

Static Exteriors and the Black-Hole Problem on the Recognition Lattice

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Abstract

A static vacuum metric does not automatically give a vacuum of the recognition lattice: fifteen squared-edge classes carry ten metric components and five additional combinations. We give the exact uniform tidal correction in the timelike-hinge regime and an explicit obstruction at the next order in amplitude. At the flat identification $\kappa = 1$, a static cubic harmonic field with its internal correction solves every linearized equation on $\mathbb{Z}^4$, but its second-order vertex Ward defect is $756\sqrt{3}$. No quadratic correction to that fixed tangent solves the thirty equations incident to the selected record. This result also holds for the Lorentzian action extended to spacelike hinges near that flat field. It does not exclude every static exterior. We distinguish this obstruction from the unresolved general radial class-Jacobian formula, describe the corresponding determinant polynomial, and state the further conditions required for a lattice black hole. Horizon geometry, entropy normalization and radiation on prescribed backgrounds remain distinct from a solution of those conditions.

1 The strong-field question

The physical question is whether the recognition lattice supports an isolated compact body, and ultimately a black hole, through its own gravitational equations. A continuum line element supplies a useful first description. The equations, however, act on squared edges. The extra edge combinations can carry a curvature source even when the continuum Einstein tensor vanishes. One must cancel that source and then satisfy the compatibility conditions of the full equations.

We study the Lorentzian Kuhn realization of the recognition lattice described in [1]. The choice $\kappa = 1$ below is a physical identification of the flat reading; it is not derived by the calculations here. Our principal exact result concerns one fixed static tangent at this reading. It establishes a failure of the second-order vertex identity for the discrete action. It leaves open the existence of solutions reached by a different tangent, a different action, matter support, or a construction outside the specified expansion.

We use the following status distinctions. A theorem has a mathematical proof; independent verification is complete for the explicit cubic obstruction, the uniform tidal identities and the

radial results proved here. A numerical observation supports a statement only at its evaluated inputs. An identification fixes a physical reading, a model supplies an object for investigation, and an open target states an obligation still to be proved. These distinctions matter especially when a local calculation is used to motivate an exterior construction.

2 Edges, action and the five additional combinations

2.1 The carrier and flat reading

Records are $x \in \mathbb{Z}^4$. A nonempty subset $S \subseteq \{0,1,2,3\}$ defines a posting displacement $v_S = \sum_{a\in S} e_a$ and a squared edge $\ell(x,S)$ from x to $x + v_S$. Each unit four-cube is subdivided into the 24 simplices

$$x, \quad x + e_{\sigma(0)}, \quad x + e_{\sigma(0)} + e_{\sigma(1)}, \quad \dots, \quad x + \mathbf{1},$$

where σ runs over permutations of the accounts. Write $m = |S|$. The constant reading is

$$\eta_\kappa(v,w) = v \cdot w - \kappa(\mathbf{1}\cdot v)(\mathbf{1}\cdot w), \qquad \ell_\kappa(x,S) = m(1-\kappa m). \tag{1}$$

At $\kappa = 1$, the diagonal translation $d = \mathbf{1}$ has squared norm -12. The orthonormal spatial coordinates are

$$\xi_1 = \frac{x_0 + x_1 - x_2 - x_3}{2}, \quad \xi_2 = \frac{x_0 - x_1 + x_2 - x_3}{2}, \quad \xi_3 = \frac{x_0 - x_1 - x_2 + x_3}{2}. \tag{2}$$

A static field obeys $\ell(x+d,S) = \ell(x,S)$. This is an exact lattice symmetry; continuous spherical symmetry is not a symmetry of the carrier. Account permutations give its tetrahedral spatial symmetry.

For a triangle with doubled Gram determinant $H < 0$, use area $A = \sqrt{-H}/4$. In the timelike-hinge regime the action is

$$S(\ell) = \sum_h A_h(\ell)\,\epsilon_h(\ell), \qquad \epsilon_h = 2\pi - \sum_{\Delta \supset h} \theta_{\Delta h}. \tag{3}$$

This is the Regge area-deficit action [3]. The overall action scale is omitted: multiplying it by a nonzero constant changes no vacuum equation or obstruction. On the infinite lattice, a row $R_{x,S}$ is the derivative of the local action containing the complete stars that depend on the edge (x,S). This finite definition avoids assigning a value to an infinite total action. All differentiations below take place in a neighborhood where the relevant simplex, facet and hinge Gram determinants are nonzero, with the stated Lorentzian signs.

2.2 Why a sampled vacuum metric needs a correction

A symmetric 4×4 matrix G gives fifteen edge values $v_S^T G v_S$. These depend on ten metric components. The remaining five combinations are captured by the subset transform

$$\mu_U(c) = \sum_{\emptyset \neq S \subseteq U} (-1)^{|U|-|S|} c(S), \qquad |U| = 3 \text{ or } 4. \tag{4}$$

Every metric class function is annihilated by these five rows. Denote the resulting 5×15 matrix by Π. For a varying metric, this decomposition does not remove the extra equations. It identifies the edge combinations that must be solved along with the metric equations.

Here is the exact curvature-order calculation. Evaluate potentials at the spatial coordinates of the edge midpoint $x + v_S/2$. Define the sampled static metric tangent

$$h_\kappa[\Phi](x, S) = -2\Phi\left(m + (\kappa - \tfrac{1}{2})m^2\right).$$

For a uniform traceless tidal potential write

$$\Phi = E_{12}\xi_1\xi_2 + E_{13}\xi_1\xi_3 + E_{23}\xi_2\xi_3 + e_1(\xi_1^2 - \xi_2^2) + e_2(\xi_2^2 - \xi_3^2).$$

The three class vectors p_{ij} are fixed by the following table. Subsets are ordered by the binary number $\sum_{a\in S} 2^a$; the entries displayed are $16p_{ij}/15$.

S	0	1	01	2	02	12	012	3	03	13	013	23	023	123	0123
$16p_{12}/15$	−1	1	0	1	0	−2	1	−1	2	0	−1	0	−1	1	0
$16p_{13}/15$	−1	1	0	−1	2	0	−1	1	0	−2	1	0	−1	1	0
$16p_{23}/15$	−1	−1	2	1	0	0	−1	1	0	0	−1	−2	1	1	0

Here 0 means the singleton $\{0\}$, and similarly for the other entries.

Theorem 2.1 (Uniform tidal cancellation)**.** *For $\kappa > 1/2$, let H_κ be the flat linearized row operator. At the fifteen edges based at the origin, the diagonal tidal potentials have zero source, while each off-diagonal potential has source $s_\kappa p_{ij}$, where*

$$s_\kappa = \frac{2(4\kappa - 1)(6\kappa - 1)}{15\sqrt{4\kappa - 1}}, \qquad H_\kappa p_{ij} = \lambda_\kappa p_{ij}, \qquad \lambda_\kappa = -\frac{2(2\kappa - 1)}{\sqrt{4\kappa - 1}}.$$

Consequently $h_\kappa[\Phi] + c_\kappa \sum_{i<j} E_{ij}p_{ij}$ has zero source in all fifteen rows there, with

$$c_\kappa = \frac{(4\kappa - 1)(6\kappa - 1)}{15(2\kappa - 1)}. \tag{5}$$

Proof. Differentiate the simplex action twice and assemble the cells containing each edge. The flat ten-edge vector is $a_{ij} = (j - i)[1 - \kappa(j - i)]$; the Gram recipe in Appendix A applies with these inputs. With exterior angles the assembled Hessian is the sum of $D\vartheta_t(X)\, DA_t(Y)$ over all simplex hinges. Substituting the five quadratic potentials gives zero for the two diagonal sources and $s_\kappa p_{ij}$ for the three mixed sources. The same assembled constant-class matrix maps each p_{ij} to $\lambda_\kappa p_{ij}$. These finite identities use only the Gram minors of the 24 simplex types; their common square-root factor is $\sqrt{4\kappa - 1}$. Linearity and $s_\kappa + c_\kappa\lambda_\kappa = 0$ give the result. □

The restriction is substantive. The correction diverges as $\kappa \downarrow 1/2$, where the pair edges become null. The theorem gives uniform tidal cancellation at one record. Gradients of a varying tidal tensor and higher orders in amplitude require further equations; an exact static exterior does not follow from (5).

3 An exact obstruction at second order

3.1 The vertex pairing

At ℓ_1, displacing one record y by $w \in \mathbb{R}^4$ induces a squared-edge tangent $g_{y,w}$: on outgoing edges it is $-2\eta_1(v_S, w)$, and on incoming edges it is $+2\eta_1(v_S, w)$. Thus only thirty edges are involved. For row values r define

$$\mathscr{W}_{y,w}(r) = \sum_{S \neq \emptyset} 2\eta_1(v_S, w)\big(r(y - v_S, S) - r(y, S)\big). \tag{6}$$

The flat Hessian has the exact vertex identity

$$D^2 S_{\ell_1}(u, g_{y,w}) = 0 \tag{7}$$

for every edge field u, interpreted through the same finite local support. A necessary condition for a quadratic family

$$\ell(t) = \ell_1 + th + \tfrac{1}{2}t^2 q \tag{8}$$

to solve the incident equations is therefore

$$D^3 S_{\ell_1}(h, h, g_{y,w}) = 0. \tag{9}$$

Indeed, differentiating the equations twice gives $D^2 S(q, g) + D^3 S(h, h, g) = 0$, and the first term vanishes by (7). This argument tests a compatibility condition rather than attempting to solve separately for fifteen correction components.

3.2 A fixed static cubic tangent

Let $\Phi(\xi) = \xi_1^3 - 3\xi_1\xi_2^2$ and define, at the edge midpoint,

$$h(x, S) = -2\Phi(\xi)\left(m + \frac{m^2}{2}\right) - 6\xi_2 p_{12}(S). \tag{10}$$

The second term is the slaved mixed-tidal correction in this normalization. The field is static because (2) is unchanged by $x \mapsto x + d$.

Theorem 3.1 (Fixed-cubic Ward obstruction)**.** *The field* (10) *solves every linearized row on* $\mathbb{Z}^4$*. At* $y = (2, 1, -1, 0)$ *its second-order Ward defect is*

$$D^3 S_{\ell_1}(h, h, g_{y,w}) = 756\sqrt{3}\,(w_0 + w_1 - w_2 - w_3). \tag{11}$$

In particular, for $w = e_0$ *no edge field* q *makes* (8) *solve all thirty rows incident to* y *for all sufficiently small* t*.*

Proof. Both assertions reduce to finite identities. The midpoint field is cubic and its internal correction is linear. Substitution into the assembled flat Hessian makes every row polynomial in the base record; every coefficient vanishes. No periodic extension of the polynomial is required.

For the third variation use the Schläfli identity in each simplex and the zero flat deficits. Put $A_t = \sqrt{-H_t}/4$ at each of its ten hinges, and let ϑ_t be the exterior angle, the supplement of the interior angle in (3). The cell trilinear form is

$$\mathscr{T}(X,Y,Z) = \sum_t \left[D^2\vartheta_t(X,Y)\, DA_t(Z) + D\vartheta_t(X)\, D^2A_t(Y,Z) + D\vartheta_t(Y)\, D^2A_t(X,Z) \right]. \tag{12}$$

This expression is the third derivative of the action after assembly; the Schläfli identity removes the higher angle derivatives. Its evaluation is specified explicitly in Appendix A. Only the $24 \times 5 = 120$ simplex incidences containing y see g_{y,e_0}. Their exact numerator sum in units $1/(1152\sqrt{3})$ is 2612736. Hence

$$\sum \mathscr{T}(h,h,g_{y,e_0}) = \frac{2612736}{1152\sqrt{3}} = 756\sqrt{3}.$$

The same calculation with the four account displacements gives the coefficient vector $(756, 756, -756, -756)\sqrt{3}$. Equation (7) and $756\sqrt{3} \neq 0$ contradict the necessary condition (9). □

This theorem applies to the specified quadratic curve and tangent. It is not a theorem that every static field has a nonzero defect. It does show that adding an arbitrary second-order edge correction to this first-order solution cannot restore the equations. Local cancellation of the tidal source does not guarantee nonlinear compatibility.

3.3 Point-mass Taylor fields and the sampled field

There is a second useful exact witness. Define the slaved field $\mathcal{S}[\Phi]$ by the metric term in (10) plus $\sum_{i<j}(\partial_i\partial_j\Phi)p_{ij}$. The cubic Taylor potential for a unit source a unit distance down the negative ξ_1 axis is

$$\Phi_A(a,b,c) = -1 + a - a^2 + \tfrac{1}{2}(b^2+c^2) + a^3 - \tfrac{3}{2}a(b^2+c^2).$$

For $h_A = \mathcal{S}[\Phi_A]$, the exact defect at the origin is

$$\mathscr{W}''_{0,w}(R(\ell_1 + th_A)) = 126\sqrt{3}\,(w_2 + w_3 - w_0 - w_1). \tag{13}$$

Here primes mean derivatives with respect to t at zero, with no factor $1/2$. This also obstructs a quadratic correction at that record.

The actual sampled potential for inverse source distance $s > 0$ is

$$\Phi_s(a,b,c) = -\frac{s}{\sqrt{(1+sa)^2 + s^2b^2 + s^2c^2}}.$$

Its cubic spatial Taylor field is $-s + s^2a + s^3[-a^2 + (b^2+c^2)/2] + s^4[a^3 - 3a(b^2+c^2)/2]$. Formula (13) concerns the polynomial witness, not a replacement of Φ_s by that witness. At the origin with $w = e_0$, establishing the leading $-126\sqrt{3}\,s^7$ defect of the actual sampled field, with a uniform $O(s^8)$ remainder on the finite reads, is an open target. A field error bound alone must be accompanied by the relevant cancellations in the quadratic Ward form. We do not use the unproved sampled-field conclusion to exclude a point-mass exterior.

4 The radial determinant question

Strong fields require the extra-component equations at non-flat local frames. For a constant symmetric matrix G, let $J(G)$ be the derivative of the fifteen rows at a record with respect to uniform changes in the fifteen edge classes. Under the nondegeneracy assumptions of Section 2 this is an actual Hessian of the action, evaluated on a uniform-class variation and one local edge variation. It is not determined by its flat value alone. *Status: Theorem: independently verified Hessian correspondence.*

The proposed formula is

$$J(G) \stackrel{?}{=} -\frac{\Pi^T K(G)\Pi}{2\sqrt{-\det G}}, \qquad K(G) = \begin{pmatrix} O - D & O\mathbf{1} \\ \mathbf{1}^T O & \sum_{a<b} G_{ab} \end{pmatrix}, \tag{14}$$

where $D = \operatorname{diag} G$ and $O = G - D$. For arbitrary symmetric G whose Kuhn cells have timelike hinges and facets and negative simplex Gram determinant, (14) is an open target. The isotropic family $G = I - \kappa \mathbf{1}\mathbf{1}^T$, $\kappa > 1/2$, has an exact derivation. The general formula requires the full geometric derivative identity and finite class sum.

Consider the radial family

$$\begin{gathered} G = T\left(I - \tfrac{1}{4}\mathbf{1}\mathbf{1}^T\right) + (F - T)nn^T - \tfrac{3F}{4}\mathbf{1}\mathbf{1}^T, \\ \mathbf{1}^T n = 0, \qquad n^T n = 1, \qquad 0 < T \le F. \end{gathered} \tag{15}$$

Its determinant is $-3F^2T^2$.

Proposition 4.1 (Radial frame admissibility)**.** *Every Kuhn simplex of* (15) *has negative triangle, tetrahedron and full simplex Gram determinants. This holds in every direction n satisfying the stated constraints, including $T = F$.*

Status: Theorem: independently verified. A proof covering all 24 simplex frames is given in Appendix C. Thus the timelike-cell premise needed to apply (14) is satisfied throughout this radial family.

Proposition 4.2 (Zero deficits for constant radial frames)**.** *On the periodic realization with eight records per account, the constant edge field induced by* (15) *has zero deficit at every hinge.*

Status: Theorem: independently verified.

Proof. Fix n and join the native flat frame to the given frame by

$$F_t = 1 + t(F - 1), \qquad T_t = 1 + t(T - 1), \qquad 0 \le t \le 1.$$

Then $T_t > 0$ and $T_t \le F_t$. Proposition 4.1 keeps every hinge, facet and simplex Gram determinant negative along this path. For each geometric hinge star, the oriented unit normals in its closed link make an integer number of turns. Its deficit is therefore an integer multiple of 2π. This conclusion alone does not determine the integer. The angles, and hence the deficit, are continuous along the path, so that integer is constant. At $F_0 = T_0 = 1$ the native flat star has zero deficit. It remains zero at $t = 1$. □

This is a statement about constant frames. It supplies neither zero deficits for a spatially varying radial field nor the remaining finite derivative identity in (14).

Choose the five complementary edge classes

$$\{0\}, \quad \{0,1,2\}, \quad \{0,1,3\}, \quad \{0,2,3\}, \quad \{1,2,3\},$$

and let W be their inclusion matrix. Then $\det(\Pi W) = 2$. Write $B = W^T J(G) W$, $\tau = T/F$, $u = \tau - 1$ and $p_j = \sum_a n_a^j$. Define

$$\begin{aligned} P(\tau, n) = & -72 + 60u + (33 - 84p_4)u^2 + 4p_3^2 u^3 \\ & + \left[4p_3^2 - 3(2p_4 - \tfrac{1}{2})^2\right] u^4. \end{aligned} \tag{16}$$

The finite algebra gives

$$4 \det K(G) = F^5 P(\tau, n). \tag{17}$$

Status: Theorem: independently verified determinant algebra. The algebraic certificate is given in Appendix B. If (14) holds for (15), it follows that

$$\det B = -\frac{P(\tau, n)}{(2\sqrt{3}\,T)^5}. \tag{18}$$

This implication retains its premises.

The useful sign bound is

$$P(\tau, n) \le 12(\tau - 2)(\tau + 5) < 0 \qquad (0 < \tau \le 1). \tag{19}$$

The first inequality holds for $0 \le \tau \le 2$. To prove it, set $a = p_3^2$ and $b = p_4 - 1/4$. Cauchy's inequality gives $b \ge 0$ and $0 \le a \le b$, using $p_3 = \sum_j n_j (n_j^2 - 1/4)$. Subtracting the right side of (19) from P gives

$$u^2[-84b + 4au(1 + u) - 12b^2 u^2] \le 0,$$

since $-1 \le u \le 1$ implies $u(1 + u) \le 2$. This proves the polynomial bound; its proof has been independently verified. Thus the candidate five-block has positive determinant throughout the stated radial family. The associated quadratic-form calculation gives four positive and one negative direction for K, and hence one positive and four negative directions for the candidate B. These algebraic results do not by themselves prove the proposed matrix is the actual derivative. Nor does a uniform, zero-frequency inverse establish an inverse for the full spatially varying exterior equations.

5 The action and the horizon problem

5.1 Spacelike hinges cannot be dropped

Extending the timelike term in (3) by the real-valued factor $\sqrt{\max\{0, -H\}}/4$ makes it vanish at spacelike hinges $H > 0$. A Lorentzian geometry requires the corresponding boost term instead [4]. This matters before one asks whether a black-hole solution exists.

An explicit example uses a constant-flow flat reading. For $m = |S|$ and $s = |S\cap\{0,1\}| - |S\cap\{2,3\}|$, its edge values are

$$\ell_v(S) = m - m^2 - \frac{\sqrt{3}}{2} vms + \frac{3}{4} v^2 m^2.$$

The triangle $x, x+e_0, x+e_0+e_1$ has hinge determinant

$$H(v) = 2(3v^2 - 2\sqrt{3}\, v - 2).$$

At $v = 0$ it is timelike, while at the horizon flow $v = -1$ it is spacelike. In fact it changes sign at $v = (\sqrt{3}-3)/3$. The original timelike-only expression therefore omits a term in a flat geometry of the relevant causal type. This is a theorem about that hinge and that action, not evidence of a physical singularity.

Here regular Lorentzian means a negative four-simplex Gram determinant and nonzero Gram determinants for every facet and hinge. An extension takes area $\sqrt{|H|}/4$, the ordinary angle for timelike hinges and an oriented boost β for spacelike hinges. With the quantities U, V, N of Appendix A, its boost convention is

$$\beta = \begin{cases} \operatorname{sign}(N)\operatorname{arcosh}\big(|N|/(2\sqrt{UV})\big), & UV > 0, \\ \operatorname{arsinh}\big(N/(2\sqrt{-UV})\big), & UV < 0. \end{cases}$$

Regularity excludes $UV = 0$. The flat angle share is $2\pi/N_h$ for timelike hinges and zero for spacelike hinges. Its slot term is

$$L_{\Delta h} = \frac{\sqrt{|H|}}{4} \begin{cases} 2\pi/N_h - \theta_{\Delta h}, & H < 0, \\ -\beta_{\Delta h}, & H > 0. \end{cases} \tag{20}$$

The Lorentzian Schläfli identity holds with these signs on each regular simplex. It gives rows of the form $\sum_h \epsilon_h\, \partial A_h/\partial \ell_e$. Consequently closing-up of every flat hinge star implies flat stationarity. General closing-up for every regular uniform Lorentzian reading is still open. The timelike isotropic subfamily $\kappa > 1/2$ is established, and Proposition 4.2 covers the stated constant radial family. Neither settles stars containing spacelike hinges.

The extension agrees with the original action in a timelike neighborhood of ℓ_1. Every fixed quadratic curve (8) stays in that neighborhood on each finite box for sufficiently small t. Theorem 3.1 therefore applies to the extended action as well. Adding its spacelike terms does not repair this particular near-flat obstruction.

5.2 A target that allows a horizon

A definition of a static exterior that requires the diagonal translation to remain timelike cannot establish a black hole by construction. The appropriate open target instead asks for a static, tetrahedrally symmetric, asymptotically ℓ_1 exterior with all fifteen extended vacuum rows at records outside a core of spread R, and regular Lorentzian simplices whose base records lie outside the core of spread $R-2$. It additionally asks for a record outside the core where

$$\ell(x, \{0,1,2,3\}) \geq 0. \tag{21}$$

The target requires such exteriors for arbitrarily large core spreads, so the feature survives at sizes large compared with a lattice spacing. Flat space does not meet (21), since its diagonal norm is -12.

This is a useful minimal Killing-norm target. A non-timelike diagonal at one record is weaker than an event-horizon or trapped-region theorem. Such a theorem also needs a specified causal reading and a proof about escape. In particular, the componentwise posting order and the interval order of a configuration are distinct objects. In the posting order every monotone observer that posts on each account without bound has the whole lattice in its past, and a nonempty region closed under all forward postings cannot be spatially bounded. In the configuration's interval order, spacelike postings can remove escape routes. One must not identify these two orders silently.

Exact Minkowski embeddings of the Rindler and river wedges illustrate static Killing-norm horizons and local horizon kinematics [2]. The Rindler embedding degenerates on its horizon; neither construction here establishes a regular, zero-deficit Regge geometry across the horizon. They do not supply an asymptotically flat, spatially localized vacuum with a compact core. The black-hole existence target above remains open.

6 What an exterior must still accomplish

The uniform tidal identity, the local obstruction and a global exterior address different questions. A complete construction must connect them without changing their carriers or hypotheses.

First, a slowly varying exterior must solve the fifteen rows to the desired asymptotic order, including the additional edge combinations. The uniform result in Theorem 2.1 supplies its curvature-order cancellation. Bounds on derivatives, the next residual and a compatible global inverse are further obligations. Related nonlinear exterior work is in preparation; its restricted constructions are not used here as an all-direction existence theorem.

Second, one must meet the exact compatibility conditions beyond the continuum Einstein order. Theorem 3.1 shows that one fixed regular perturbative route fails already at second order in amplitude. A finite lattice artifact can be asymptotically smaller than the leading equations and still prevent an exact vacuum solution. Shrinking a residual is different from proving that it can be canceled.

Third, if that artifact is not canceled within the vacuum equations, there are elevated physical targets to pursue: an action with exact vertex symmetry, or matter whose equations and stress carry the required load. These are open constructions. An added source cannot be called vacuum, and an altered action must be checked against the same continuum and physical requirements.

Finally, a black-hole construction must pass from a timelike exterior to regular Lorentzian cells of the necessary causal types using the extended action, solve the exterior equations, and establish the relevant horizon property. This is stronger than inserting a continuum Schwarzschild line element, finding a zero of a sampled lapse, or obtaining a uniform radial five-block inverse.

7 Entropy and radiation remain downstream

The black-hole companion [2] separates horizon cuts, state counting and test-background radiation. The same separation is required here. An area coefficient obtained by choosing independent bits per unit area is a model. A physical quarter-law requires the ledger's actual independent states and an action-scale condition derived independently of the desired count. Different horizon orientations and different spacing identifications change elementary per-record counts; they cannot be suppressed by referring only to area.

For radiation, exact stationary-band identities and numerical scattering on prescribed horizon backgrounds do not constitute emission from a lattice black hole solving the gravitational equations. The latter requires existence of that geometry, a prepared state, the appropriate limiting two-point function and a justified observable. A vacuum condition imposed at every positive sampled frequency is obstructed by aliasing; a constructed finite-band flat vacuum avoids that particular defect. Collapse preparation and its late-time behavior remain open. The scalar's growing record-time modes are a separate obstacle to a stable particle interpretation. We therefore make no claim here of a physical Hawking spectrum, evaporation law or Page curve for a solved compact lattice object.

A The finite third-variation evaluation

The finite recipe behind (11) uses squared edges, not a continuum differential operator. Number each simplex's vertices $0, \dots, 4$. At the flat reading their squared separations are $a_{ij} = (j - i)(1 - (j - i))$ for $i < j$. Given any assignment a, its doubled Gram matrix based at a vertex is

$$Q_{ij} = a_{0i} + a_{0j} - a_{ij}, \qquad Q_{ii} = 2a_{0i}.$$

For a selected triangle, relabel its vertices 0, 1, 2 and the opposite vertices 3, 4. Put $H = \det Q_{\{1,2\}}$. Let $U = \frac{1}{2}\det Q_{\{1,2,3\}}$, $V = \frac{1}{2}\det Q_{\{1,2,4\}}$, and $N = \det Q_{(1,2,3),(1,2,4)}$. The exterior angle in (12) is

$$\vartheta = \arccos \frac{N}{2\sqrt{UV}}.$$

These formulas fix the normalization of every derivative in the calculation. In each incident simplex take the ten reads $X_e = h(x_e, S_e)$ and $Z_e = g_{y,w}(x_e, S_e)$. Evaluate the mixed derivatives in (12) at its flat ten-edge vector. The 24 permutations and the five possible positions of y specify all 120 terms; a simplex with y at position k has base $y - \sum_{j<k} e_{\sigma(j)}$. Thus the recipe fixes every rational input from (2), (10) and the displayed p_{12} table. The resulting terms are rational multiples of $1/(1152\sqrt{3})$; their coefficients sum to 2612736. The finite row calculation uses the second derivative of the same action with the same ten-edge ordering. No continuum field equation is substituted for those rows.

B A certificate for the radial polynomial

Here is a short algebraic certificate for (17). Set $F = 1$, eliminate $n_3 = -(n_0 + n_1 + n_2)$ and put $p_2 = \sum n_a^2$, without yet setting $p_2 = 1$. Direct expansion of the determinant gives

$$\begin{aligned} 4\det K(G) - P = (p_2 - 1)\big[&(-3 + 3p_2^2 - 3p_2^3 + 12p_2p_4)u^4 \\ &+ (\tfrac{9}{2} + 15p_2^2 - 30p_4)u^3 + (60 + 42p_2)u^2 - 30u\big]. \end{aligned}$$

The right side vanishes at $p_2 = 1$. Since K is linear in G and $G(F,T,n) = FG(1,T/F,n)$, its five-dimensional determinant scales as F^5. This proves (17). Conditional on (14), congruence by ΠW, whose determinant is 2, and $\sqrt{-\det G} = \sqrt{3}FT$ give (18). This calculation proves the determinant identity of the candidate matrix; the equality of that matrix with the general class derivative remains a separate target.

C All radial simplex frames

Proof of Proposition 4.1. Put $d = F - T \geq 0$, $c = (T + 3F)/4$ and

$$Q(v) = T\|v\|^2 - c(\mathbf{1}^T v)^2 + d(n^T v)^2.$$

For $s = \mathbf{1}^T v$ and $w = v - s\mathbf{1}/4$, Cauchy's inequality gives

$$Q(v) = T\|w\|^2 + d(n^T w)^2 - \tfrac{3F}{4}s^2 \leq F(\|v\|^2 - s^2).$$

In particular, an indicator vector with $M \geq 2$ nonzero coordinates has $Q(v) \leq F(M - M^2) < 0$.

Consider disjoint nonempty coordinate blocks with indicators u_i, cardinalities m_i and radial sums $a_i = n^T u_i$. Their Gram matrix is

$$g(u_i, u_j) = Tm_i\delta_{ij} - cm_i m_j + da_i a_j.$$

Set $q(M,V) = TM - cM^2 + dV^2$ and

$$B(M,V,D) = Tq(M,V) + d(T - cM)D.$$

For a combined indicator with $M \geq 2$, we have $q(M,V) < 0$ and $T - cM \leq T - 2c \leq -F < 0$. Thus $B(M,V,D) < 0$ whenever $D \geq 0$.

For two blocks of sizes m, k and radial sums a, b, expansion gives

$$4 \det \mathrm{Gram}(u_1, u_2) = \frac{4mk}{m+k} B\left(m+k, a+b, \frac{(ka-mb)^2}{mk}\right) < 0.$$

For three blocks of sizes m, k, l and radial sums a, b, h, put

$$D = \frac{(ka-mb)^2}{mk} + \frac{(la-mh)^2}{ml} + \frac{(lb-kh)^2}{kl}.$$

Then

$$4 \det \mathrm{Gram}(u_1, u_2, u_3) = \frac{4Tmkl}{m+k+l} B(m+k+l, a+b+h, D) < 0.$$

Both expansions also follow from $D = M\sum_i a_i^2/m_i - (\sum_i a_i)^2$. Their prefactors are positive, and their D are nonnegative.

For any coordinate permutation σ, the simplex vertices are $p_j = \sum_{r<j} e_{\sigma(r)}$, $0 \leq j \leq 4$. Three ordered vertices have successive differences u_1, u_2 that are disjoint nonempty coordinate blocks. Their edge basis is $(u_1, u_1 + u_2)$, a determinant-one change from (u_1, u_2). Four ordered vertices similarly give the basis $(u_1, u_1 + u_2, u_1 + u_2 + u_3)$. These observations cover all ten triangles and five tetrahedra of each of the 24 simplices, with exactly the Gram determinants computed above.

Finally, G has eigenvalues $-3F, F, T, T$ on $\mathbf{1}, n$ and their common orthogonal complement. The cumulative coordinate edge basis of a full simplex has determinant ± 1. Its Gram determinant is therefore $-3F^2T^2 < 0$. The triangle, tetrahedron and full simplex Gram polynomials used in the action are respectively $4, 4, 16$ times these ordinary Gram determinants. Every required strict sign follows, without division by $F - T$. □

References

[1] J. Washburn, *Quantum Gravity from the Recognition Ledger: Weighted One-Loop Graviton Responses with Finite Real Parts, Einstein Gravity at Long Wavelengths, and the Massless-Graviton Problem*, version 1.2 (2026), doi:10.5281/zenodo.23839819.

[2] J. Washburn, *The Recognition Ledger of a Black Hole: Horizon Records, Entropy Counts and Lattice Hawking Radiation*, version 1.2 (2026), doi:10.5281/zenodo.23839821.

[3] T. Regge, General relativity without coordinates, *Nuovo Cimento* **19**, 558–571 (1961).

[4] R. D. Sorkin, Lorentzian angles and trigonometry including lightlike vectors (2019), arXiv:1908.10022.