

Sourced Einstein equations from the recognition lattice

Linearized gravity about the flat lattice

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Abstract

The flat recognition lattice is a periodic four-dimensional Kuhn triangulation whose edges have Lorentzian squared lengths $n(1 - \kappa n)$. Linearizing its Regge action about this flat lattice, we derive the linearized Einstein equations with their matter sources, for every $\kappa > 1/2$. Conservation of a lattice source at first order in the wave vector is exactly conservation of the stress tensor, and every conserved stress tensor has an exactly conserved lattice source. The long-wavelength field is $h_{\mu\nu} = (16\pi G/q)(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$ with $G = 1/(8\pi s)$, where s is the action scale. For a static mass the post-Newtonian parameter γ equals 1 gauge invariantly at the level of the long-wavelength symbol, and in the scaling limit the de Donder field has $h_{00} \rightarrow 2GM/r$ and $h_{ij} \rightarrow (2GM/r)g_{ij}$. At fixed lattice spacing h_{00} has Newton's coefficient pointwise, against probes and in Gauss's law. These far-field theorems use band-limited sources: at one static wave, given in closed form for every κ , the static operator has a non-gauge zero mode and an unsmeared worldline source has no solution.

1 The question

General relativity says that matter tells spacetime how to curve through $G_{\mu\nu} = 8\pi G T_{\mu\nu}$. To first order about flat space this is a statement anyone can check: the metric perturbation of a static mass is $h_{00} = 2GM/r$ in its time-time component and $h_{ij} = (2GM/r)\delta_{ij}$ in its spatial components. The first number is Newton's potential. The second is what bends light and delays radar echoes, and the Cassini measurement pins the post-Newtonian parameter γ that compares the two to $|\gamma - 1| < 2.3 \times 10^{-5}$ [1, 2].

Recognition Science places spacetime on a lattice of recognition records. Distances on it are squared edge lengths, and gravity is the Regge action of those lengths [3]. Regge calculus is known to reproduce the linearized Einstein operator in the continuum limit [4, 5]. On the recognition lattice that is not enough, because the lattice spacing is physical: we can't send it to zero. The questions are sharper. Does a matter source that is conserved in the continuum sense couple to the lattice at all, given that the lattice Bianchi identity demands conservation at every wavelength? Do the spatial components come out with $\gamma = 1$? Does a source of fixed profile, sitting on a lattice of fixed spacing, have Newton's far field? All three have affirmative answers at linear order, the far-field ones for band-limited sources, for the true Regge operator of the lattice and for every value $\kappa > 1/2$ of its length parameter.

The result: *Linearized about the flat recognition lattice, the Regge equations sourced by any conserved stress tensor are the linearized Einstein equations at long wavelength, and a band-limited static mass has, in de Donder gauge and in the scaling limit, the linearized Schwarzschild far field $h_{00} \rightarrow 2GM/r$, $h_{ij} \rightarrow (2GM/r)g_{ij}$ with $G = 1/(8\pi s)$.*

The time-time component of this statement, for static matter, gives Newton's law as a scaling limit ([Theorem 6.1](#)). Newton's law alone leaves out the coupling of general conserved matter, including pressure and motion; the spatial components and $\gamma = 1$; far fields at fixed lattice spacing; the obstruction at one static wave, at every κ , that leads every far-field theorem to use band-limited sources; and the smoothness of the action that makes its second variation a genuine Hessian. Each is established below. What remains is the nonlinear theory and the derivation of the lattice geometry itself, which [Section 8](#) states.

Contributions.

1. The Regge action is real analytic near the flat lattice for every $\kappa > 1/2$, so its Hessian is a genuine symmetric second derivative ([Theorem 2.1](#)).
2. Lattice conservation of a source at first order is continuum conservation $T^{\mu\nu}u_\nu = 0$; every conserved stress family has an exactly conserved lattice source, a gauge-orthogonal projection that differs from the naive source by $O(|u|^2)$; no conserved lattice source can escape continuum conservation ([Theorems 3.1 to 3.3](#)).
3. A static point source carries no pressure at linear order; moving matter couples on its comoving cone ([Theorems 3.4 and 3.5](#)).
4. The long-wavelength response to every conserved source is the trace-reversed stress tensor times $16\pi G/q$, for moving mass too; a static mass has $\gamma = 1$, gauge invariantly at the level of the long-wavelength symbol ([Theorems 4.2 to 4.6](#)).
5. The static operator has a non-gauge zero mode at every $\kappa > 1/2$ at an explicit wave $\cos t_* = 1 - 6(2\kappa - 1)/(\kappa(4\kappa - 1)(16\kappa - 5))$, and an unsmeared static worldline source has no solution at that wave ([Theorem 5.1](#)).
6. A band-limited static body has $h_{00} \rightarrow 2GM/r$ and $h_{ij} \rightarrow (2GM/r)g_{ij}$ as a scaling limit; at fixed spacing h_{00} has Newton's far field pointwise, against probes, and in Gauss's law ([Theorems 6.1 to 6.3](#), [6.5](#) and [6.6](#)).

2 The lattice and its action

2.1 Record coordinates and the Kuhn triangulation

Vertices are points $x \in \mathbb{Z}^4$ (on a periodic torus $(\mathbb{Z}/N)^4$ when an action has to be finite). Each unit cube is cut into 24 four-simplices by Kuhn's rule [[7](#), [8](#)]: for each ordering (a_1, a_2, a_3, a_4) of the axes, the simplex has vertices

$$x, \quad x + e_{a_1}, \quad x + e_{a_1} + e_{a_2}, \quad x + e_{a_1} + e_{a_2} + e_{a_3}, \quad x + (1, 1, 1, 1).$$

Table 1. Terms used throughout.

Term	Object	Meaning
Edge class d	$d \in \{0,1\}^4 \setminus \{0\}$, $ d $ = number of ones	The 15 edge directions of the Kuhn triangulation
Time edge	$d = (1,1,1,1)$	The edge along which static matter sits
Static wave	covector u with $\sum_a u_a = 0$	A wave constant along the time direction
Vertex move	covector field ξ on vertices	The lattice form of a coordinate change
Conserved source	$\Gamma(u)^\top b(u) = 0$	A source orthogonal to every vertex move
Residual zero mode	null vector of B orthogonal to vertex moves	A lattice mode with no continuum counterpart
Band-limited source	spectrum $\hat{\rho}(Lk)$ in $ k < \delta$	A smooth profile spread over many lattice spacings

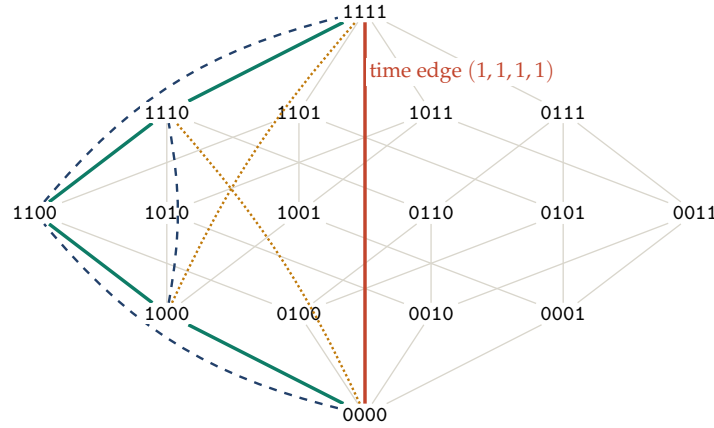


Figure 1. The unit cube $\{0,1\}^4$ ordered by inclusion of bits. A Kuhn simplex is a chain from 0000 to 1111; the highlighted one is $0000 < 1000 < 1100 < 1110 < 1111$. Its ten edges join any two vertices of the chain and flip 1 (solid teal), 2 (dashed), 3 (dotted) or 4 bits. An edge that flips n bits has squared length $n(1 - \kappa n)$. Every one of the 24 chains contains the time edge (red), the body diagonal.

Every edge joins x to $x + d$ for one of the 15 edge classes $d \in \{0,1\}^4 \setminus \{0\}$ (Fig. 1). The background metric in these record coordinates is

$$g_\kappa = I - \kappa J, \quad J_{ab} = 1, \quad g_\kappa(d, d) = |d| (1 - \kappa |d|), \quad \det g_\kappa = 1 - 4\kappa. \quad (1)$$

So an edge that flips n bits has squared length $n(1 - \kappa n)$. For $\kappa > 1/2$ every edge of two or more bits is timelike, and the one-bit axis edges are spacelike for $1/2 < \kappa < 1$, null at $\kappa = 1$ and timelike for $\kappa > 1$. Every Kuhn triangle contains an edge of at least two bits, so every hinge is timelike and carries a real rotation deficit. The time direction is the body diagonal: the unit future time vector is

$$\hat{t} = \frac{(1,1,1,1)}{2\sqrt{4\kappa-1}}, \quad g_\kappa(\hat{t}, \hat{t}) = -1, \quad t^b = g_\kappa \hat{t}.$$

A covector u is static when $u(1,1,1,1) = \sum_a u_a = 0$. The wave symbol is $q(u) = g_\kappa^{-1}(u, u)$; on static covectors it is $q(u) = \sum_a u_a^2 > 0$.

Where the lattice comes from. In Recognition Science the vertices are recognition records and their coordinates are record counts. Three spatial dimensions are forced by the retained linking conditions [10], and time is placed along the body diagonal, the direction in which every record coordinate advances together. The specific triangulation, the length family $n(1 - \kappa n)$, the deformability of those lengths and the placement of time on the diagonal are taken here as the given lattice. Deriving them from the recognition ledger is the remaining step underneath every theorem below, and nothing here depends on a particular κ beyond $\kappa > 1/2$.

2.2 The Regge action is smooth near the flat lattice

A field is an assignment of a squared length ℓ_e^2 to every edge e . The flat field is $\ell_0^2(x, d) = |d|(1 - \kappa|d|)$. For a triangle (hinge) with squared side lengths a, b, c write $H = 4ab - (a + b - c)^2$; for a timelike triangle $H < 0$ and its area is $A = \frac{1}{4}\sqrt{-H}$. Each pair (simplex σ , hinge $h \subset \sigma$) is a slot; n_h counts the slots on h and $\theta_{\sigma,h} = \arccos c_{\sigma,h}$ is the dihedral angle of σ at h , computed from the squared lengths of σ . The Regge action is

$$S(\ell^2) = \sum_{\text{slots } (\sigma,h)} A_h(\ell^2) \left(\frac{2\pi}{n_h} - \theta_{\sigma,h}(\ell^2) \right) = \sum_h A_h \varepsilon_h, \quad (2)$$

Regge's area times deficit with no extra factor.

Theorem 2.1 (Smoothness). *Let U be the set of fields for which every simplex has negative facet determinants, negative hinge determinants and negative simplex Gram determinant. Then U is open, the zero field is not in U , and for every torus side N and every $\kappa > 1/2$ the flat field lies in U . On U the action S is of class C^n for every n , including $n = \infty$ and real analyticity. Consequently the Hessian $B(h, h') = \partial_t \partial_s S(\ell_0^2 + th + sh')|_{t=s=0}$ is the genuine Fréchet second derivative of S at the flat field, and it is symmetric, $B(h, h') = B(h', h)$.*

Mechanism. The only non-polynomial operations in (2) are the square root in the area and in the denominator of $c_{\sigma,h}$, division by the hinge determinant, and arccos. On U each is evaluated strictly inside its smooth domain: $H \neq 0$, $-H > 0$, the product of the two apex norms is positive, and $c_{\sigma,h} \neq \pm 1$. A finite sum of products of such functions is real analytic, and openness of U follows from continuity of the finitely many determinants. Symmetry of the second derivative of a C^2 function finishes the proof. $\square$

The flat lattice is a regular point of the action: no square root or arccosine sits at the edge of its domain, so the linearized theory below is the honest second-order expansion of the Regge action, and its operator is symmetric by construction.

2.3 The operator, vertex moves and the Bianchi identity

On a torus of side $N \geq 3$ the Hessian is translation invariant. On a plane wave with phase $z_a = e^{iu_a}$ it acts through a 15×15 matrix $B_\kappa(z)$ indexed by edge classes, Hermitian on the unit torus, and the static second-variation form of one time slab acts through the same matrix at static phases.

A vertex move displaces each vertex by a covector ξ_x , which changes squared lengths by $\delta \ell^2(x, d) = 2 \sum_a d_a (\xi_{x+d} - \xi_x)_a$. On a plane wave $\xi_x = \hat{\xi} z^x$ it acts through the gauge columns

$$\gamma_a(z)_d = 2 d_a (z^d - 1), \quad z^d = \prod_b z_b^{d_b}. \quad (3)$$

These do not depend on κ . Their linear part at small u is $2d_a(d \cdot u) i$, the edge reading of the continuum gauge strain $h = u \otimes \xi + \xi \otimes u$.

Theorem 2.2 (Lattice Bianchi identity). *For every $\kappa > 1/2$, every unit phase z , every axis a and every edge class j ,*

$$\sum_d \overline{\gamma_a(z)_d} B_\kappa(z)_{dj} = 0.$$

Hence a source τ for which $B_\kappa(z)h = \tau$ is solvable satisfies $\sum_d \overline{\gamma_a(z)_d} \tau_d = 0$ for every a : solvable sources are conserved, at every wavelength. More precisely, $B_\kappa(z)h = \tau$ is solvable exactly when τ is orthogonal to the null space of $B_\kappa(z)$, and exactly when τ is conserved and orthogonal to the residual null space, the null vectors orthogonal to every vertex move.

The identity is the discrete form of $\nabla^\mu G_{\mu\nu} = 0$: the action is unchanged by moving vertices, so its second variation annihilates vertex moves. On the lattice it binds at every wavelength, so coupling matter to the lattice takes work.

2.4 Long wavelength: the Fierz–Pauli form

Write a field amplitude at wave u in the midpoint frame, $w_d = e^{i d \cdot u/2} b_d$, which places the amplitude of edge class d at the midpoint of its edge. In this frame the operator is a real symmetric matrix $M_\kappa(u)$ and the gauge columns are $e^{i d \cdot u/2} i \Gamma(u)_{da}$ with $\Gamma(u)_{da} = 4 d_a \sin(d \cdot u/2)$. A constant strain h_{ab} changes squared lengths by $(Vh)_d = d^a d^b h_{ab}$; this is the strain embedding $V : \mathbb{R}^{10} \rightarrow \mathbb{R}^{15}$. The matrix has the expansion $M_\kappa(u) = M_0 + M_2(u) + O(|u|^3)$ with $M_0 V = 0$, and M_0 is invertible on a complement W of the strains: the determinant of that 5×5 block is $C(2\kappa - 1)^8 (3\kappa - 1)^{10} (4\kappa - 1)^{16} \neq 0$ on $(1/2, \infty)$. On strains the quadratic term is the linearized Einstein operator:

$$V^T M_2(u) V = -\frac{1}{4} \sqrt{|\det g_\kappa|} \mathcal{F}_{g_\kappa}(u), \quad (4)$$

where $\mathcal{F}_g(u)$ is the Fierz–Pauli form [6] with all indices contracted by g^{-1} : for strain basis matrices B_i, B_j and $\bar{u} = g^{-1}u$,

$$\begin{aligned} \mathcal{F}_g(u)_{ij} = & q \operatorname{tr}(g^{-1} B_i g^{-1} B_j) - 2 \bar{u}^T B_i g^{-1} B_j \bar{u} + \operatorname{tr}(g^{-1} B_j) \bar{u}^T B_i \bar{u} \\ & + \operatorname{tr}(g^{-1} B_i) \bar{u}^T B_j \bar{u} - q \operatorname{tr}(g^{-1} B_i) \operatorname{tr}(g^{-1} B_j). \end{aligned}$$

Away from the light cone the operator is nondegenerate up to gauge near $u = 0$: for every $\eta > 0$ there is $\delta > 0$ such that for $0 < |u| < \delta$ with $\eta |u|^2 \leq |q(u)|$, a source is solvable exactly when it is conserved.

3 Coupling matter

3.1 The coupling and the gauge charge

Matter with stress tensor $T^{\mu\nu}$ couples to a strain h through $c_\kappa h_{\mu\nu} T^{\mu\nu}$ with $c_\kappa = \frac{1}{2} \sqrt{|\det g_\kappa|} = \frac{1}{2} \sqrt{4\kappa - 1}$, the Lagrangian $\frac{1}{2} \sqrt{-g} h_{\mu\nu} T^{\mu\nu}$ read on one coordinate cell. Its canonical edge source is $c_\kappa E(T)$, where $E(T)$ puts $T^{aa} - \sum_{b \neq a} T^{ab}$ on the axis edge e_a , T^{ab} on the face diagonal $e_a + e_b$, and nothing on edges of three or four bits. It reproduces T on constant strains: $\sum_d E(T)_d (Vh)_d = h_{ab} T^{ab}$.

A source localized at one site, with amplitude τ_d^0 on the edge of class d leaving it, has midpoint amplitude $b_d = \cos(d \cdot u/2) \tau_d^0$. Its gauge charge is $Q_a(u) = \sum_d \Gamma(u)_{da} b_d$, and a source is conserved when $Q(u) = 0$.

Theorem 3.1 (The charge). *For every one-site source τ^0 and every wave u ,*

$$Q_a(u) = \sum_d 2d_a \sin(d \cdot u) \tau_d^0.$$

If τ^0 reproduces $c_\kappa T$ on constant strains, its first-order charge is $Q_a^{(1)}(u) = 2c_\kappa T^{ab} u_b$, and $|Q_a(u) - Q_a^{(1)}(u)| \leq 4\|u\|_1^3 \sum_d |\tau_d^0|$. In particular, for $\kappa > 1/4$ the canonical source $c_\kappa E(T)$ is conserved at first order at u exactly when $T^{\mu\nu} u_\nu = 0$.

Mechanism. $4 \sin(x/2) \cos(x/2) = 2 \sin x$ gives the exact charge. Its linear part is $\sum_d 2d_a (d \cdot u) \tau_d^0$, which is the pairing of τ^0 with the constant strain $u \otimes e_a + e_a \otimes u$, and reproduction on strains turns that into $2c_\kappa (Tu)_a$. The remainder is the cubic Taylor bound of the sine. $\square$

So the lattice asks for nothing new at first order: its conservation law is the continuum one. The difference is at higher order. A static point mass written on the face diagonals by $E(T)$ is conserved at first order, yet at $\kappa = 5/4$ there is a static wave at which its charge is nonzero. The same mass placed on the time edge is exactly conserved on every static wave, because the time edge has $d \cdot u = \sum_a u_a = 0$ and its gauge column vanishes there.

3.2 A conserved lattice source for every conserved stress

Theorem 3.2 (The conserved repair). *The Gram matrix of the gauge columns obeys $|\Gamma(u)\xi|^2 \geq \frac{1}{11} |\xi|^2 |u|^2$ whenever $|u|^2 \leq 1/256$, so it is invertible for small $u \neq 0$. The gauge-orthogonal projection*

$$\Pi(u)b = b - \Gamma(\Gamma^\top \Gamma)^{-1} \Gamma^\top b$$

is exactly conserved at every wave, and for $|u|^2 \leq 1/256$ it moves each component by at most $(\Pi b - b)_d^2 \leq (11/|u|^2) |\Gamma(u)^\top b|^2$. Let $T(u)$ be a family of stress tensors, continuous at $u = 0$ and conserved, $T(u)^{\mu\nu} u_\nu = 0$, for every u in a set P . Then the repaired source $\Pi(u)$ applied to the canonical source of $T(u)$ is exactly conserved at every u , equals $c_\kappa E(T(0))$ at $u = 0$, tends to it as $u \rightarrow 0$ in P , and its strain projection at $u = 0$ is the coupling $c_\kappa T(0)$.

Mechanism. The Gram bound reduces to a polynomial inequality on the ten axis and face-diagonal classes. On P the canonical source is conserved at first order by [Theorem 3.1](#), so its charge is $O(|u|^3)$ and the projection moves it by $O(|u|^2)$; continuity of T gives the limits. $\square$

Theorem 3.3 (No escape from continuum conservation). *Let $\kappa > 1/4$. Suppose a source family $b(u)$ is exactly conserved for all $u \neq 0$ in a set P near 0, converges to b_0 as $u \rightarrow 0$ in P , and b_0 reproduces $c_\kappa T$ on constant strains. Then $T^{\mu\nu} p_\nu = 0$ for every nonzero p whose ray $\{\varepsilon p : \varepsilon > 0\}$ lies in P .*

The two theorems together say that the lattice couples exactly the conserved stress tensors. No choice of how matter is spread over the edges of a cell can couple a nonconserved one, and every conserved one couples with a correction of second order in the wave vector. The coupling is a statement in wave-vector space. The repaired source is a projection that depends on the wave, so in position space it is nonlocal, and it is constructed rather than derived from a lattice matter action. The one source that is local, follows from an action and is a ledger posting stress is the static point mass on the time edge (Section 6), whose source is the gradient of the tick action $-M\tau$.

3.3 Pressure and motion

Theorem 3.4 (Static pressure obstruction). *Let $\kappa > 1/2$ and let $T = \rho \hat{t} \otimes \hat{t} + p (g_\kappa^{-1} + \hat{t} \otimes \hat{t})$ be a static perfect fluid at one site. On static waves $T^{\mu\nu} u_\nu = p u^\mu$, because $g_\kappa^{-1} u = u$ and $\hat{t} \cdot u = 0$ there. Hence T is conserved on every static wave exactly when $p = 0$; no source family that is exactly conserved near 0 on static waves and converges to a source reproducing $c_\kappa T$ exists unless $p = 0$; and for $p = 0$ one exists, and the fluid is the point mass $\rho \hat{t} \otimes \hat{t}$.*

Reading. At linear order a static point source carries no pressure. This is the lattice form of the von Laue condition [9]: a static body in linearized gravity has $\partial_i T^{ij} = 0$, so pressure at a point has nothing to push against. A star holds its pressure against its own gravity, and that balance enters the field equations only at second order in the field. The theorem is the linear statement; the star is its interpretation.

For moving matter take $T = M w \otimes w$ for a timelike vector w . It is conserved on its comoving cone $\{u : w \cdot u = 0\}$.

Theorem 3.5 (Moving matter). *For every vector w and every M , the repaired source of $M w \otimes w$ is exactly conserved at every wave, tends to $c_\kappa E(M w \otimes w)$ along the comoving cone, and couples through $c_\kappa M w \otimes w$. For $\kappa > 1/2$, if $g_\kappa(w, w) < 0$ then every nonzero covector on the comoving cone has $q(u) > 0$. The vector*

$$w = \alpha(1, 1, 1, 1) + (1, -1, 0, 0), \quad \alpha^2 = \frac{3}{4(4\kappa - 1)},$$

is a unit timelike vector, $g_\kappa(w, w) = -1$, that moves relative to $\hat{t}$.

Finally, two lattice sources of the same static mass can differ and still agree where it matters.

Proposition 3.6 (Two sources, one coefficient). *Let $\kappa > 1/2$ and $m \neq 0$. The repaired face-diagonal point mass of cell stress m is exactly conserved and has the same strain projection as the time-edge source $c_\kappa m / (16\kappa - 4)$ placed on $(1, 1, 1, 1)$. On every static wave the repaired source carries nothing on the time edge while the time-edge source does, so they differ. Both give the same Newton coefficient: every solution of the lattice equation with either source has time-edge amplitude $h_{\hat{t}}$ with $q(u) h_{\hat{t}}(u) \rightarrow (16\kappa - 4)m/s$ as $u \rightarrow 0$ along static waves.*

The far field does not see how a cell distributes its mass over its edges. It sees the strain projection, which is the stress tensor.

4 The field: Einstein's linearized equations

4.1 The general sourced response

With action scale s the lattice field equation at wave u is

$$s B_\kappa(z) h = -\tau(u), \quad z_a = e^{iu_a}. \quad (5)$$

Fix de Donder gauge by adding $\ell(u)Q\ell(u)^\top$ to the midpoint matrix, where $\ell(u)$, linear in u , reads the de Donder (harmonic gauge) condition off edge amplitudes, $Q = 2\lambda_\kappa g_\kappa^{-1}$ and $\lambda_\kappa = -\frac{1}{4}\sqrt{4\kappa-1}$. Call the result $A(u)$. On strains its quadratic part is a multiple of one constant matrix,

$$V^\top (M_2(u) + \ell(u)Q\ell(u)^\top) V = q(u) C', \quad C' = \lambda_\kappa \mathcal{G}_{g_\kappa}, \quad (6)$$

where $\mathcal{G}_g(X, Y) = \text{tr}(g^{-1}Xg^{-1}Y) - \frac{1}{2}\text{tr}(g^{-1}X)\text{tr}(g^{-1}Y)$ is the DeWitt form. Newton's law and its relativistic completion live in the blow-up of $A(u)^{-1}$ at $u = 0$.

Lemma 4.1 (Blow-up). *Let $A(u) = M_0 + N(u)$ with $M_0V = 0$, $V^\top M_0 = 0$, $W^\top M_0W$ invertible, $\|N(u)\| \leq K|u|^2$, and $V^\top N(u)V = q(u)C' + E(u)$ with C' invertible, $q(u) \geq \lambda|u|^2$ and $\|E(u)\| \leq K|u|^3$. Then near 0, $A(u)$ is invertible and*

$$\|A(u)^{-1} - q(u)^{-1}VC'^{-1}V^\top\| \leq C/|u|.$$

If in addition a directional derivative DA obeys $\|DA\| \leq K|u||v|$ and $\|V^\top DAV - dq \cdot C'\| \leq K|u|^2|v|$, then $\|A^{-1}DA A^{-1} - (dq/q^2)VC'^{-1}V^\top\| \leq C|v|/|u|^2$. On a closed cone where $q > 0$ off 0, $q(u)a^\top A(u)^{-1}b \rightarrow (V^\top a)^\top C'^{-1}(V^\top b)$.

Mechanism. Put $\varepsilon = \sqrt{q(u)}$ and conjugate by the scaled frame $T_\varepsilon = [\varepsilon^{-1}V \mid W]$. Because M_0 kills the strains, $T_\varepsilon^\top AT_\varepsilon = \text{diag}(C', W^\top M_0W) + \Delta$ with $\|\Delta\| \leq K_1|u|$. A Neumann estimate controls its inverse, and $A^{-1} = T_\varepsilon(T_\varepsilon^\top AT_\varepsilon)^{-1}T_\varepsilon^\top$ returns the ε^{-2} strain part. In $A^{-1}DA A^{-1}$ the only term of order $|u|^{-3}$ is the product of two Newton parts, and on strains $V^\top DAV = dq \cdot C'$ up to $O(|u|^2)$, which is exactly the derivative of $q^{-1}VC'^{-1}V^\top$. $\square$

Theorem 4.2 (General sourced response). *Let $\kappa > 1/2$, $s \neq 0$, and let P be a closed cone of covectors with $q > 0$ on $P \setminus \{0\}$. Let $T(u)$ be continuous at 0 and conserved on P , and let $b(u)$ be its repaired source (Theorem 3.2). Then there is $\delta > 0$ such that for $u \in P$ with $0 < |u| < \delta$ the gauge-fixed field $h(u) = -s^{-1}A(u)^{-1}b(u)$ solves the lattice equation (5), every other solution differs from it by a vertex move, and its strain part satisfies*

$$q(u) h_{\text{strain}}(u) \longrightarrow -s^{-1}C'^{-1}\sigma_\kappa(T(0)) \quad (u \rightarrow 0, u \in P \setminus \{0\}),$$

where $\sigma_\kappa(T)$ is the coupling $h \mapsto c_\kappa h_{\mu\nu} T^{\mu\nu}$ in strain coordinates. The static waves and the comoving cone of the vector w of Theorem 3.5 satisfy the hypotheses.

The constant matrix C'^{-1} applied to the coupling is the trace reversal.

Theorem 4.3 (Trace reversal). *For every $\kappa > 1/2$ and every stress tensor T , the strain $C'^{-1}\sigma_\kappa(T)$ is $-2(gTg - \frac{1}{2}\text{tr}(gT)g)$, which is $-2(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$ with indices lowered by g_κ . Hence the response $h = -(sq)^{-1}C'^{-1}\sigma_\kappa(T)$ is*

$$h_{\mu\nu} = \frac{16\pi G}{q(u)} \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right), \quad G = \frac{1}{8\pi s},$$

and for conserved T it solves the long-wavelength symbol equation $s K_\kappa(u)h + \sigma_\kappa(T) = 0$, $K_\kappa = -\frac{1}{4}\sqrt{|\det g|} \mathcal{F}_g$: the response is in de Donder gauge. For a nonzero point mass the coefficient is forced: if its response equals $(c/q)(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$ for some constant c , then $c = 16\pi G$.

Proof. Write $X = \alpha(gTg - \frac{1}{2}\text{tr}(gT)g)$. Then $g^{-1}Xg^{-1} = \alpha(T - \frac{1}{2}\text{tr}(gT)g^{-1})$ and $\text{tr}(g^{-1}X) = -\alpha\text{tr}(gT)$, so for every strain Y

$$\mathcal{G}_g(X, Y) = \alpha \left[\text{tr}(TY) - \frac{1}{2}\text{tr}(gT)\text{tr}(g^{-1}Y) \right] + \frac{1}{2}\alpha\text{tr}(gT)\text{tr}(g^{-1}Y) = \alpha\text{tr}(TY).$$

The coupling is $\sigma_\kappa(T) \cdot Y = c_\kappa\text{tr}(TY)$, so $C'X = \sigma_\kappa(T)$ when $\lambda_\kappa\alpha = c_\kappa$, that is

$$\alpha = \frac{\frac{1}{2}\sqrt{4\kappa - 1}}{-\frac{1}{4}\sqrt{4\kappa - 1}} = -2.$$

Then $-(sq)^{-1}X = (2/(sq))(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T)$ and $2/s = 16\pi G$. By (6) the response solves the gauge-fixed symbol equation; for conserved T the de Donder condition holds on it, so it also solves the equation without the gauge-fixing term. $\square$

Theorem 4.4 (The linearized Einstein equation for every conserved source). *In the setting of Theorem 4.2, the gauge-fixed solution of the lattice equation sourced by the repaired source of $T(u)$ satisfies*

$$q(u)h_{\mu\nu}(u) \longrightarrow 16\pi G \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right), \quad T = T(0), \quad G = \frac{1}{8\pi s},$$

as $u \rightarrow 0$ in $P \setminus \{0\}$, with indices lowered by g_κ .

Proof. Theorem 4.2 gives the limit $-s^{-1}C'^{-1}\sigma_\kappa(T)$, and Theorem 4.3 gives

$$C'^{-1}\sigma_\kappa(T) = -2 \left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T \right).$$

The product of the two factors is $(-s^{-1})(-2) = 2/s = 16\pi G$. $\square$

These are the linearized Einstein equations in de Donder gauge, with the coefficient fixed by the Regge action scale alone and independent of κ .

4.2 A static mass: $\gamma = 1$

For a static point mass of cell stress m , $T = m\hat{t} \otimes \hat{t}$. Then $gTg = m t^b \otimes t^b$ and $\text{tr}(gT) = -m$, so Theorem 4.3 gives, for all vectors e, f ,

$$q(u)h(e, f) = 8\pi G m (2(t^b \cdot e)(t^b \cdot f) + g(e, f)). \quad (7)$$

Three readings follow at once: $q h(\hat{t}, \hat{t}) = 8\pi G m$ (Newton's coefficient); $h(e, e) = h(\hat{t}, \hat{t}) g(e, e)$ for every spatial e , $g(e, \hat{t}) = 0$ (that is $\gamma = 1$); and $h(\hat{t}, e) = 0$ (no mixed component). Every solution of the symbol equation is this de Donder response plus a pure gauge strain $u \otimes \xi + \xi \otimes u$, whose (e, e) component $2(u \cdot e)(\xi \cdot e)$ vanishes when $u \cdot e = 0$. That gives a gauge-invariant form.

Theorem 4.5 ($\gamma = 1$, gauge invariantly). Let $\kappa > 1/2$, $s \neq 0$, let $u \neq 0$ be static, and let h be any solution of $s K_\kappa(u)h + \sigma_\kappa(m \hat{t} \otimes \hat{t}) = 0$. For every vector e with $u \cdot e = 0$ and $g(e, \hat{t}) = 0$,

$$q h(\hat{t}, \hat{t}) = 8\pi G m, \quad q h(e, e) = q h(\hat{t}, \hat{t}) g(e, e), \quad h(\hat{t}, e) = 0.$$

The hypotheses are met by $u = (1, -1, 0, 0)$ and $e = (1, 1, -2, 0)$, for which $g(e, e) = 6$.

The time-time and space-space components of a static mass's field are the same function times the metric. That is the statement light-bending experiments measure, and on the lattice it is a theorem about the Regge operator at every κ , with no parameter adjusted to produce it.

4.3 Moving matter

Theorem 4.6 (The field of a moving mass). Let $\kappa > 1/2$, $s \neq 0$, $M \in \mathbb{R}$, and let w be the moving unit timelike vector of [Theorem 3.5](#). On its comoving cone the gauge-fixed field of the repaired source of $M w \otimes w$ satisfies

$$q(u) h_{\mu\nu}(u) \longrightarrow 16\pi G M \left(w_\mu^b w_\nu^b + \frac{1}{2} g_{\mu\nu} \right), \quad w^b = g_\kappa w.$$

In the mass's own frame the limit gives $q h(w, w) \rightarrow 8\pi G M$, Newton's coefficient.

For any unit timelike w the trace-reversed tensor of $M w \otimes w$ is $M(w^b \otimes w^b + \frac{1}{2} g)$, and at rest, $w = \hat{t}$, it reproduces (7). A uniformly moving mass sources the boosted field.

5 The static resonance

Long-wavelength theorems need small u . The reason is a feature of the static operator away from $u = 0$: it has zero modes that are not vertex moves.

Theorem 5.1 (Resonance at every κ). Let $\kappa > 1/2$ and define $t_* \in (0, \pi]$ by

$$\cos t_* = 1 - \frac{6(2\kappa - 1)}{\kappa(4\kappa - 1)(16\kappa - 5)}.$$

At the static wave $u_* = (-t_*, t_*, 0, 0)$ there is an explicit nonzero vector W in the null space of B_κ that is orthogonal to every vertex move, with time-edge entry $8(4\kappa - 1)^2(4\kappa + 1)(16\kappa - 5) > 0$. Consequently the static operator is not nondegenerate modulo gauge, and for every $m \neq 0$ the time-edge worldline source is conserved and orthogonal to every vertex move at u_* , yet the lattice equation $B_\kappa h = \tau$ has no solution there.

Mechanism. On the static line $z = (v^{-1}, v, 1, 1)$ the operator is $N(\kappa, v)/D(\kappa)$ with $D(\kappa) \neq 0$ and N a Laurent polynomial read from integer polynomials of the cell table. Every row of $v^2 N W$ and every gauge row is an exact polynomial multiple of $Qv^2 - 2Pv + Q$ with $Q = \kappa(4\kappa - 1)(16\kappa - 5)$ and $P = Q - 6(2\kappa - 1)$. That quadratic has the root $v_* = e^{it_*}$ on the unit circle, because $P/Q \in [-1, 1]$; the lower bound is $64\kappa^3 - 36\kappa^2 - \kappa + 3 > 0$. At v_* the vector W is a null vector orthogonal to the gauge columns. It moves the time edge, so the worldline source is not orthogonal to the null space, and the Fredholm alternative of [Theorem 2.2](#) rules out a solution. $\square$

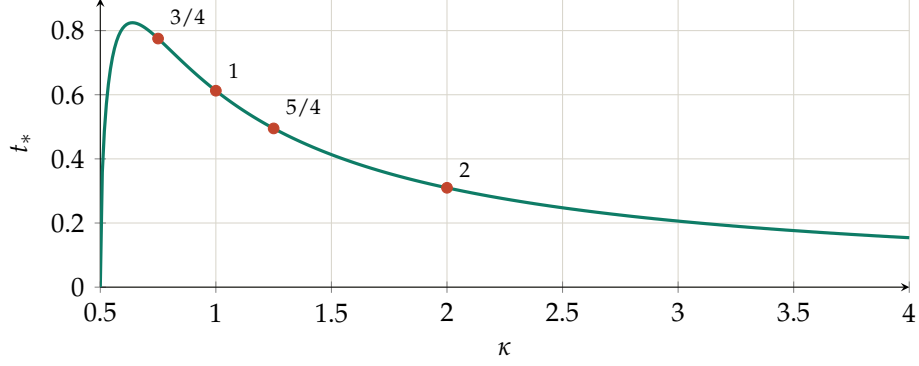


Figure 2. The resonance wave $u_* = (-t_*, t_*, 0, 0)$ as a function of κ , from the closed form of [Theorem 5.1](#). Red points mark the couplings where a second, independent determinant-sign argument also locates it. The curve tends to 0 at both ends of the range: $t_*^2 \approx 8(2\kappa - 1)$ as $\kappa \rightarrow 1/2^+$ and $t_*^2 \approx 3/(8\kappa^2)$ as $\kappa \rightarrow \infty$. These asymptotics are read off the closed form.

At $\kappa = 3/4, 1$ and 2 (spacelike, null and timelike axis edges) the same conclusions also follow by an independent route: exact rational LDL^* factorizations at the two ends of a narrow window on the path $v(s) = (1 + is)/(1 - is)$ give opposite determinant signs, and a perturbation bound keeps the block without the time edge invertible across the window. At $\kappa = 5/4$ the window is $s \in [1579/6250, 5053/20000]$, around $s_* = \tan(t_*/2) \approx 0.2526$.

Reading. What is proved is a statement at one Bloch wave: at u_* the null space of B_κ contains a mode that is not a vertex move, and the worldline source has no solution at that wave. No coupling is free of it. This does not settle whether a point source has a position-space solution, and on a finite torus u_* is generically not an allowed wave. The far-field theorems below avoid the question by using exactly band-limited profiles, whose spectrum is supported in a ball $|k| < \delta$ around $k = 0$; such profiles extend over the whole lattice. A body of compact support has a spectrum that is generically nonzero at u_* , and its far field is an open question. The resonance sits at a finite wave vector, and its position approaches $u = 0$ only at the two ends of the range ([Fig. 2](#)).

6 Newton and the far field

6.1 The band-limited static field

The static slice is $\mathbb{Z}^3$; a static wave is $u = (k_1, k_2, k_3, -k_1 - k_2 - k_3)$ for a Bloch vector $k \in [-\pi, \pi]^3$, and $q = |ek|^2 = k_1^2 + k_2^2 + k_3^2 + (k_1 + k_2 + k_3)^2$ with $|\det e| = 2$. Proper distance on the slice is $r_g(X) = |(e^{-1})^\top X|$.

A particle of proper mass M whose worldline runs along time edges has action $-M\tau$ per tick, with proper time $\tau = \sqrt{16\kappa - 4}$. Varying the metric $g \mapsto g + \varepsilon H$ changes it by $MH(d, d)/(2\tau)$ at first order, $d = (1, 1, 1, 1)$, and on constant strains this equals the coupling of the cell stress $m = 2M$. The factor is $\tau/\sqrt{|\det g|} = 2$: one slice cell has proper volume $1/2$, so a mass M per site is a density $2M$.

Take a profile $\hat{\rho}$ on $\mathbb{R}^3$, continuous with compact support, and a scale L . The lattice density $\rho_L(y) = (2\pi)^{-3} \operatorname{Re} \int_{[-\pi, \pi]^3} \hat{\rho}(Lk) e^{-ik \cdot y} dk$ describes a body about L spacings across. By the

standard Fourier inversion on $\mathbb{Z}^3$ its total mass $\sum_y \rho_L(y)$ is $\hat{\rho}(0)$; this is the physical reading of $\hat{\rho}(0)$, and the body has total proper mass $M_{\text{tot}} = M\hat{\rho}(0)$. Its source puts $\rho_L(y) c_\kappa 2M/(16\kappa - 4)$ on the time edge at y . The band-limited field is the Bloch synthesis of $\hat{\rho}(Lk)(2\pi)^{-3}$ times the gauge-fixed solution at k , kept on the ball $0 < |k| < \delta$; its h_{00} is the time-edge value divided by $16\kappa - 4$. For every $\kappa > 1/2$ and $s \neq 0$ there is a band radius $\delta > 0$ on which the Bloch solution exists with every component bounded by a multiple of $|k|^{-2}$, and whenever the spectrum $\hat{\rho}(L \cdot)$ lies in that ball, the field solves the static lattice field equation of the Regge action in position space: for every finitely supported static test field Φ , $s B_{\text{st}}(H, \Phi) + \sum_y \sum_j \rho_L(y) \tau_j \Phi_y(j) = 0$. The band radius is existential: some $\delta > 0$ works, and no explicit relation between δ and the resonance wave t_* is proved. The band-limited field is one solution of the position-space equation; uniqueness is not proved, since homogeneous solutions, including the resonant mode, are not excluded.

6.2 Newton's law as a scaling limit

Theorem 6.1 (Scaling-limit Newton, every κ). *Let $\kappa > 1/2$, $s \neq 0$, $M \in \mathbb{R}$, and let $\hat{\rho}$ be C^1 with compact support. There is $\delta > 0$ such that the band-limited field $h^{(L)}$ solves the static lattice field equation for all large L , and*

$$r_g(X) \cdot L \cdot h_{00}^{(L)}(\text{round}(LX)) \longrightarrow 2G M \hat{\rho}(0), \quad G = \frac{1}{8\pi s},$$

in the iterated sense: for every $\eta > 0$ there is R_0 such that every X with $|X| \geq R_0$ has an L_0 with error below η for all $L \geq L_0$. The potential $\Phi = -h_{00}/2$ has the limit $-GM\hat{\rho}(0)$. If $M \neq 0$, $\hat{\rho}(0) \neq 0$ and some G' gives the same double limit with $2G'M\hat{\rho}(0)$, then $G' = 1/(8\pi s)$. The coefficient does not depend on κ .

Mechanism. Substitute $k = p/L$. The response obeys $q R \rightarrow c$ with $c = 8\pi G \cdot 2M$ ([Proposition 3.6](#) with $m = 2M$), so $L h^{(L)}$ tends to c times the continuum potential

$$\Psi(X) = (2\pi)^{-3} \int \cos\langle X, p \rangle \frac{\hat{\rho}(p)}{q(p)} dp$$

by dominated convergence, since $1/|p|^2$ is integrable near 0 in three dimensions. Then $r_g(X)\Psi(X) \rightarrow \hat{\rho}(0)/(4\pi|\det e|)$, and $c/(4\pi \cdot 2) = 2GM$. $\square$

6.3 Every component of the far field

Read the strain component $h(e, f)$ of the band field by averaging the edge of class d at y with the edge of class d whose midpoint is the reflection through y . On a constant strain this centered readout returns $h(e, f)$ exactly, and on the time edge it is the h_{00} readout.

Theorem 6.2 (Spatial components, $\gamma = 1$ in the far field). *In the setting of [Theorem 6.1](#), in de Donder gauge, for all vectors e, f the centered readout of the band field has the double limit*

$$r_g \cdot L \cdot h(e, f) \longrightarrow 2GM\hat{\rho}(0) (2(t^\flat \cdot e)(t^\flat \cdot f) + g(e, f)).$$

So $h_{00} \rightarrow 2GM/r$, $h(e, e) \rightarrow (2GM/r) g(e, e)$ for every spatial e , and $h(\hat{t}, e) \rightarrow 0$.

The symmetric pairing of edges is what makes this work: the centered readout of the band field is exactly the scaling-limit field of a cosine-weighted response, and the cosines tend to 1 at $k = 0$. A gauge-invariant version in position space would need a lattice readout that annihilates the exact gauge columns $4d_a \sin(d \cdot u/2)$; the condition $u \cdot e = 0$ of [Theorem 4.5](#) does so only at leading order.

6.4 Fixed lattice spacing

The scaling limit grows the body with the reading distance. The physical question keeps both fixed: one source profile, one lattice, read farther and farther away. The profile is band-limited, so it is smooth and spread over the lattice, not compactly supported.

Theorem 6.3 (Newton at fixed spacing, pointwise). *Let $\kappa > 1/2$, $s \neq 0$, $M \in \mathbb{R}$. There is $\delta > 0$ such that for every C^1 profile $\hat{\rho}$ supported inside the ball $|k| < \delta$, the band-limited field at scale $L = 1$ solves the static lattice field equation and*

$$r_g(x) h_{00}(x) \longrightarrow 2GM\hat{\rho}(0) \quad \text{as the site } x \in \mathbb{Z}^3 \text{ leaves every finite set,}$$

so $\Phi(x) = -h_{00}(x)/2$ has $r_g(x)\Phi(x) \rightarrow -GM\hat{\rho}(0)$.

Mechanism. Write the time-time response as $R = c/q + R_1$. [Lemma 4.1](#) with the explicit derivatives of the cosine table gives $|R_1(k)| \leq C/|k|$ and $\|\nabla R_1(k)\| \leq C/|k|^2$ near 0. The Newton part $c\hat{\rho}/q$ is the continuum potential read at the lattice site. The remainder $g = R_1\hat{\rho}$ and its axis derivatives are bounded by multiples of $|k|^{-2}$, integrable in three dimensions. Integrating by parts along the axis of the largest coordinate of x turns $x_j \int \cos\langle x, k \rangle g$ into a sine moment of $\partial_j g$, which tends to 0 by the Riemann–Lebesgue lemma. Only the Newton part reaches the $1/r$ tail, and lattice anisotropy enters only through e . $\square$

A far field is read by a detector of finite size, which averages the field against a probe. Extend h_{00} off the lattice by its own Fourier integral,

$$h_{00}^{\text{ext}}(y) = (2\pi)^{-3} \int_{[-\pi, \pi]^3} \cos\langle y, k \rangle R(k) \hat{\rho}(Lk) dk,$$

which equals the lattice field at every site. A probe is a function φ on $\mathbb{R}^3$ that is continuous, bounded and integrable, with integrable cosine transform; the Gaussian and every smooth compactly supported function are probes.

Lemma 6.4 (Coulomb pairing). *For a linear equivalence e of $\mathbb{R}^3$, $q(k) = |ek|^2$ and every probe φ with cosine transform $\hat{\varphi}$,*

$$(2\pi)^{-3} \int \frac{\hat{\varphi}(k)}{q(k)} dk = \int \varphi(X) \frac{1}{4\pi|\det e| r_g(X)} dX.$$

Mechanism. Regularize $1/q$ by $e^{-q/t^2}/q$. Its cosine transform is $t\Phi_e(tX)$, and the Coulomb coefficient $r_g(Y)\Phi_e(Y) \rightarrow 2\pi^2/|\det e|$ gives the pointwise limit $2\pi^2/(|\det e| r_g(X))$. Dominated convergence on both sides of Fubini's theorem as $t \rightarrow \infty$ identifies the two integrals; $(2\pi)^{-3} \cdot 2\pi^2 = 1/(4\pi)$. $\square$

Theorem 6.5 (Weak far field at fixed spacing). *Let $\kappa > 1/2$, $s \neq 0$, $M \in \mathbb{R}$, and $\hat{\rho}$ continuous with compact support. There is $\delta > 0$ such that $\hat{\rho}(L \cdot)$ lies in the ball $|k| < \delta$ for all large L , and for every such L and every probe φ ,*

$$r^{-2} \int_{\mathbb{R}^3} h_{00}^{\text{ext}}(y) \varphi(y/r) dy \longrightarrow 2GM\hat{\rho}(0) \int \frac{\varphi(X)}{r_g(X)} dX \quad (r \rightarrow \infty).$$

The source and the spacing are fixed; only the probe is dilated. After $k = p/r$ the probe's transform concentrates the integral at $p/r \rightarrow 0$, where $q(p/r)R(p/r) \rightarrow c$, and dominated convergence plus [Lemma 6.4](#) gives the limit. The resonance surface is never reached.

Theorem 6.6 (Gauss's law at fixed spacing). *Let Δ_e be the constant-coefficient operator on $\mathbb{R}^3$ with symbol $-q(k)$. In the setting of [Theorem 6.5](#), for every smooth compactly supported φ ,*

$$-\frac{1}{2|\det e|} r^{-2} \int h_{00}^{\text{ext}}(y) (\Delta_e \varphi)(y/r) dy \longrightarrow 4\pi G M\hat{\rho}(0) \varphi(0) \quad (r \rightarrow \infty).$$

Reading. With $\Phi = -h_{00}/2$ and proper volume $d\text{vol} = |\det e|^{-1} d^3y$, whose Laplace–Beltrami operator is Δ_e , the left side is $\int \Phi \Delta_e[\varphi(\cdot/r)] d\text{vol}$; two integrations by parts make it $\int (\nabla^2 \Phi) \varphi(\cdot/r) d\text{vol}$, a cutoff that tends to $\varphi(0)$ everywhere. So the theorem is $\int \nabla^2 \Phi d\text{vol} = 4\pi G M_{\text{tot}}$: the total flux of the gravitational field, read from the far field of the lattice operator at fixed spacing. The integration by parts is the reading; the proved object is the pairing.

7 Newton's constant

Every coefficient above is fixed by one number, the Regge action scale s : $G = 1/(8\pi s)$. [Theorem 6.1](#) shows that no other value matches the far field, and [Theorem 4.3](#) that no other constant multiplies the trace-reversed stress. Attraction needs $s > 0$, and units are the lattice units of the normalization $n(1 - \kappa n)$.

In Recognition Science the action scale is the posting scale, $s = \hbar \ln 2 / (2\pi \lambda_{\text{rec}}^2)$, the cost of one bit per recognition area. With it, $1/(8\pi s) = \lambda_{\text{rec}}^2 c^3 / (4\hbar \ln 2)$, which is the Recognition Science value of G . The two expressions are built from the same $\hbar$ and λ_{rec} , so their agreement checks the identification $s = \text{posting scale}$ for consistency; it does not determine G by itself. The numerical value of G follows from the native values of $\hbar$ and λ_{rec} . Deriving the action scale from the lattice dynamics, without identifying it with the posting scale, is part of the remaining step named in [Section 2](#).

8 Scope and what remains

[Figure 3](#) collects the chain. Its scope:

- *Linearized.* Every statement is about the second variation of the Regge action at the flat lattice. The nonlinear sourced equations, including the gravitational self-interaction that holds a star's pressure, are not addressed.
- *Sources.* Coupling holds for every conserved stress family ([Theorems 3.2](#), [4.2](#) and [4.4](#)). The far-field theorems are for static mass; moving matter is covered at the level of the

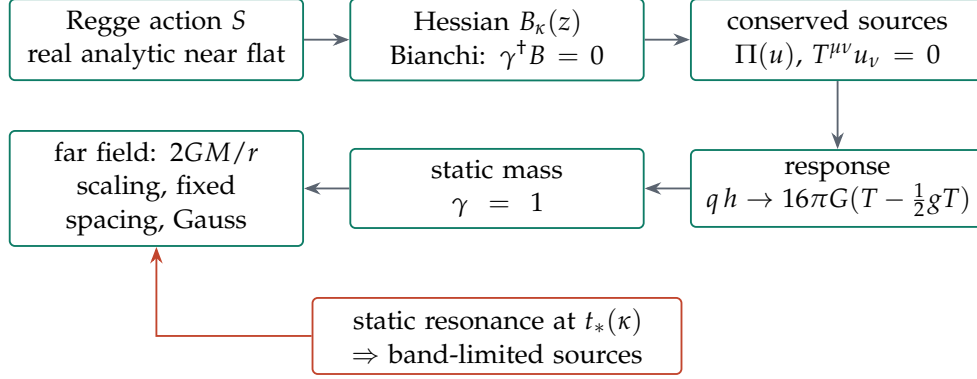


Figure 3. The chain of results. Each arrow is one of the theorems above; the resonance is the reason every far-field statement uses band-limited sources.

long-wavelength field on its comoving cone, for the explicit unit vector of [Theorem 3.5](#) ([Theorem 4.6](#)).

- *Band-limited.* The far-field theorems are proved for exactly band-limited profiles, whose spectrum lies in a ball $|k| < \delta$ of existential radius and which have infinite spatial extent. Compactly supported bodies, whose spectrum is generically nonzero at the resonance wave of [Theorem 5.1](#), are open, and so is the unsmeared point source.
- *Uniqueness.* The band-limited field solves the position-space equation; uniqueness is not proved, because homogeneous solutions, including the resonant mode, are not excluded.
- *Coupling.* For general conserved T the lattice source is a wave-dependent, nonlocal projection, constructed in wave-vector space. Only the static point mass on the time edge is local and derived from an action.
- *Gauge.* h_{00} is gauge invariant on static waves. The spatial far field of [Theorem 6.2](#) is a de Donder statement; its gauge-invariant form is the symbol-level [Theorem 4.5](#).
- *Fixed spacing.* The pointwise fixed-spacing theorem is for h_{00} . The weak field and Gauss's law pair the Fourier extension of h_{00} with probes over $\mathbb{R}^3$.

The open questions are concrete. The far field of a compactly supported body, and uniqueness of position-space solutions, need control of the resonant mode. A position-space, gauge-invariant far field for h_{ij} needs a transverse lattice readout that kills the exact gauge columns. The weak field as a lattice sum $\sum_{y \in \mathbb{Z}^3} h_{00}(y) \varphi(y/r)$ needs Poisson summation on $\mathbb{Z}^3$. Convergence and stability of lattice solutions beyond the band, the light cone itself, and the nonlinear theory remain. Underneath all of them is the derivation of the lattice: the triangulation, the length family, the deformability of lengths and the time direction.

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