k, j, a, b; c, m\rangle = |k, j, a, b; c, 1 - m\rangle.$$

It is a self-adjoint involution, so (B.2) supplies its positive contact and exact finite-time write. It retains the whole 96-mode vector and every control field. With the twelve control sectors and the two values of the bit, the extended carrier has $96 \cdot (3 \cdot 2 \cdot 2) \cdot 2 = 2304$ modes. Call the four account and routing stages the payload, and enable them exactly when

$$\chi(c) \quad \Longleftrightarrow \quad i = r(q) \text{ and } \nu = 1.$$

In each enabled control sector use the four stages defined above; in every disabled sector use the identity. This block construction is unitary on the complete extended carrier. On the prepared address sector $b = q$, its enabled account generator can have a nonzero entry only on row $r(q) = i$, retaining location and address. The prepared enabled contact thus acts on the prepared origin's own row, and its balanced book path has that same sole author.

Write first, then execute the four guarded payload pulses. Combining the five endpoint calculations gives

$$Z(\mathcal{B}^0; q, 0) \otimes |c, 0\rangle \longmapsto \begin{cases} Z(\mathcal{B}^r(q); q, 2) \otimes |c, 1\rangle, & \chi(c), \\ Z(\mathcal{B}^0; q, 0) \otimes |c, 1\rangle, & \neg\chi(c). \end{cases} \tag{B.3}$$

The emitted record is decoded from the retained c and the bit actually written: ready means no release event, while written means release(q, i, ν). No independently authenticated event is inserted into the dynamics; attribution rests on the prepared origin sector.

B.8.6 The original cell transition still decides closure

Append that written event at index 4, preserving all four earlier events. The release rule checks the designated releaser, the current revision, an active term, and an open cell. The designated releaser for this application is $r(q)$. The last two conditions already hold at cutoff 4, so its exact result is

$$s'_q = \begin{cases} (1, 1, \emptyset, \emptyset, \text{true}, F_q), & \chi(c), \\ (1, 1, \emptyset, q, \text{false}, F_q), & \neg\chi(c). \end{cases}$$

The tuple lists revision, last nonce, pending proposal, active term, closed flag and formation record. The other cell is unchanged in full. This follows by substituting (q, i, ν) into the release rule: its two remaining checks are precisely $i = r(q)$ and $\nu = 1$. Decoding the produced event and applying that rule therefore commute with the original cell update.

Together with the endpoint (B.3), this gives the contact's result. Write $z_0(q) = Z(\mathcal{B}^0; q, 0)$ for the opening book vector at service location zero and $z_1(q) = Z(\mathcal{B}^r(q); q, 2)$ for its prepared and reflected book vector at location two.

Proposition B.7 (The contact produces the fields of its release event). *For the prepared controls and opening vector just specified, the completed five-pulse sequence has endpoint*

$$z_0(q) \otimes |c, 0\rangle \longmapsto \begin{cases} z_1(q) \otimes |c, 1\rangle, & \chi(c), \\ z_0(q) \otimes |c, 1\rangle, & \neg\chi(c). \end{cases}$$

Decode the written sector as release(q, i, ν) *and apply it to the retained four-event history. The original transition closes exactly cell q when $\chi(c)$ holds. Otherwise both complete original cells remain unchanged. In every case their revision, nonce and formation records, and the earlier history, are retained.*

Proof. The first pulse implements W, changing only the bit. Each subsequent operator preserves the control sector, so its block for c acts on the book vector. If $\chi(c)$ holds, the four coordinate calculations above give $z_1(q)$. Otherwise each block is the identity, giving $z_0(q)$. All five factors are unitary.

The decoder reads the bit actually written and the fields retained by that same sequence. At cutoff four the addressed cell is open and has active terms. Its release checks therefore reduce to the designated author $i = r(q)$ and the current revision $\nu = 1$, exactly $\chi(c)$. On success the original rule clears pending and active content and marks the cell closed; on failure it retains the complete cell. It does not change the other cell or erase any earlier event. □

A wrong-origin request (Carol asking to release Bob's claim, say) or a stale revision-0 request writes its actual attempted event, retains both original cells, and leaves the entire opening vector at mode 0. Its allocation is unassigned. An enabled request closes only its addressed cell and assigns the original unit to $\bar{q}$. Every currently active cell is satisfied exactly in the enabled case: the released cell has no active term, and the remaining cell receives the one unit. In the disabled case the addressed original term is active and unmet. None of this converts a service label into food or derives Alice's material labor or her reserve payment.

Closure grants no permission, and it leaves the released recipient's original service term unperformed. The service comparison records that. It has two bonds $A \to r(s)$, $s \in \{0, 1\}$, with multipliers

$$m_s^{\mathrm{svc}}(a) = \begin{cases} 1, & a(u) = s, \\ 2, & a(u) \neq s. \end{cases}$$

It puts served terms at balance and every unserved term at one common positive multiplier; the chart adopts 2, a convention of this comparison rather than a derived value. Hence the released recipient retains service-comparison cost $J(2) = 1/4$: after Bob's release, Carol's term is at balance and Bob's has cost $1/4$. This quarter shares only its value with the care ledger's $J(2)$ of Chapter 6, the cost of a bond that exposure has doubled under the care environment that chapter adopts; here the 2 is a labeling convention of the comparison. A changed waiver record and a satisfied remaining cell neither erase that retained shortfall nor establish the participant's initial consent.

Because the writer precedes the payload, a written record can be exposed before the response and routing finish, and the construction does not make release and service one uninterruptible step.

B.9 A completed record certifies the completed payload

Put the writer after the four payload pulses instead of before them. Then, at every moment of the evolution, the written sector holds only a payload that has already finished, with its original origin, claim and revision. The order uses the same 2304-mode carrier with the same control c, ready bit and five contacts.

Let U_1, U_2, U_3, U_4 be the controlled flows for $\hat{K}$, $\hat{\Sigma}$, $\hat{T}_{01}$, and $\hat{T}_{12}$, respectively. In an enabled control sector each has the flow in Equation (B.2); in a disabled sector it is the identity at every time. None of these four flows uses the record bit as a guard. Write

$$V(p,r,f,s) = U_4(s)U_3(f)U_2(r)U_1(p).$$

The prepared opening and completed payload are

$$z_0(c) = Z(\mathcal{B}^0;q,0), \qquad z_*(c) = \begin{cases} Z(\mathcal{B}^r(q);q,2), & \chi(c), \\ z_0(c), & \neg\chi(c). \end{cases}$$

The preceding account and routing calculations give $V(d,d,d,d)(z_0(c)\otimes|c,0\rangle) = z_*(c)\otimes|c,0\rangle$. For the same duration $d = \hbar\pi/(2g) > 0$, put

$$\alpha(t) = \frac{1+e^{-2igt/\hbar}}{2}, \qquad \beta(t) = \frac{1-e^{-2igt/\hbar}}{2}.$$

The independent writer has the flow of Proposition 7.6, $U_W(t) = \alpha(t)I + \beta(t)W$.

Use the following five consecutive phase operators, each for $0 \le t \le d$:

$$\begin{aligned} Q_1(t) &= V(t,0,0,0), \\ Q_2(t) &= V(d,t,0,0), \\ Q_3(t) &= V(d,d,t,0), \\ Q_4(t) &= V(d,d,d,t), \\ Q_5(t) &= U_W(t)V(d,d,d,d). \end{aligned}$$

Earlier pulses are finished, the indicated pulse is evolving, and later pulses have time zero. These are actual partial-time products of the flows defined above, so no single time-independent Hamiltonian for the whole switched contact is needed. Each $Q_j(t)$ is unitary. They start at $Q_1(0) = I$ and meet exactly: $Q_j(d) = Q_{j+1}(0)$ for $j = 1, 2, 3, 4$. The first three boundary equalities follow from the displayed parameters; the fourth uses $U_W(0) = I$.

Define the written-sector projection independently of any payload endpoint:

$$(\Pi_1\psi)(x, c, m) = \begin{cases} \psi(x, c, m), & m = 1, \\ 0, & m = 0. \end{cases}$$

Let $\psi_j(t; c) = Q_j(t)(z_0(c) \otimes |c, 0\rangle)$.

Proposition B.8 (Completion-record soundness). *For every prepared control c and every time in each of the five phases,*

$$\begin{aligned} \Pi_1\psi_j(t; c) &= 0 && (j = 1, 2, 3, 4), \\ \psi_5(t; c) &= \alpha(t)z_*(c) \otimes |c, 0\rangle + \beta(t)z_*(c) \otimes |c, 1\rangle, \\ \Pi_1\psi_5(t; c) &= \beta(t)z_*(c) \otimes |c, 1\rangle. \end{aligned}$$

The final state is exactly $z_*(c) \otimes |c, 1\rangle$, equal to the prepared final endpoint obtained by putting the independent writer first.

Proof. Every payload pulse is block diagonal in (c, m) and therefore preserves $m = 0$. The projection onto $m = 1$ consequently annihilates each of the first four phase states, even when a payload pulse has only partly run.

At the fourth endpoint the numerical account calculation and the two location exchanges above give $z_*(c) \otimes |c, 0\rangle$. This includes disabled controls: their four payload pulses are identity operations. Applying the writer's exact partial-time expression to this state gives the displayed two terms. Projecting keeps precisely the second term. At $t = d$, $\alpha(d) = 0$ and $\beta(d) = 1$, proving the final endpoint.

For the writer-first order, the writer endpoint first gives $z_0(c) \otimes |c, 1\rangle$. The payload pulses are independent of m, so the same four endpoint operations give $z_*(c) \otimes |c, 1\rangle$ again. This proves equality of the prepared endpoints without requiring a change of carrier. □

Whenever the written record has nonzero weight, its payload vector is proportional to the completed payload and its control is the original c, so origin, claim and version are unchanged. This holds for the written component even while the bit is in superposition.

The event decoder still reads (i, q, ν) only from a written bit, so a current request from the right recipient gives the same release event, cell update and single-unit service as before, though at different intermediate times. Wrong-origin and stale requests have the identity payload throughout: they can record the rejected attempt while the accepted cell stays unchanged, so a written bit alone does not mean a release was accepted.

Completed records are thus truthful about their completed payload. The evolution is not atomic: during the first four phases the payload can change while the bit is still unwritten.

B.10 One retained acceptance record controls a distinct resource route

When Carol's acceptance is added to Alice's undertaking with Bob, Bob's resource must not disappear if Carol declines. Six packet exchanges on one carrier keep the two routes apart. The first three move Alice's source packet to Bob in every branch; the last three move Carol's separate packet to the recording apparatus only when the acceptance memory holds one. Every contact keeps the full holding Hamiltonian, the evolution each part has when no contact acts.

The complete carrier and its holding. Write a register label as $m = (i, a, \rho) \in \{0, \dots, 7\}^3$: the event, acceptance-memory and reservoir coordinates. Its diagonal holding energy is

$$E_M(m) = \varepsilon(i) + \varepsilon(a) + \varepsilon(\rho), \qquad \varepsilon(k) = \frac{\hbar \pi k}{4\tau_0}, \quad \tau_0 > 0.$$

Write H_M for the diagonal operator with these entries. The control is the memory coordinate a itself, read in its sector $a = 1$; taking that sector as an authored acceptance requires the prepared encoding and provenance of the record, since the number 1 alone identifies no author.

Each resource is a *material packet*. It stands in for Alice's meal so that the routing is checked on matter with its own energy; calibrating a packet to

nourishment is one of the open connections of Section B.14. The routing argument uses one property of the packet: a self-adjoint holding H_N, the same on all five packets, so it holds for any self-adjoint holding used alike on all five. The particular body matters only in the sections that follow: Section B.12 carries it on the probe's own channels with nothing discarded, and Section B.13 needs its spectrum. The body is a small piece of matter built from the same four-level clocks as the probe of Section B.1, with its own holding. It has two species of opposite charge on one open edge. Each species can sit at either end of the edge and carries two four-level clocks, so a packet has $(2{\cdot}4{\cdot}4)^2 = 1024$ levels. Let $\mathcal{K}$ be this complete packet space, with its unchanged self-adjoint holding H_N: both species' clock terms and the electric field cost along the edge, with no rest-energy subtraction or spectral projection. Use five copies: a shuttle T and four counted sites $0, 1, 2, 3$. Site 0 holds the packet that plays Alice's meal for Bob, site 1 receives for Bob, site 2 holds Carol's separate resource, and site 3 receives for the recording apparatus. Thus

$$\mathcal{H} = \mathbb{C}^{8^3} \otimes \mathcal{K}_T \otimes \mathcal{K}_0 \otimes \mathcal{K}_1 \otimes \mathcal{K}_2 \otimes \mathcal{K}_3, \qquad H_0 = H_M \otimes I + I \otimes \sum_{j \in \{T,0,1,2,3\}} H_N^{(j)}.$$

The matrix $H_N = M$ is given in Section B.12, in dimensionless clock units, so taking the sum H_0 as one physical energy requires a common unit with the memory's quarter-turn scale. The operator identities below hold for the displayed matrices as they stand.

For a joint basis label $x = (m, t, b)$, where b gives the four site labels, let s_j exchange the shuttle and site j. Define two operations:

$$S_{0,j}|m,t,b\rangle = |m, s_j(t,b)\rangle, \qquad S_{1,j}|m,t,b\rangle = \begin{cases} |m, s_j(t,b)\rangle, & a = 1, \\ |m,t,b\rangle, & a \neq 1. \end{cases}$$

The first subscript distinguishes unconditional and controlled contacts. Both operations retain all three register labels. Each is a self-adjoint involution: exchanging the same two packets twice restores the state, and the controlling register is unchanged by the exchange.

Moreover $[H_0, S_{c,j}] = 0$. The two exchanged packets have identical copies of H_N, so their sum is exchange invariant, and the other packet terms are untouched. The projector onto $a = 1$ is diagonal in the register energy basis and commutes with H_M, in the inactive sectors as in the active one.

A contact with positive duration and retained holding. For $\gamma > 0$, take the microscopic interaction

$$K_{c,j} = H_0 + \frac{\gamma}{2}(I - S_{c,j}), \qquad \delta = \frac{\pi\hbar}{\gamma} > 0.$$

The added term is the contact of Proposition 7.6 with strength $\gamma/2$ and $S_{c,j}$ in place of the swap. That proof uses only $S^* = S$ and $S^2 = I$, so the term is positive semidefinite and its flow at the half-period $\hbar\pi/(2 \cdot \gamma/2) = \delta$ is exactly $S_{c,j}$. Since the holding and exchange commute, the two flows factor:

$$e^{-i\delta K_{c,j}/\hbar} = e^{-i\delta H_0/\hbar} e^{-i\pi(I-S_{c,j})/2} = D(\delta) S_{c,j}, \qquad D(t) = e^{-itH_0/\hbar}.$$

On an inactive controlled sector, $S_{1,j} = I$ and the pulse is $D(\delta)$, so elapsed time is not suppressed. The nonnegative added term belongs to the contact; it is not identified with a debit in any of Carol's accounts (Tables 13.3 and B.2).

Six contacts, with the original delivery in both branches. Apply the following contacts in chronological order:

$$(0,0),\ (0,1),\ (0,0),\ (1,2),\ (1,3),\ (1,2).$$

Let W be their exchange product, with later operations acting on the left. Three shuttle exchanges with sites j, k, j exchange the contents of j and k and restore the shuttle. Follow the three contents: (t, b_j, b_k) becomes (b_j, t, b_k), then (b_k, t, b_j), then (t, b_k, b_j). The first triple therefore exchanges sites 0 and 1 in every memory sector. The second exchanges sites 2 and 3 exactly when $a = 1$. Consequently the resulting label map F is

$$F(m,t,b) = (m,t,b'), \qquad b'_0 = b_1, \quad b'_1 = b_0,$$

$$(b'_2, b'_3) = \begin{cases} (b_3, b_2), & a = 1, \\ (b_2, b_3), & a \neq 1. \end{cases}$$

All exchanges commute with $D(t)$. The product of the six actual contacts is therefore exactly

$$U_\gamma = D(6\delta) W.$$

Its elapsed duration is $6\delta > 0$ in both memory branches.

Choose distinct spectral labels $g \neq e$ of H_N for the resource's ready and loaded states. Both states belong to the full material body; *unoccupied* and *occupied* below mean these two resource states, not the absence or presence of the body. Prepare the shuttle in g and the four sites in (e,g,e,g). The endpoints are

$$(g;e,g,e,g) \longmapsto \begin{cases} (g;g,e,g,e), & a = 1, \\ (g;g,e,e,g), & a \neq 1. \end{cases}$$

The entry before the semicolon is the shuttle. Bob receives Alice's source packet in both cases. The apparatus receives Carol's separate packet only in the active case. Exactly two of the four sites are occupied before and after, and the empty shuttle is restored, so the complete five-packet bank also contains exactly two occupied packets. The second route does not make a second copy of Alice's resource.

The resource outcomes are observables of the actual timed state. It remains to connect the label calculation to the full packet states. Let U_N be a unitary eigenbasis for H_N and set

$$\Omega = I_M \otimes U_N^{\otimes 5}.$$

Then $\Omega^*\Omega = I$ and $\Omega^* H_0 \Omega$ is diagonal. For any function q on joint spectral labels, define the diagonal observable

$$O_q = \Omega \, \mathrm{diag}(q) \, \Omega^*.$$

For an occupation measurement, q is the real indicator of the desired site and spectral level. The same formula also allows joint indicators that retain correlations between sites and the memory.

All O_q commute with H_0, because both are diagonal in the same complete basis. Hence $D(t)^* O_q D(t) = O_q$ for every t. Also $W\Omega = \Omega W$: applying the same U_N to all five packets commutes with their exchanges, while the register factor is unchanged. The columns of the actually evolved preparation are $E_\gamma = U_\gamma \Omega = D(6\delta) W \Omega$. Thus

$$\begin{aligned} E_\gamma^* O_q E_\gamma &= \Omega^* W^* D(6\delta)^* O_q D(6\delta) W \Omega \\ &= W^* \, \mathrm{diag}(q) W \\ &= \mathrm{diag}(q \circ F). \end{aligned}$$

The final equality follows by applying the permutation to each column: $W|x\rangle = |F(x)\rangle$. This is an identity of complete matrices, not a calculation for the two prepared labels alone. The resource endpoints above are therefore observed values of the actual preparation after six finite contacts.

One acceptance-memory coordinate thus controls a separate resource route while Bob's own route runs in both branches. The interaction, the holding, the elapsed time and the observables are explicit; what switches the six contacts on in turn is not derived here. A written mode, a positive contact term or a delivered packet does not become an independent source of moral priority.

B.11 Writing the record before routing the resources

The route of the previous section reads an acceptance memory that is already written. Here the memory is written first, by the conditional memory–reservoir exchange of Section B.5, and the six contacts then read the freshly written coordinate. All seven stages run on one carrier with their full holding evolution, so the history is fixed at every time, not only at its endpoint.

Use the complete carrier, register labels $m = (i, a, \rho)$ and holding of Section B.10:

$$\mathcal{H} = \mathbb{C}^{8^3} \otimes \mathcal{K}^{\otimes 5}, \qquad H_0 = H_M \otimes I + I \otimes \sum_{j \in \{T,0,1,2,3\}} H_N^{(j)}.$$

The first stage writes the original memory. Define the register permutation

$$w(i, a, \rho) = \begin{cases} (i, \rho, a), & i = 1, \\ (i, a, \rho), & i \neq 1. \end{cases}$$

Lift it to $\mathcal{H}$ by leaving all five packets unchanged, and call the resulting permutation operator Ξ. It retains i, satisfies $\Xi^* = \Xi$ and $\Xi^2 = I$, and preserves E_M: when an exchange occurs, the last two terms of the same sum exchange places. Consequently $[H_0, \Xi] = 0$. The packet terms are unchanged, and the memory energy is preserved even in sectors outside the prepared source event.

For $\gamma > 0$, put

$$\delta = \frac{\pi\hbar}{\gamma} > 0, \qquad K_w = H_0 + \frac{\gamma}{2}(I - \Xi), \qquad D(t) = e^{-itH_0/\hbar}.$$

The added operator is the writer of Proposition 7.6 with strength $\gamma/2$, so it is positive semidefinite and its flow at δ is exactly Ξ. Since Ξ commutes with H_0, the write pulse has the exact endpoint

$$U_w = e^{-i\delta K_w/\hbar} = D(\delta)\Xi.$$

This is the original writer, now with the complete common holding and its energy phases.

Take a source label $s \in \{0,1\}$ and prepare

$$m_s = (s,0,1).$$

Then $w(m_0) = (0,0,1)$ and $w(m_1) = (1,1,0)$. In the second branch the memory gains one register quantum while the reservoir loses one; in the first branch both keep their initial values. The writer does not authenticate the event's author.

The following stages use the newly written coordinate. After U_w, apply the six full-holding contacts of Section B.10 in chronological order

$$(0,0),\ (0,1),\ (0,0),\ (1,2),\ (1,3),\ (1,2).$$

Each has Hamiltonian $K_{c,j} = H_0 + (\gamma/2)(I - S_{c,j})$ and duration δ. The $c = 1$ contacts read the actual memory coordinate $a = 1$. They read neither the original source coordinate directly nor an independently supplied switch, and all six retain the written record.

During the first stage the propagator is $e^{-itK_w/\hbar}$, $0 \le t \le \delta$. If P is the product of the stages already completed, the next stage is $e^{-itK_{c,j}/\hbar}P$ on the same interval. At $t = 0$ this is P; at $t = \delta$ it is the product with the next contact on the left. These expressions therefore meet at each boundary, and every stage includes H_0.

Let W denote the six exchange operators in their execution order, with the latest factor on the left. The actual seven-stage propagator is

$$U_{\text{staged}} = U_\gamma U_w = D(6\delta)WD(\delta)\Xi = D(7\delta)W\Xi.$$

The final equality uses $[H_0, S_{c,j}] = 0$ for every resource contact; it does not require Ξ to commute with the controlled route, and the write remains first. The total elapsed time is $7\delta > 0$ in both source branches.

The complete record and packet outcomes. For a joint label $x = (m, t, b)$, the resulting label map is

$$G(x) = F(w(m), t, b),$$

where F is the six-contact map already derived. The memory and reservoir at the endpoint are therefore precisely $w(m)$; the shuttle returns to t; sites 0 and 1 exchange in every source branch; and sites 2 and 3 exchange exactly when the *written* memory equals 1.

Choose distinct complete spectral labels $g \neq e$ and prepare the shuttle and sites as $(g; e, g, e, g)$. Including the register, the two outcomes are

$$(m_0; g; e, g, e, g) \longmapsto (0, 0, 1; g; g, e, e, g),$$

$$(m_1; g; e, g, e, g) \longmapsto (1, 1, 0; g; g, e, g, e).$$

Bob receives Alice's original resource in both branches. The apparatus receives Carol's separate resource only after the source-one write. Exactly two sites contain e before and after, and the restored shuttle contains g. No packet is duplicated.

These are observed values of the actual state, including its holding phases. Use the eigenbasis Ω and observables O_q of Section B.10. The lifted writer commutes with Ω, since it acts on the register alone; so does the route, and $D(t)$ preserves every O_q. For the actual evolved preparation $E = U_{\text{staged}}\Omega$,

$$\begin{aligned} E^* O_q E &= \Omega^* \Xi^* W^* O_q W \Xi \Omega \\ &= \Xi^* W^* \operatorname{diag}(q) W \Xi \\ &= \operatorname{diag}(q \circ G). \end{aligned}$$

The last equality follows by applying the permutation to each basis vector. The formula holds for every function q on the joint labels, so it retains the correlations among event, written memory, reservoir, shuttle and packet occupations.

The composition connects the acceptance-memory write to the controlled delivery on one carrier, with all seven holding intervals retained. Being a longer history than the write pulse, it does not meet a requirement on that pulse's history merely by ending with the same record.

Which interaction must the apparatus supply?. Let P be the projector on the full memory space that selects the middle coordinate $a = 1$, and let

T_j exchange the complete shuttle and site-j packets. Then

$$S_{1,j} = (I - P) \otimes I + P \otimes T_j, \qquad I - S_{1,j} = P \otimes (I - T_j).$$

To prove the first identity, apply both sides to any memory and packet basis vector. If $a = 1$, both exchange the packets; otherwise both leave them in place. Linearity gives the full operator identity, including superpositions of the memory sectors. Subtraction gives the second. The necessary added interaction is therefore exactly

$$K_{1,j} - H_0 = \frac{\gamma}{2} P \otimes (I - T_j).$$

Its availability requires an apparatus that supplies this memory-selective whole-packet interaction, or a sequence with the same effect.

One way to supply it is from the probe's own parts, so that the memory and the matter sit on the same kind of register as the probe: a step toward the single carrier Section B.14 asks for. That route adds a constraint, which is why Section B.12 builds a larger carrier instead of relabeling this one. One channel of the probe's cell is an eight-level register, an occupation bit times a four-level clock: a count of the cell, separate from the eight entries of Section 2.2. A carrier made of such registers has dimension a power of 8. The present carrier has three eight-level registers and five complete 1024-level packets, so its dimension is

$$8^3\, 1024^5 = 2^{59}.$$

It cannot be relabeled as a complete tensor of n eight-level registers for any integer $n \geq 0$: such a tensor has dimension $8^n = 2^{3n}$, and $59 = 3 \cdot 19 + 2$ is not a multiple of 3. A change of coordinates alone therefore cannot carry the construction onto such registers; an enlarged apparatus must contain added physical sectors and account for them, as Section B.12 does. The count concerns relabeling only; it says nothing against control on the present carrier.

The exact remaining question can be stated without naming a particular control scheme. Let $\mathcal{U}$ be a set of positive interaction terms supplied by an independently specified apparatus on this same carrier, each to be added to the holding. Retain $\gamma > 0$, $\hbar > 0$ and $\delta = \pi\hbar/\gamma$. Are there finitely many terms $Y_1, \ldots, Y_k \in \mathcal{U}$ and positive durations $t_1, \ldots, t_k$ such that

$$\sum_{\ell=1}^{k} t_\ell = \delta, \qquad e^{-it_k(H_0+Y_k)/\hbar} \cdots e^{-it_1(H_0+Y_1)/\hbar} = D(\delta) S_{1,2}?$$

Every factor retains the original holding. The set $\mathcal{U}$ must be derived from the apparatus independently of this desired equality. An approximate endpoint on an enlarged carrier would answer a different, useful question, with its additional resources and error stated.

B.12 The same material operation inside a larger physical carrier

The packet of Section B.10 is a separate material system. It can also be carried by eight-level registers, the channels of the probe's cell, with nothing discarded: the holding keeps every extra state of those channels rather than dropping it or giving it an arbitrary constant energy, and the controlled packet exchange and its whole evolution restrict exactly to the original ones. The routing result, Bob served in every branch and nothing copied, therefore holds unchanged on the larger carrier. Three facts carry the section: the frame that places a packet on the channels is an isometry, the full holding on the channels is positive, and it restricts exactly to the packet's holding M; the contact and its whole evolution then restrict to the originals. The clock operator and the coupling weight below are the material body's, and its field scale 4 is adopted rather than derived; they are written out in full so that each of the three facts can be checked.

Write $V = \mathbb{C}^8$ for one channel. A single species in the material packet sits at the left or right end of its edge and has two internal clocks, so its space is $\mathcal{K}_{\mathrm{sp}} = (\mathbb{C}^4 \otimes \mathbb{C}^4) \oplus (\mathbb{C}^4 \otimes \mathbb{C}^4)$. The two-species packet is $\mathcal{K} = \mathcal{K}_{\mathrm{sp}} \otimes \mathcal{K}_{\mathrm{sp}}$, of dimension 1024. Let M be its complete material holding, including its electric term. Each species gets one channel at each end of the edge. At the left end it fills its left channel and leaves its right channel empty, and the reverse at the right end; both channels keep their clocks.

The four-register embedding writes this exactly, starting from one frame. Let $Y : \mathbb{C}^4 \to V$ be the empty occupation frame, the channel's four empty states. In coordinates $t = 0, \dots, 7$ and $j = 0, \dots, 3$ it is

$$Y_{tj} = \frac{\delta_{t,2j} - i d_j \delta_{t,2j+1}}{\sqrt{2}}, \quad (d_0, d_1, d_2, d_3) = (1, -1, -1, 1),$$
$$S = \mathrm{diag}(1, -1, -1, 1, -1, 1, 1, -1).$$

Put

$$\mathrm{P} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \qquad O = SY\mathrm{P}^{*}.$$

Here P is a signed permutation, and the columns of O are the channel's four occupied states. The two entries in each column of Y have squared moduli $1/2$ and disjoint support from the other columns. Thus $Y^{*}Y = I$. On that pair, $SY_j = (d_j, i)/\sqrt{2}$, and $Y_j^{*}SY_j = (d_j - d_j)/2 = 0$. The displayed P is unitary. Consequently $O^{*}O = I$ and $Y^{*}O = 0$; the eight columns of $[\,Y\ O\,]$ form an orthonormal basis, giving

$$Y^{*}Y = O^{*}O = I_{\mathbb{C}^4}, \qquad Y^{*}O = O^{*}Y = 0, \qquad YY^{*} + OO^{*} = I_{V}.$$

The occupied channel therefore keeps its four-dimensional internal clock. Define

$$\iota_1 = [\,O \otimes Y \quad Y \otimes O\,] : \mathcal{K}_{\mathrm{sp}} \longrightarrow V \otimes V, \qquad \iota = \iota_1 \otimes \iota_1 : \mathcal{K} \longrightarrow V^{\otimes 4}.$$

The order is positive species left and right, followed by negative species left and right. Multiplication of the two block columns gives $\iota_1^{*}\iota_1 = I$: the diagonal blocks are $I \otimes I$ and the off-diagonal blocks contain $Y^{*}O$ or $O^{*}Y$. Consequently $\iota^{*}\iota = I_{\mathcal{K}}$.

The full holding on $V^{\otimes 4}$ must also act on the states a packet never occupies, so it comes from the complete occupation chart. For each species use the four Fock states $0, \mathrm{L}, \mathrm{R}, \mathrm{LR}$ (neither end occupied, the left, the right, both), retaining both internal clocks in every state. The single-channel unitary frame is $[\,Y\ O\,]$; its two-channel tensor product, reordered as $0, \mathrm{L}, \mathrm{R}, \mathrm{LR}$, gives the species frame. Taking the two species gives a unitary F on the full four-register carrier. In that source chart put

$$f_L = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad f_R = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

For an internal clock matrix D and a hopping matrix T, which moves the species from one end to the other, the complete species row is

$$\mathrm{R}(D, T) = I \otimes D + (f_R^{*}f_L) \otimes T + (f_L^{*}f_R) \otimes T^{*}.$$

The rows are built from the positive clock operator Q of one channel, written in its complete circulant chart. Its entries are

$$Q_{ij} = \frac{7\delta_{ij} - (1 - \delta_{ij})}{2} + \frac{i}{2}\{u^{(1)}_{i-j} + \sqrt{2}\, u^{(2)}_{i-j}\},$$

where indices are modulo eight, $u^{(1)} = (0, 1, 1, -1, 0, 1, -1, -1)$ and $u^{(2)} = (0, 1, 0, 1, 0, -1, 0, -1)$. To check its positivity directly, put

$$U_{jk} = \frac{\exp(-2\pi i j k/8)}{\sqrt{8}}.$$

The finite geometric sum of eighth roots gives $U^*U = I$, and the displayed entries follow by substitution in $8^{-1}\sum_{k=0}^{7} k \exp[-2\pi i(i-j)k/8]$, so $Q = U\,\mathrm{diag}(0, \dots, 7)U^* \geq 0$. Let $D_0 = Y^*QY$, $\kappa = 1/\sqrt{2}$, and set

$$X_L = [Y \otimes I \quad O \otimes \mathrm{P}^*], \qquad X_R = [I \otimes Y \quad \mathrm{P} \otimes O],$$

$$K_L = (1 - \kappa)\,\mathrm{diag}(D_0 \otimes I, D_0 \otimes I) + \kappa X_L^*(Q \otimes I)X_L,$$
$$K_R = (1 - \kappa)\,\mathrm{diag}(I \otimes D_0, I \otimes D_0) + \kappa X_R^*(I \otimes Q)X_R.$$

Each is a convex sum of positive matrices. Their diagonal blocks are $D_L = D_0 \otimes I$ and $D_R = I \otimes D_0$, respectively; write their lower off-diagonal blocks as T_L, T_R. Use $\mathrm{R}(D_L, T_L) + \mathrm{R}(D_R, T_R)$ for each species and sum over both species. This defines H_{clock}. In the Fock order above each row has the empty block D, the original one-particle block $\left(\begin{smallmatrix} D & T^* \\ T & D \end{smallmatrix}\right)$, and the double block D. All these blocks are positive for the fixed material rows, so $H_{\mathrm{clock}} \geq 0$; the empty and double clocks remain present.

The electric term is the field energy on three edges: the packet's own edge between its two ends, and an added exterior edge beyond each end. If n_L, n_R are the occupation numbers at the two positions, let q_L and q_R be positive-species occupation minus negative-species occupation at those positions, and put $Q_{\mathrm{tot}} = q_L + q_R$. At exterior flux $m \in \mathbb{Z}$ the three edge fields are $(m, m - q_L, Q_{\mathrm{tot}} - m)$. With the field scale 4 of the material body, let

$$d(x) = x\,\mathrm{arsinh}(x/2) - \sqrt{x^2 + 4} + 2,$$
$$V_m = \mathrm{diag}\{4d(m) + 4d(m - q_L) + 4d(Q_{\mathrm{tot}} - m)\}.$$

The full physical holding used here is

$$H_4 = F(H_{\mathrm{clock}} + V_0)F^*.$$

To see that it is positive, write $x = 2\sinh r$. Then $d(x) = 2(r\sinh r - \cosh r + 1)$. This is even in r, is zero at $r = 0$, and has derivative $2r\cosh r \geq 0$ for $r \geq 0$. Hence $d(x) \geq 0$ for every real x. Therefore $H_4 \geq 0$ on every occupation and clock state, including charged states. Here $m = 0$ fixes the *left* exterior flux. The right edge still carries Q_{tot} in a charged sector; it is not set to zero. Using the same scale on the added exterior edges is part of this extension.

The field density d, whose multiple $4d$ is each edge's cost, is the Legendre conjugate of twice the recognition cost in its logarithmic coordinate, $r \mapsto 2J(e^r)$. The two are therefore a convex-conjugate pair, related as a Hamiltonian and a Lagrangian are under the Legendre transform; the edge energy $4d$ and the bond cost J are fixed multiples of them.

Proposition B.9 (The field density is conjugate to the recognition cost). *For all real x and r,*

$$xr - 2J(e^r) \leq d(x),$$

with equality at $r = \operatorname{arsinh}(x/2)$. Hence $d(x) = \max_{r\in\mathbb{R}}\{xr - 2J(e^r)\}$.

Proof. Since $J(e^r) = \cosh r - 1$ and cosh is convex with derivative sinh, its graph lies above each of its tangent lines: $\cosh r \geq \cosh s + \sinh s\,(r - s)$ for all real r and s. Take $s = \operatorname{arsinh}(x/2)$, so that $\sinh s = x/2$ and $\cosh s = \sqrt{1 + x^2/4} = \frac{1}{2}\sqrt{x^2 + 4}$. Then

$$\begin{aligned} xr - 2J(e^r) &= xr - 2\cosh r + 2 \\ &\leq xr - 2\cosh s - x(r - s) + 2 \\ &= xs - \sqrt{x^2 + 4} + 2 = d(x), \end{aligned}$$

and every step is an equality at $r = s$. □

The relation is exact. Taking the edge flux as the variable conjugate to a bond's logarithmic ratio would be a physical identification not made here.

Now restrict H_4 to the material subspace, the image of ι, where each species is present once. There the clock term is $(K_L + K_R)\otimes I + I \otimes (K_L + K_R)$. Its material electric matrix is

$$\Phi_{xy} = \delta_{xy} \begin{cases} 0, & \text{the two species share a position,} \\ 4\{\operatorname{arsinh}(1/2) - \sqrt{5} + 2\}, & \text{their positions differ.} \end{cases}$$

Thus $M = (K_L + K_R) \otimes I + I \otimes (K_L + K_R) + \Phi$. In this sector $Q_{\mathrm{tot}} = 0$. At equal positions $q_L = 0$, so the three fields are zero. At different positions $q_L = 1$ or -1, so the only nonzero field is the middle edge and its cost is $4d(1)$, precisely the second entry of Φ. Hence V_0 restricts to Φ. The block has no matrix entries coupling it to the other occupation sectors. The frame therefore gives the reducing identity

$$H_4 \iota = \iota M. \tag{B.4}$$

Now keep the original three eight-level registers, with their original quarter-turn energy

$$D_{\mathrm{mem}}|a,b,c\rangle = \frac{\hbar\pi}{4\tau_0}(a+b+c)|a,b,c\rangle, \qquad a,b,c \in \{0,\dots,7\}, \quad \hbar, \tau_0 > 0.$$

The packet bank consists of its original shuttle and four sites. Its enlarged carrier and material frame are

$$\mathcal{H} = V^{\otimes 3} \otimes (V^{\otimes 4})^{\otimes 5}, \qquad \mathcal{I} = I_{V^{\otimes 3}} \otimes \iota^{\otimes 5}.$$

The five packet blocks are consecutive, shuttle first, so $\mathcal{H}$ is a complete twenty-three-register carrier in which the memory is unchanged, each original packet occurs once and every extra occupation direction is retained. Factorization of inner products gives $\mathcal{I}^*\mathcal{I} = I$ and $\|\mathcal{I}\| = 1$.

The identities below are exact for the displayed matrices; taking a sum as one physical energy needs the common unit of Section B.10, now for M, H_4 and the memory's scale. Put

$$H = D_{\mathrm{mem}} \otimes I + I \otimes \sum_{k=0}^{4} (H_4)_k, \qquad H_{\mathrm{old}} = D_{\mathrm{mem}} \otimes I + I \otimes \sum_{k=0}^{4} M_k.$$

Both sums count the shuttle as well as the four sites. Positivity of the summands proves $H \geq 0$. Applying (B.4) in one packet factor at a time, with ι in every other packet factor, gives

$$H\mathcal{I} = \mathcal{I} H_{\mathrm{old}}. \tag{B.5}$$

For site r let P_r exchange its whole four-register packet with the shuttle. The conditional exchange W_r applies P_r when the original middle memory coordinate is 1, and is the identity otherwise. This is a definition on the whole carrier, not only on prepared material states. The memory

projector is $I_8 \otimes |1\rangle\langle 1| \otimes I_8$; its rank on the complete memory is 64. An unconditional contact applies P_r in every memory sector. Write W for either contact and W_{old} for exactly the corresponding whole-packet contact on the original material carrier.

The *contact price* of an exchange W at strength γ on a holding H is the positive part of $WHW - H - \gamma W$, the change the exchange makes to the holding less γW; a contact's full target is the holding plus its price. For a self-adjoint involution commuting with the holding and $\gamma \geq 0$, the proof below shows that the price is $\gamma(I - W)/2$. The definition thus names, for any exchange, the term a contact adds to its holding; for the exchanges used here it is the added term $\frac{\gamma}{2}(I - W)$ of every contact in the preceding sections.

Proposition B.10 (The enlarged contact restricts to the original). *For each of these contacts, $W = W^*$, $W^2 = I$, $HW = WH$, and*

$$W\mathcal{J} = \mathcal{J}W_{\mathrm{old}}.$$

For $\gamma \geq 0$ the full target

$$T_\gamma = H + \frac{\gamma}{2}(I - W)$$

is positive and self-adjoint. It has precisely the original material target as a reducing restriction:

$$T_\gamma \mathcal{J} = \mathcal{J}\left(H_{\mathrm{old}} + \frac{\gamma}{2}(I - W_{\mathrm{old}})\right). \tag{B.6}$$

Proof. Exchanging two equal packet factors twice is the identity. The permutation matrix is consequently its own adjoint. The condition reads the memory and does not change it, so the same statements hold in each memory block. The sum of the five equal packet holdings is unchanged by their exchange; the memory holding is diagonal in the unchanged memory coordinates. Hence $HW = WH$.

In packet coordinates the bank frame has entries $\prod_{k=0}^{4} \iota(y_k, x_k)$. Exchanging the same two factors in y and x only permutes this product. Therefore the rectangular frame commutes with the exchange. In memory blocks with unequal coordinates both sides vanish; in equal blocks the conditional decision is identical on both carriers. This proves $W\mathcal{J} = \mathcal{J}W_{\mathrm{old}}$, including the inactive memory sectors.

The self-adjoint involution W has eigenvalues 1 and -1, so $I - W \geq 0$. Thus $T_\gamma \geq 0$. The contact price is the positive part of $WHW - H - \gamma W$. Since $W^2 = I$ and $HW = WH$, this difference is $-\gamma W$, whose positive part is $\gamma(I - W)/2$. Adding the retained holding gives exactly T_γ. The same computation applies to the original contact. Finally distribute $T_\gamma \mathcal{J}$ and use (B.5) and the exchange identity. This gives (B.6). □

In particular the exchange with site 2 controlled by middle mode 1 has the required material target inside the enlarged carrier. A positive-strength contact requires $\gamma > 0$; its original pulse duration is $\hbar\pi/\gamma$, the half-period of Proposition 7.6 at strength $\gamma/2$.

B.12.1 The material evolution is retained at every time

The embedding preserves the complete evolution generated by the contact target, not only its endpoint. Keep the isometry $\mathcal{J}$ and the two targets just constructed:

$$T_\gamma = H + \frac{\gamma}{2}(I - W), \qquad T_{\gamma,\mathrm{old}} = H_{\mathrm{old}} + \frac{\gamma}{2}(I - W_{\mathrm{old}}).$$

Here W is the unconditional or the memory-controlled whole-packet exchange, with the same choice and site in W_{old}. The construction gave

$$\mathcal{J}^*\mathcal{J} = I, \qquad T_\gamma \mathcal{J} = \mathcal{J} T_{\gamma,\mathrm{old}}.$$

Proposition B.11 (The enlarged evolution restricts to the original). *For $\hbar > 0$, every real t, and each of the specified contacts,*

$$e^{-itT_\gamma/\hbar}\mathcal{J} = \mathcal{J}e^{-itT_{\gamma,\mathrm{old}}/\hbar}, \tag{B.7}$$

and consequently

$$\mathcal{J}^* e^{-itT_\gamma/\hbar}\mathcal{J} = e^{-itT_{\gamma,\mathrm{old}}/\hbar}. \tag{B.8}$$

These identities hold for every real γ. The nonnegative-strength condition $\gamma \geq 0$ is needed for the earlier positivity and contact-price interpretation, and a positive-duration exchange uses $\gamma > 0$.

Proof. Set $\Gamma = -itT_\gamma/\hbar$ and $\Gamma_{\mathrm{old}} = -itT_{\gamma,\mathrm{old}}/\hbar$. Multiplying the target identity by the same scalar gives $\Gamma\mathcal{J} = \mathcal{J}\Gamma_{\mathrm{old}}$. Induction gives $\Gamma^n\mathcal{J} = \mathcal{J}\Gamma_{\mathrm{old}}^n$ for every integer $n \geq 0$. For $n = 0$ this is the identity matrix. For the inductive step,

$$\Gamma^{n+1}\mathcal{J} = \Gamma(\Gamma^n\mathcal{J}) = \Gamma\mathcal{J}\Gamma_{\mathrm{old}}^n = \mathcal{J}\Gamma_{\mathrm{old}}^{n+1}.$$

The exponential series converges in the finite-dimensional operator norm. Left and right multiplication by the fixed matrix $\mathcal{J}$ are continuous, so the power identities can be summed term by term:

$$e^{\Gamma}\mathcal{J} = \sum_{n=0}^{\infty} \frac{\Gamma^n \mathcal{J}}{n!} = \mathcal{J} \sum_{n=0}^{\infty} \frac{\Gamma_{\text{old}}^n}{n!} = \mathcal{J} e^{\Gamma_{\text{old}}}.$$

This proves (B.7). Multiplication on the left by $\mathcal{J}^*$ and use of $\mathcal{J}^*\mathcal{J} = I$ proves (B.8). □

A vector in the material image therefore stays there, and its memory and packet vector follow the original evolution exactly.

B.13 The energy match a reservoir recharge would require

Writing an acceptance spends one quantum of the reservoir, and delivering a packet does not put it back. A transfer from packet to reservoir must respect both spectra. The criterion below states that requirement without changing either carrier or choosing convenient units.

Let M be the complete 1024-level material holding operator of Section B.12. Choose an orthonormal eigenbasis $\{u_p\}$, with $Mu_p = \lambda_p u_p$. The reservoir levels are $r = 0, \dots, 7$, with

$$q_r = \frac{\hbar \pi r}{4\tau_0}, \qquad E_0 = q_1 > 0.$$

For a real coefficient α, retain the full joint carrier and define its holding by

$$H_\alpha = (\alpha M) \otimes I_8 + I_{1024} \otimes \operatorname{diag}(q_0, \dots, q_7).$$

The coefficient α is part of the operator. Choosing it supplies no physical calibration between the complete material operator and the reservoir energy scale.

Choose two material eigenlabels g, e. In the joint spectral basis, let S exchange exactly the two states

$$|e, 0\rangle \longleftrightarrow |g, 1\rangle$$

and fix every other basis state. All 8192 joint levels remain in the construction; the material body is not replaced by a two-level system. Let

$\mathcal{U} = U_N \otimes I_8$, where the columns of U_N are the eigenvectors u_p. The corresponding operator in the original physical basis is $X = \mathcal{U} S \mathcal{U}^\dagger$. The idea is simple: exchanging two basis states leaves a diagonal holding unchanged exactly when the two states have the same energy.

Proposition B.12 (The recharge energy match). *The selected exchange is unitary. It commutes with the full holding exactly when*

$$[H_\alpha, X] = 0 \quad \Longleftrightarrow \quad \alpha(\lambda_e - \lambda_g) = E_0.$$

Its selected forward endpoint is $(e,0) \mapsto (g,1)$*, its reverse endpoint is* $(g,1) \mapsto (e,0)$*, and all other spectral basis states are retained.*

Proof. Write θ for the permutation exchanging $(e,0)$ and $(g,1)$. Its permutation matrix has entries

$$S_{yx} = \begin{cases} 1, & x = \theta(y), \\ 0, & \text{otherwise.} \end{cases}$$

The permutation is its own inverse. Consequently $S^\dagger = S$ and $S^2 = I$, so $S^\dagger S = I$. For a diagonal matrix Δ with entries w_y,

$$(\Delta S)_{yx} = w_y S_{yx}, \qquad (S\Delta)_{yx} = S_{yx} w_x.$$

Every fixed state satisfies these equations with equal diagonal entries. The two exchanged entries agree precisely when $w_{(e,0)} = w_{(g,1)}$. Conversely, that equality makes every entry of ΔS equal to the corresponding entry of $S\Delta$.

The joint holding is diagonal in this basis, with entries $w_{(p,r)} = \alpha\lambda_p + q_r$. Since $q_0 = 0$ and $q_1 = E_0$, equality on the exchanged pair is

$$\alpha\lambda_e = \alpha\lambda_g + E_0,$$

which is the stated criterion. The spectral decomposition gives $\mathcal{U}\Delta\mathcal{U}^\dagger = H_\alpha$. Unitary conjugation preserves both the commutation equation and the identity $S^\dagger S = I$, proving the assertions in the original physical basis. The two endpoints and retention of every other level follow directly from θ. If further coordinates are carried as spectators by the map $(t,x) \mapsto (t, \theta(x))$, their values are retained. This last assertion is a coordinate statement, not a construction of an admitted contact on a larger interacting apparatus. □

Two different operators, two different tests. For the literal coefficient $\alpha = 1$, the criterion is $\lambda_e - \lambda_g = E_0$. For the explicitly different holding with $\alpha = E_0$, positivity of E_0 reduces it to $\lambda_e - \lambda_g = 1$. Neither substitution is a physical common unit. The elementary clock has its own energy-scale identification; extending it to the complete material holding, electric term included, needs its own argument. The criterion is the first exact energy equality this recharge route needs, and any implementation on a common carrier must also preserve the acceptance record and the meal already selected.

B.14 What connects the contacts into one execution

Each construction above is exact on its own carrier. The accepted record already controls every selection in the fixed four-interval policy (Section 7.8.4). Joining that controller to all the delivered packets, paid work, physiological intake and renewed preparation in one available execution remains unfinished. The missing pieces are specific.

The dynamics needs an admitted schedule. The exact question is stated in Section B.11: a finite product of available contacts, each keeping the full holding, that equals the controlled exchange $D(\delta)S_{1,2}$ in total time δ. The release contact also needs an execution that survives interruption; the completed-record order makes finished records truthful without making the sequence atomic.

The energy accounting needs three connections. The packet holdings and the memory's quarter-turn scale need a common unit before their sum is one energy. The reservoir recharge needs a packet pair with $\alpha(\lambda_e - \lambda_g) = E_0$, an available exchange coupling and its execution in elapsed time. And the release contact's work must be funded on the same ledger: affordable from the actual origin's retained capacity, debited from it by exactly the work supplied, with everyone else's capacity unchanged. A positive Hamiltonian establishes neither that debit nor the availability of the work.

The physical identifications complete the account. The account postings and the preparation of the source occupation from them must be realized within one admitted carrier and interaction, then connected to the memory pulse and to paid execution. The delivered packets must be calibrated to Carol's paid record work and to Bob's physiological intake on the care calendar. And the additional undertaking needs its own gov-

erning argument, including how claims that compete for one unit are ordered; Section 13.11 states what that derivation must establish.

Terms and distinctions

Each entry gives the meaning a term carries throughout, with the chapters that introduce and develop it. Where a term rests on a step still to be derived or on an identification adopted as a premise, the entry says so once; Chapter 17 keeps the register of both.

Action. A transformation of the physical relations, described by the strains it applies to bonds. Its actor, the people it affects, the person who authorized it and the one who pays for it can all be different participants. A ledger action fixes only the strains; a recipe, its payment and the authority to execute it are further conditions. Chapters 3, 8 and 13.

Admissible. In the two-person economy of Chapter 15, said of a transition whose command satisfies custody and at each of whose authors the moral decision rule holds; custody and the decision rule together form that economy's admission rule. It is a different notion from *compliant*.

Apparatus. The comparison apparatus of Chapter 2, a model of one physical comparison: a carrier holding both operands, an operation acting on them (the join), a measured residual energy and a reference built from the same runs. What it reports is *native*. One route to J leaves its operation, operand placement and reference open (see *recognition cost*).

Authorship. The participants whose own events make up a proposal, acceptance, release or other authored occurrence. The history retains it; an action's specification and physical effect alone may not recover who authored it. Chapter 8.

Bond. A directed physical relation that brings two positive quantities of the same kind into comparison; its multiplier is their quotient,

first over second, and read from the other end the same bond has multiplier $1/x_b$. Each kind of relation supplies its own pair: two signal amplitudes in the comparison of Chapter 2, a claim and an acknowledgment on the account books of Chapter 8, an actual and a required signal in provision (Chapter 14). An internal bond has one participant at both ends and compares their response under *exposure*. For a bond between two people, see *pair identification*. Chapter 3.

Budget. Under the charged law of Chapter 5 (Table 5.2), the amount that renews a participant's reserve each interval. A retained reserve can fund a temporary charge above that renewal. Under the stated constant-charge law, the renewal bounds the charge sustainable indefinitely from a positive opening reserve. A continuing extension's renewable budget is the same kind of renewal (Chapter 7). Temperance divides a commitment, not a budget (Chapter 10).

Calibration. The fixed correspondence between a physical report and the quantity it represents, including its reference and units. A calibration fixed for an argument cannot change from case to case to obtain a desired answer. Chapters 2 and 17.

Capacity. See *reserve*.

Carrier. The physical system a construction runs on, together with the coordinates that describe its states. The comparison runs on a cycle of eight entries, one complex amplitude for each joint state of three binary coordinates, one for each dimension of the three-dimensional realization of the apparatus that Recognition Science's foundation constructs (Section 2.2); its cost uses only the half-turn, which exchanges opposite entries, so the count leaves no trace in J. The set of labels on which a protocol runs is its participant set. Chapters 2, 7 and 13.

Catalogue. The list of fourteen named virtues, each with a proposed rule for its operation, that the virtue chapters take as their starting list and test. Some rules survive as exact constructions, some carry chosen parameters, and some fail, as the Sacrifice, Wisdom and Hope rules do (Table 11.2). The catalogue also proposed that the

fourteen, as maps on lists of participant states, generate every transformation that keeps the total skew. Each map keeps the length of the list, so no sequence of them reproduces a skew-preserving merge that drops a participant, and the proposal fails (Chapter 11). Fourteen is the size of the list, not a derived number. Chapters 5 and 9–11.

Cell. In the versioned protocol of Chapter 13, the small record each undertaking keeps of its present state, including a *revision* number counting accepted versions of the terms and the last proposal's *nonce*, a fresh number naming that proposal. An acceptance must come from the recipient and match the pending nonce, revision and terms.

Charge. Under the charged law of Chapter 5, the recognition cost of the bonds a participant keeps, debited from their reserve unit for unit every interval the bonds are kept. It is one of the three rules that debit a reserve (Table 5.2), with the *restoration cost* and *maintenance*. Chapters 5, 7, 9, 10 and 17.

Checkpoint, maintained and missed. A checkpoint is the end of a care interval. It is maintained for a person when the repair is paid and some capacity remains: the response is balanced and the reserve strictly positive. Otherwise it is missed, and it stays missed: later care can restore the reserve but cannot make that earlier checkpoint maintained. A history is maintained when every checkpoint in it is maintained for everyone it concerns. Chapters 6 and 7.

Closure of the authored reliance. An action closes the authored reliance when it leaves Bob, the recipient, where both formation interventions say he would have stood had the undertaking never formed. In the care case it is equivalent to Bob's continuation, to a common positive repair margin and to performing the accepted terms. Chapter 7.

Coherence. Defined under *reading*. Coherence, completeness and an available performance do not identify the governing reading: the helper-only reading of Chapters 7 and 13 has all three. Grounding by the installed record does, and so, equivalently, does completeness together with faithfulness to the restoration comparison.

Comparator. The finite device that evaluates accepted terms against a candidate action's result and writes a verdict. Inside the accepted window it passes exactly when Bob's checkpoint is maintained; outside it the terms ask nothing. A comparator implements the consequence reading of its pass condition. Chapter 7.

Comparison multiplier. See *multiplier*.

Comparison port. The pulse that holds a comparison's two operand amplitudes u and w on a cycle of at least four entries. Its energy is $u^2 + w^2 = 2uw(J(u/w) + 1)$, so recognition cost is the energy above $2uw$ in units of $2uw$. Chapter 3.

Comparison profile. The log value $p_s(b)$ of each bond's multiplier in economic state s. A natural profile compares an actual signal with a required one, $p_s(b) = \log(a_s(b)/q_s(b))$; which profile holds is for the *pair identification* to supply. The economic transition and the ethical action have exactly the same effect for whatever profile is supplied. Chapter 14.

Compliant. An action is compliant when it is physically legal and has every property each installed reading contains: it does everything the installed agreement requires, and avoiding what it forbids is not enough. A property in every installed reading is obligatory. Chapter 15's notion for its economy is *admissible*. Chapter 8.

Consent. An authored event with definite content, participants and scope. A favorable physical effect does not establish consent, and an observer's description cannot replace the author's event. Its authorship is derived; whether an acceptance was also informed and uncoerced is still to be derived. Chapter 8.

Conserving repair. Repair of two bonds that keeps the product of their multipliers fixed, which is to keep their combined logarithmic flow; that flow is a different quantity from either participant's skew. Its unique cheapest endpoint is Love's. Chapter 5.

Contact. An interaction that couples parts of a carrier for a set time, on top of their holding evolution: the probe contact that registers Bob's acceptance, a writer's exchange, the exchanges that move a packet. Appendix B.

Contact price. For an exchange W at strength γ acting on a holding H, the positive part of $WHW - H - \gamma W$: the change the exchange makes to the holding, less γW. A contact's full target is the holding plus its price. Appendix B.

Contextual equivalence. See *live resource tuple*.

Continuation. A future policy with a renewable budget under which a person's reserve stays strictly positive at every *stage*, the present included (Chapter 7's continuing extension); a person who has one can continue. A reserve admits a continuation exactly when it is positive, since zero already fails the present checkpoint. The boundary concerns whether some funded future exists, not the person's actual future or whether the reserve alone can pay the next repair: from $1/8$ Bob cannot pay the next $J(2) = 1/4$ by himself, yet he has a continuation; from zero he has none. Chapters 1, 6 and 7.

Control. The physical capacity to select or execute an action through a given controller. Preserving an undertaking through a change of controller includes preserving whose events can propose, accept or release it. Chapters 7 and 13.

Controller. The physical system whose configuration selects a participant's next action. In the care case it is a register whose action coordinate Alice's first policy reads: before the accepted write it selects idle, after it, feeding. With the prepared service clock and reservoirs, the integrated writer selects every action of the fixed four-interval schedule (Section 7.8.4). The installed register settles what the installed controller selects, not what Alice does: she can still run another controller, and omission is running one that selects idle. The model assumes she can and does not represent what in her does so. The governing reading still rejects the omission such a controller chooses (Chapter 13), and responsibility attaches to whoever runs it (Chapter 8). Chapters 7, 13 and 16.

Cost. Unqualified, recognition cost. Whole-ledger cost $C(L)$, a participant's incident cost $C_i(L)$ and an allocated payment answer different questions, and a reduction in any of them does not by itself give the energy an actuator paid. Chapters 3 and 5.

Custody. In the economy of Chapter 15, the convention that a transfer is authorized exactly when its actor is the donor. It is adopted for that economy; which entitlements hold belongs to the open governing derivation of Chapter 13.

Cutoff. The point up to which a history is read when a state or a judgment is evaluated. Chapter 13.

Exact representation. A smaller report of an economy whose every actual step projects to a reported step and whose every reported step lifts to an actual step from the actual source state, and which keeps authorship, each participant's observations and requirements, and which transitions are admissible. Plans, joint requirements and safety then agree between report and economy. Chapter 15.

Exposure. In the care chapters, the load that getting through an interval puts on a person's own bond: each interval's exposure doubles its multiplier, and *maintenance* repairs it. Which two physical quantities of the person the bond compares belongs to the derivation of the care environment, still to be made. Chapters 3 and 6.

Faithful and sound reading. A reading is faithful to the restoration comparison when its performances are exactly the legal actions that restore the reference fixed by the formation interventions, and sound when every performance restores it. Soundness is one direction of faithfulness; with completeness, faithfulness determines the whole reading. Chapter 13.

Feasibility. Availability under the resources, actions, access, timing and history of the case at hand. A feasible omission can still violate a governing requirement, and when no action meets every claim, that physical fact does not by itself say which claim has priority. Chapters 6, 7 and 13.

Formation. The proposal and acceptance that put an undertaking's terms in place. A formation intervention prevents one of them before the recipient acts; in the care case both interventions agree on where Bob would have stood, which fixes the reference he is owed. Chapters 7 and 13.

Golden ratio. $\phi = (1 + \sqrt{5})/2$, the positive root of $\rho^2 = \rho + 1$. It is forced as the join's growth factor: when sizes add, comparisons survive the update and the first new level is a costly join, each level's size is ϕ times the one before. It does no work in the value of J: any operation that keeps the comparison's sum part and multiplies its mismatch part by a fixed factor other than one gives the same report, since the factor cancels against the reference. Golden scaling returns once among the moral results, conditionally: Temperance's cap is $1/\phi$ of a commitment that is divided self-similarly. Every other constant built from ϕ in the virtue rules, Temperance's reserve limit included, is a parameter until a law of the reserve or the response fixes it. Chapters 2, 5 and 9–11.

Governing requirement. A requirement fixed by the physical relation under examination. The governing criterion asks three things of it: some grounded coherent reading has a legal performance, every grounded coherent reading contains it, and some legal action fails it. Since a legal action can fail it, to govern an action does not mean to control it. In the care case it runs from proposal and acceptance through the retained terms and the comparator to the installed first selection and, with the prepared clock and reservoirs, the complete four-interval schedule. That the governing requirement is what Alice owes rests on an adopted identification: the requirements of an installed agreement are the ends of the reading its apparatus implements. Chapters 1, 7 and 13.

Half-share. On the account books of Chapter 8, half a participant's incident cost: each relation's cost is split equally between its two ends, a convention under which the half-shares add up to the total cost. The factor two changes no sign.

Harm. The signed change in a participant's incident cost under an action: a positive change adds to their burden, a negative change is relief. It measures burden carried in relations. A fall in reserve, an ended continuation and a broken undertaking are separate damages with their own quantities: abandoning Bob in Table 1.1 changes no incident cost at that checkpoint and exhausts his reserve. One interval later the omission reaches the ledger, as a quarter-unit of bond cost

that feeding would have left at zero (Chapter 7). Chapters 3, 4, 5, 14 and 15.

History. The ordered occurrences a description retains. Equal endpoints can come from different authorship, releases or earlier missed care, so which distinctions a history must keep depends on the judgment it has to support. Chapters 8 and 17.

Holding. The evolution each part of a carrier has when no contact acts, and the energy it keeps there. The exact write conserves the summed holding energies of its memory and reservoir. Chapter 7 and Appendix B.

Identification of a device with the people. A premise that ties a device's states to particular people: that the converter's two comparisons are Alice's and Bob's internal bonds (Chapter 9, used again by Courage in Chapter 10), that a converter's comparison is an internal bond of its owner (Chapter 14), that each participant's preparation contact acts only in that participant's own origin sector, with the address equal to the retained claim (Chapter 13), or that the care register stands for Alice's means of action and Bob's act of accepting prepares the occupation the probe reads (Chapter 7). Each is adopted where it is used, and none is an *installation*.

Incident cost. A participant's incident cost $C_i(L)$ is the sum of J over the bonds touching them: the burden they carry. A bond between two participants counts in both their incident costs while remaining one bond in the whole. Burden lies in relations, so a giver's can fall: in the provision example of Chapter 14, the meal that balances Alice's provision bond to Bob lowers her incident cost with his. Chapter 3.

Installed configuration and grounded reading. A controller configuration is installed when the completed accepted write has reached it, and a reading is grounded when an installed controller implements it: the rule the accepted agreement writes into the means of action. Installation fixes the governing reading before Alice acts, so it is not chosen after the outcome. Under the grounded reading, feeding Bob performs the undertaking, and staying away, still physically legal, violates it. At the contact used, with probe and target at the same position, the acceptance probe detects Bob's acceptance on a

branch of weight one half, and only there is the selection installed. On the other branch nothing in the construction governs Alice's action, and who answers for the terms there is open. Chapters 7 and 13.

Join and level. A level is a stage of a record; the join forms each new level from the two before it, and each level has a positive size. The simplest join holds two whole counts and adds them, $(p, q) \mapsto (p + q, p)$: from $(0, 1)$ it runs through $(1, 0)$, $(1, 1)$, $(2, 1)$, $(3, 2)$, and so on, with size $\lambda p + q$ for a fixed $\lambda > 0$. Its growth factor is the *golden ratio*. Chapter 2.

Justice. Making relations carry exactly the correction that is owed, to the right people and no one else. The record fixes the content: once the accounts are fixed, the correction that makes a claim truthful to the recorded events is unique (Chapter 8). Conservation fixes the execution: on two bonds running between two people in opposite directions, a correction that keeps the pair's total flow and moves one person's skew by δ strains the bonds by $\delta/2$ and $-\delta/2$, moves the other's skew by $-\delta$ and leaves everyone else unchanged (Chapter 5). Connecting the two is still to be derived.

Ledger. The set of active bonds with their multipliers: the shared physical record of relations. A conventional account book represents it only when the economic representation is connected to the same physical transformations. Chapters 3 and 14.

Level. See *join and level*.

Live resource tuple. For a stock $s = (r, f, h, c, e, k)$, the quantities that enable operations, $\ell(s) = (r, f, h, e)$; consumed material and spent effort enable none. Two stocks are contextually equivalent, executing the same operation lists after every counted supply, exactly when their live tuples agree. Chapter 14.

Love. Conserving repair of a relation. Keeping the sum of logarithmic flows keeps the product of two multipliers; among those endpoints, their geometric mean is the unique minimum-cost endpoint and treats both alike, so $(1, 4)$ becomes $(2, 2)$. When the two bonds run

between the same two people in opposite directions and their multipliers differ, it relieves both and leaves everyone else unchanged. On two people's own bonds the same move is Sacrifice. Chapter 5.

Maintenance. In the care chapters, the *restoration cost* of each interval's repair, paid by the bond's owner (Table 5.2): *exposure* doubles a person's own bond, and from balance its repair costs $J(2) = 1/4$. The integer reserves of Chapter 15 have a maintenance of their own, two units an interval. Chapters 5, 6 and 7.

Moral decision rule. The rule evaluated at each declared author of an action in the economy of Chapter 15, adopted for that economy together with custody. With zero harm allowance it admits the action when the absolute bond sum $\sum_b |\log x_b|$ over active bonds does not increase and no listed participant in the actor's scope, the actor and those sharing an incident bond with the actor, has an increase in incident cost. It therefore rejects any action that raises the actor's own burden, Alice's Sacrifice among them.

Multiplier. The positive number x_b a bond carries, the quotient of the two quantities it compares; also called its comparison multiplier. $x_b = 1$ is balance, and its log value $\log x_b$ is the signed departure. The same ratio can occur at different physical energy scales. Chapters 2 and 3.

Mutual information and the half-order measure. Two calibrated measures of how much a joint record of agent and environment says about the environment: mutual information and $R = -2 \log \beta$, where the affinity β is the overlap of the record's square-root amplitudes with those of its product-marginal reference. Both meet the composition, refinement, mixture and calibration conditions, and they differ on a noisy binary record. When each cell's energy is its probability, comparing a record with a supplied reference reports R; assembling calibrated branch reporters under a retained flag reports the branch average, which mutual information equals on the flagged record and R does not. Whether a value law's information term must keep that average is open. Chapter 12 and Appendix A.

Native. Belonging to the physical comparison or the recognition ledger itself, as opposed to a description of it. What the apparatus reports

for two whole counts is native; a cost function written down to summarize such reports is a description, and native reporting asks it to say what the apparatus says wherever the apparatus speaks. Chapter 2.

Nonce. See *cell*.

Packet. A finite material system, kept under one fixed holding evolution, that stands for a unit of resource, such as Alice's unit of food, in the routing constructions. Timed exchanges move packets between counted sites, each packet counted once. No construction converts a packet into food. Chapter 13 and Appendix B.

Pair identification. The open identification on which every harm a ledger assigns to a person rests, as does every agreement's cost: for a bond between two people, which physical quantities form its pair, the actual condition of one compared with what the other's undertaking or need requires, and whose relation carries the comparison. In provision, and in Chapter 14's comparison profile, the pair is an actual signal and a required one. The chapters compute exactly what follows once the pair is supplied. Section 3.2.

Participant and subject. A participant is one who can stand at an end of a bond: Alice or Bob, and in the moral chapters, a person. Participants are told apart by how their records respond to actions, and can be affected without authoring anything. Being a subject concerns undergoing effects. Under the dual-aspect premise of Chapter 4, every occurrence of recognition has an experiential inside; which organizations of recognition make up one subject's experience, and how subjects compose into a whole, are still to be derived. Chapters 3, 4 and 16.

Physical witness. A two-bond state built to carry a prescribed skew τ: a bond at multiplier e^{τ} and a return bond at 1. It realizes the number; it does not repair any existing relation. Chapters 9 and 11.

Physically legal. An action is physically legal when it passes the operation check (the technology admits it at that time, and its material balances), the resource checks (food in stock; each person's labor and reserve can absorb the work asked) and the history checks

(work covered by an accepted *work grant*; only food units the record holds; each actual delivery credited once, toward an active undertaking with quantity still due). Authorization enters legality only as a matter of record. In the care case both feeding and staying away are legal, so the governing result concerns a requirement that can be broken. Chapter 7.

Reading and performance. A reading is a set of properties required of an action, its ends. It is coherent when it contains the property that always holds and not the one that never holds, and is closed under conjunction and under consequence over the legal actions. Its performances are the physically legal actions with every property in it, and it is complete when it contains every property its performances share. The consequence reading of a condition collects everything that every legal action satisfying the condition also satisfies. Chapter 7.

Recognition. Distinguishing, comparing and retaining occurrences: telling things apart and keeping what a later comparison needs. Comparing two occurrences requires information about both. Chapter 2.

Recognition cost. The cost of a bond's departure from balance, $J(x) = \frac{x+x^{-1}}{2} - 1$: zero at $x = 1$, the same for x and $1/x$, and unbounded in both directions. It is forced three ways, each a uniqueness theorem from its own requirements. One of them leaves the apparatus's operation, operand placement and reference open. Take any linear comparison of two signals on the eight entries of the *carrier*, measured in squared-norm energy by the meter of Chapter 2 and reported against a reference built from its runs that is positive when both operands are present and zero when the second is absent. If the report is zero at balance, the same for x and $1/x$ and never smaller for a larger imbalance, it is a fixed multiple of J, and J itself at the quarter-phase reference. Of the other two, one fixes the apparatus and requires a continuous report, and one replaces the apparatus with a law for combining chained comparisons, calibrated near balance. Chapters 2 and 3.

Recognition dynamics. A reversible dynamics of recognition events, each keeping a record of the current that flowed. Appendix B adopts

one step of its basic cell, two clocked channels connected by edges that carry an integer flux, as the model of the acceptance probe of Chapter 7.

Recognition Science. The physics that starts from what recognition does, derives the cost of imbalance from how a comparison is physically reported, and follows people and their relations, capacities, resources and histories through conservation and transformation. Ethics on this account concerns the same bonds, cost, skew and transformations. Chapters 1 and 2.

Recording ledger and service comparison. Two constructions beside the care bonds when one unit of food meets two claims. The recording ledger registers on two bonds read from account books, Alice to Bob and Alice to Carol, whether a waiver was made. The service comparison gives each claim a bond at multiplier 1 if it is served and, by convention, 2 if not, so the quarter an unserved term costs, $J(2) = 1/4$, rests on that convention. No number passes between them or into the care bonds. Appendix B.

Release. An authorized event that changes what remains due or active. It leaves the earlier delivery, acceptance or reliance in the history, so it can clear a present claim without undoing a dependence already created. Chapters 8 and 9.

Reserve. The capacity a participant can spend on repair and work: what someone still has to give or to live on, counted in units of recognition cost. It can be spent, renewed or exhausted, and it is a different quantity from present recognition cost. Its debits, the *charge*, the *restoration cost* and *maintenance* (Table 5.2), take recognition cost from it unit for unit, each an adopted law at this general scope. Section 3.5.1 derives one restricted restoration debit from the accessible energy an executed export removes; a *budget* renews it. In the care chapters a unit of intake adds 1/2 to it and a unit of work costs 1/4. The material reserve of Chapter 13 (Table 13.3), the signed reserve and holding energy of Appendix B (Table B.2) and the integer reserves of Chapter 15 are separate accounts with their own numbers, and no conversion between them and the reserve is derived. Chapters 3, 5, 6, 7 and 13.

Response. In the care chapters, a person's pair (m, e): the multiplier m of their internal bond, which records how they respond to exposure, and their reserve e. The multiplier is balanced at $m = 1$, doubled by exposure each interval in the environment those chapters take, and returned to one by repair paid from the reserve; a response is balanced when its multiplier is. Closure of the authored reliance (Chapter 7) compares the whole pair: feeding leaves Bob at $(1, 1/2)$, omission at $(1, 0)$. Chapters 6, 7 and 13.

Restoration cost. The cost of the bonds a restoration returns to balance, each counted once and debited once from one designated payer by an adopted law of its own (Table 5.2): restoring a bond between Alice and Bob from multiplier 2 relieves each of them by $1/4$ and costs $1/4$. Levied each care interval with the bond's owner as payer, it is *maintenance*. Chapter 5.

Revision. See *cell*.

Sacrifice. Love's equalizing move on two people's own bonds. Alice carries only her bond, so equalizing moves imbalance from Bob's bond onto hers: from $(1, r^2)$ to (r, r) with $r > 1$, her burden rises by $J(r)$ while his falls by $J(r)(2J(r) + 3)$, and the total falls. The burden she takes on is the sacrifice. A lower total can coexist with an exhausted helper. Chapters 5 and 9.

Skew. A participant's signed net imbalance $\sigma_i(L)$, outgoing minus incoming logarithmic flow on the actual ledger. The skews of all participants sum to zero, so an action can only move skew between them. Net skew, the sum of absolute bond departures and recognition cost keep different information; correcting skew changes the relations themselves, and treating it as correcting a belief needs the physical register that holds that belief. Chapters 3 and 11.

Stage. The opening of a continuing extension, stage zero, or one of the intervals after it (Chapter 7). A level is a stage of a record (Chapter 2).

Strain. The change an action makes to a bond's log ratio: with strain v_b, the multiplier becomes $x_b' = x_b e^{v_b}$. Successive strains add, and a strain followed by its negative restores the bond. Chapter 3.

Undertaking. A relation formed by a proposal and its recipient's acceptance, retaining the participants, the accepted terms, the interval in which they operate and the events that can change or release it. Chapters 7 and 8.

Virtue. A virtue is named for a kind of physical work. For each, the chapters construct an operation that does the work and prove what it does under its conditions; Table 11.2 records, virtue by virtue, what kind of result that is, and some operations do only part of that work so far. The *catalogue* has fourteen, each named for its work: Love and Justice (Chapter 5), each with its own entry; Compassion, supplying what another lacks without consuming the provider; Sacrifice, taking part of another's burden onto oneself; Forgiveness, releasing a claim without erasing the event that created it; Gratitude, carrying a received benefit into later conduct (Chapter 9); Wisdom, choosing the available action with the best consequences; Prudence, counting the imbalance a net figure hides; Temperance, limiting what one step may take from a reserve; Courage, carrying a justified action through when it exposes the actor (Chapter 10); Patience, letting needed information arrive while those waiting stay within a feasible continuation; Humility, correcting one's own record toward evidence; Hope, keeping action directed at a future not yet present and testing it for a funded path; Creativity, adding alternatives when every listed one is inadequate (Chapter 11). That a person who has a virtue is one whose retained records reliably select its operation when its conditions hold is a step shared by all fourteen and still to be derived.

Whole and member. Two descriptions of the same shared reality. The whole must keep the member distinctions a judgment uses, and the member accounts must keep the shared constraints and count each resource once. Chapters 4, 14, 15 and 18.

Window. The active span of an undertaking's terms, such as Bob's care window over checkpoints 1 to 4 or a work grant's window; outside it the terms ask nothing. Chapter 7.

Work grant. An accepted authorization for a named worker to work within a stated window, up to a stated total; work beyond it is unauthorized however much food is in stock. Chapters 6, 7 and 10.

Sources for the formal results

The formal sources are in the *IndisputableMonolith* Lean library. Its declarations specify the objects, hypotheses and conclusions used here. The proofs in the book explain the argument; the source dossier accompanying the manuscript records the inspected source versions and retained verification receipts. Those receipts concern their named targets and pinned dependencies. They do not assert a fresh audit of the entire current library.

A declaration that begins from a prepared carrier, an admitted contact or a calibrated response retains that premise in its conclusion. A named open specification records the next connection to derive. These distinctions concern the same physical relations and transformations throughout the book.

Recognition and comparison. The library's front door is `Verdict.lean`. The accepted foundation and its verification scope are recorded in the certificate `Certificates/RS-RUNG-1-v1/CERTIFICATE.md`. Native reporting and the canonical reciprocal cost are developed in `VerdictMinimalNativeReporting.lean`. The cost's independent published characterization is cited in the bibliography. Chapter 2 states the conditions used by each route.

The physical moral state. The core objects are `Ethics/MoralState.lean`, `Ethics/Harm.lean` and the bond ledger on which they operate. An agent's incident cost, signed skew and reserve are distinct quantities. The care projection, including an exhausted participant with zero reserve, is in `Ethics/Decision/CareMoralStateRealization.lean`; its positive face recovers the positive-reserve state. Chapter 4 keeps the participant when a capacity is exhausted.

Love, Justice and Sacrifice. The native virtue constructions are under `Ethics/Virtues/`. `SharedSacrifice.lean` develops the same-ledger burden and relief result. `Ethics/Morality/FiniteCapacityLove.lean` contains `feasible_iff`, `common_level_exists`, `clipped_unique_minimum` and `continuing_minimum_exists`, corresponding to Section 5.5. Its separate positive reserves and budgets, fixed total log flow and constant charged continuation law are essential. The physical converter in Section 9.6 is formalized in `Ethics/Morality/SharedSacrificeNativeContact.lean`, notably `exact_native_action` and `actual_work_debit`. The catalogue's list-generation counterexample is in `Ethics/Virtues/CompletenessResolution.lean`; it does not refute every participant-preserving generation claim.

A physical restoration debit. Section 3.5.1 follows `Ethics/Morality/NativeExportMaintenance.lean`. Its `executed_debit` reads the actual accessible-energy change; `physical_afterSmooth` also requires a positive remaining reserve. The prepared donor bank, retained baseline, positive calibration and export-access restriction belong to this construction. Other controls can have a different reserve account.

Care through time. `Ethics/Decision/UndertakenCare.lean`, `CareCommitmentHorizon.lean` and `FiniteCapacityCareHistories.lean` supply the care and finite-horizon results. The last module's `main_existence` is an existence criterion for a schedule, while the chronological theorem checks every prefix of a specified schedule. `CareAfterViolation.lean`, `CareReleaseTiming.lean`, `CareForeclosure.lean` and `CareInformedPerformance.lean` retain missed checkpoints, event order, future feasibility and the information needed by a policy.

Authored records and governing requirements. The authored-book constructions distinguish effects, authorship, evidence and release. In `Ethics/Morality/`, `AuthoredCareBinding.lean` and `AuthoredCareRestorationInstallation.lean` establish the installed original care case. `Ethics/Decision/CareOperationalClosureSemantics.lean` identifies its faithful operational reading. The general formation and authority questions remain explicit in the restoration specifications. `Ethics/Decision/CareAcceptedRecordFullPolicyActuation.lean`, especially `accepted_physical_policy_controls_full_undertaking` and

`accepted_record_full_policy_certificate`, supplies the prepared writer for all four scheduled selections described in Section 7.8.4.

Roles, new participants and scarce claims. Under `Ethics/Monograph/`, `UndertakingRoleRecovery.lean` contains the bounded three-probe role-recovery result. `UndertakingParticipantTransport.lean` characterizes the role separation needed to preserve every finite protocol response and carries permission, satisfaction and remembered failure through that transport. Under `Ethics/Morality/`, the scarce-claim modules retain both claims and the single actual unit. `ScarceClaimAuthenticatedReleaseContact.lean` connects the prepared five-contact execution to the undertaking cell; its participant-origin installation and paid-contact identification remain named open joins. `ScarceClaimCompletionRecordContact.lean` supplies the completion-first recording order without asserting interruption tolerance.

Information and reference preparation. Appendix A develops distinct questions: what an apparatus reports, which information axioms a quantity satisfies, and what a value law must retain. In `Ethics/Monograph/`, `ContinuousRenyiCountermodel.lean` and `FinitePolynomialReporting.lean` give the continuous half-order countermodel and its uniform finite-block expected reports. The latter does not assert one-trial accuracy. `NativeCorrelatedReferencePreparation.lean` constructs the stated reference from two supplied coherent copies with two fixed stages. `NativeFlaggedRecordReporter.lean` proves that the selected detector-bank reading does not identify information across the examples used here.

Material contacts and recharge. The Monograph modules `CareAcceptanceControlledNativeResourceDelivery.lean` and `CareStagedAcceptanceNativeResourceHistory.lean` retain the full material packets, original meal and newly written control. The Morality modules `CareEnlargedNativeContactRealization.lean` and `CareEnlargedNativeContactEvolution.lean` give the enlarged-carrier restriction and every-time evolution. `Ethics/Monograph/CareNativePacketReservoirResonance.lean` separates unconditional exchange unitarity from the exact energy-matching condition for commutation with the full holding. Section B.14 states the

remaining admitted-schedule, payment, calibration and renewal connections.

Economic effects on the same ledger. `Economics/Foundation/EffectRealization.lean` identifies represented effects with native ledger actions for its supplied physical profile. `NativeLedgerRealization.lean` retains internal bonds as well as relations between distinct participants. `NativeResponseRealization.lean`, notably `readings_activeEquivalent` and `realization_of_readings`, reconstructs the active ledger from the calibrated two-step harm and signed-flow readings of Section 14.3.1. Probe access, calibration and the application-specific identification of needs remain explicit.

The source dossier records immutable Git versions for constructions whose original working directories are no longer present, including the two-copy reference and the release, completion and enlarged-contact results. It also distinguishes direct source reading, declaration-level coverage and retained remote build evidence. This makes the formal account inspectable without making a file's location stand in for its proof.

References

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[2] Claude E. Shannon. *A Mathematical Theory of Communication*. Bell System Technical Journal 27, 379–423 and 623–656, 1948. Complete paper, reprinted with corrections.

[3] Alfréd Rényi. *On Measures of Entropy and Information*. Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, volume I, 547–561. University of California Press, Berkeley, 1961. Complete paper.