

Non-Local Recognition Phases and the Massless-Graviton Target

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Abstract

A phase can depend on an entire record history and still have exactly the one-loop response of a local action. At zero deficit, a twice differentiable phase that vanishes identically there enters the Gaussian operator only through its deficit slope, the geometric variation of that slope, and its even deficit kernel. An odd phase with the Regge slope therefore has the native one-loop remainder. The specified face-holonomy phase is exactly the native phase under its posting and face-count readings. Blocking supplies a different non-local construction. At fixed fine-scale damping, edge-record blocking collapses the normalized inverse propagator, but that order also excludes hypercube block averages and does not decide the physical route. With damping fixed in coarse units, a proved lower bound at identified $\kappa = 1$ distinguishes the two: an edge-record zone average grows at least as 2^k, while hypercube averages remain bounded. This result is an averaged obstruction, not the still-open pointwise flow statement. For the specified dyadic Kuhn refinement, every nonzero equal-piece additive linear face reading fails gauge covariance; the oriented sum factors through edge records. The phase and coarse-fixed blocking proofs are complete with independent verification pending. The remaining target is a derived action with the Regge classical limit and a vanishing ordered transverse-traceless remainder; exact vertex symmetry is a sufficient structural route, also motivated by a separate classical second-order obstruction.

1 The question is the Gaussian action

The massless-graviton question asks whether the one-loop effective action of constant strains contains a term that a covariant volume response cannot produce. A non-local phase could remove that term, but non-local dependence alone does not change the Gaussian action. We determine which part of a phase can reach the one-loop response, then examine two concrete constructions: holonomy phases and blocking of edge records. The distinction between fixed fine-scale damping and fixed coarse-scale damping is essential.

The carrier is the four-dimensional Kuhn lattice, with fifteen edge classes $d \in \{0,1\}^4 \setminus \{0\}$. A configuration ℓ assigns squared lengths to edges. On a flat lattice the metric is

$$g_\kappa = I - \kappa \mathbf{1}\mathbf{1}^{\mathsf{T}}, \qquad g_\kappa(d,d) = |d|(1 - \kappa|d|).$$

For $\kappa > 1/2$ every triangle is timelike and the Lorentzian Regge action $S_{\mathrm{R}} = \sum_h A_h \delta_h$ is regular near the flat field. Here A_h is the positive timelike area and δ_h the deficit. The interpretation of the interval as the posting order selects $\kappa = 1$ only with that interval reading supplied (Identification). The foundational recognition premises alone do not select it [1, 2].

For a nonzero strained determinant $D(B)$ define, whenever the displayed derivatives exist,

$$\mathscr{R}_H[D] = \frac{i}{2}\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}\log D(tH)\Big|_0 + \frac{\tau_H}{4}\frac{\mathrm{d}}{\mathrm{d}t}\log D(tg)\Big|_0\right], \qquad (1)$$
$$\tau_H = \mathrm{tr}(g^{-1}Hg^{-1}H), \qquad \mathrm{tr}(g^{-1}H) = 0.$$

The logarithmic derivatives are local, so no global branch of the logarithm is needed. The per-site remainder sums this expression over waves and divides by the number of sites. Its ordered massless condition takes the torus side to infinity at fixed positive Feynman damping, then takes the damping to zero. Equality of the Hessians along the strained flat family gives equality of these remainders with the same gauge, regulator and weight.

All one-loop masslessness statements below concern the specified Gaussian determinant model: the native phase normalization and fixed gauge/regulator conventions of [1], rather than a completed quantum theory with a derived ghost measure (Model). Its native transverse-traceless probes satisfy $H^{\mathsf{T}} = H$, $H_{aa} = 0$ for each posting direction, and $\mathrm{tr}(g^{-1}H) = 0$. The zero-momentum remainder is a diagnostic of that model. Interpreting it as a propagating pole mass or width requires the additional record-time dynamics. We retain that distinction throughout.

2 Only a slope map and an even kernel survive

Let $d(x)$ be the full vector of deficits and $y(x)$ further geometric data; the latter may contain the entire configuration. Write the real phase as

$$S(x) = \Phi(d(x), y(x)), \qquad \Phi(0, z) = 0,$$

and define the covector W and symmetric bilinear form K by

$$W(z) = \partial_d \Phi(0, z), \qquad K(z) = \partial_d^2 \Phi(0, z).$$

The components of W can depend on distant geometry. The components of K can couple different hinges. The word non-local refers to these dependencies, not merely to a higher-order term that mentions a distant hinge.

Theorem 2.1 (The zero-deficit Hessian)**.** Proof complete, independent verification pending. *Let $d(x_0) = 0$, let d, y be C^2 at x_0, and let Φ be C^2 at $(0, y(x_0))$, with $\Phi(0, z) = 0$ for every z. Put $z_0 = y(x_0)$. For variations u, v,*

$$\begin{aligned} D^2 S(x_0)[u, v] = {} & W(z_0)[D^2 d(x_0)[u, v]] \\ & + K(z_0)[Dd(x_0)u, Dd(x_0)v] \\ & + DW(z_0)[Dy(x_0)v][Dd(x_0)u] \\ & + DW(z_0)[Dy(x_0)u][Dd(x_0)v]. \end{aligned} \qquad (2)$$

Two such phases with equal slope maps on a neighborhood of z_0 and equal even kernels at z_0 have equal Hessians there.

Proof. Differentiate the composite twice. The pure geometric first and second derivatives of Φ vanish at zero deficit because $\Phi(0, z)$ is identically zero. The remaining four terms are the pure deficit second derivative, the slope applied to the second variation of the deficits, and the two mixed derivatives. These are exactly (2). Equality of the slope maps on a neighborhood also gives equality of their derivatives; equality only at a single point would not suffice. □

Every sufficiently small constant strain of the flat lattice still has zero deficits. Consequently, equality of these data near the flat field on every torus gives equality of Hessians along the strained flat family, hence equality of the one-loop remainders. A common, separately specified weight may be retained; a change of weight is a different Gaussian operator.

Theorem 2.2 (Odd phases with the native slope)**.** Proof complete, independent verification pending. *On each torus of side $N \geq 3$, suppose Φ is odd in the entire deficit vector, vanishes at zero deficit, is C^2 at zero deficit near the flat $\kappa = 1$ field, and has the Regge slope map on a neighborhood of that field. Then its ordered massless condition on every native transverse-traceless probe holds if and only if the native phase's condition holds.*

Proof. Twice differentiating $\Phi(-d, z) = -\Phi(d, z)$ at $d = 0$ gives $K(z) = 0$. The native phase is also odd, with zero even kernel. The common slope germ and (2) give the same Hessian at each strained flat field. Their determinants, remainders and ordered conditions therefore agree. □

The native all-probe ordered condition fails at $\kappa = 1$ (theorem; *Proof complete, independent verification pending.*). Combining that obstruction with Theorem 2.2 excludes an odd phase with the native slope as a repair of this Gaussian model. This statement concerns the zero-momentum remainder; it assigns no propagating pole mass.

2.1 A precise two-probe target

The canonical phase of supplied data is

$$\Phi_{W,K}(d, z) = W(z)[d] + \tfrac{1}{2}K(z)[d, d].$$

Its slope is W and its even kernel is K when K is symmetric. Suppose W and K are each C^2 near the flat geometry on every torus. Every phase meeting the preceding C^2, zero-deficit and germ conditions then has the same ordered response as this canonical phase (theorem; *Proof complete, independent verification pending.*).

One useful test asks for zero ordered remainder on the native probes f_E and f_{T_2}, one in each of the two irreducible strain sectors. This two-probe condition is a defined target. The all-probe massless condition implies it; we do not identify the converse with a proved result here.

For a C^3 local hinge charge $C(A, \delta)$ with $C(A, 0) = 0$, the data reduce to

$$W(z)[\alpha] = \hbar^{-1} \sum_h \sigma(A_h(z))\alpha_h, \qquad K(z)[\alpha, \beta] = \hbar^{-1} \sum_h e(A_h(z))\alpha_h\beta_h,$$

where σ is the charge's deficit slope and e its even deficit curvature. For a positive integer refinement and a C^3 local even-charge profile, the Regge slope reduces the general two-probe

condition exactly to the local even-charge condition. If the profile also vanishes at the four flat hinge areas, the corresponding corner condition is an additional compatibility target in that local reduction; imposing it does not prove the full ordered condition (theorems; *Proof complete, independent verification pending.*).

3 What the specified holonomies supply

The native bridge loop carries phase $\kappa_B \delta_h$ at a hinge. Summing over its channels cancels the channel count from the phase; that count still affects the modulus. Read the loop phase from postings, and count one cycle per elementary face so that the hinge multiplicity is its area in recognition units (Identification). For the native bridge construction these readings give

$$S_{\mathrm{hol}}(\ell) = a \sum_h A_h(\ell)\delta_h(\ell) = a S_{\mathrm{R}}(\ell),$$

on every geometry of the specified construction, with the native phase scale $a = s/\hbar$ (theorem; *Proof complete, independent verification pending.*). The posting model supplies its normalization. The equality is stronger than a shared classical limit: the phase itself is the native phase. Its Hessian, cubic vertex and one-loop massless condition are consequently native ones. This candidate supplies no independent repair parameter.

Averaging deficit-linear phases over record histories is a different operation. For finitely many real slopes a_j and weights $\mu_j \geq 0$ with $\sum_j \mu_j = 1$, put

$$Z(t) = \sum_j \mu_j e^{ita_j}, \quad m = \sum_j \mu_j a_j, \quad Z_c(t) = e^{-itm} Z(t).$$

Then $Z_c(0) = 1$, $Z_c'(0) = 0$ and

$$Z_c''(0) = -\sum_j \mu_j (a_j - m)^2 \leq 0$$

(theorem; *Proof complete, independent verification pending.*). Through quadratic order the centered average supplies real attenuation, not a real even phase kernel. Equal-weight reversal pairing makes the centered factor real at every t; arbitrary histories do not ensure this.

For a C^3 function F of each signed single-hinge deficit, with $F(0) = 0$ and $F'(0) > 0$, the ordered masslessness condition of $\sum_h A_h F(\delta_h)/\hbar$ reduces to that of the local area-proportional even-curvature family under the specified positive-integer refinement normalization (theorem; *Proof complete, independent verification pending.*). This phase does not generate a coupling between distinct deficits. The currently inspected source constructions therefore do not provide the non-local real data needed for a repair (Inference). This is a conclusion about those constructions, not a theorem excluding every possible record-history functional. The remaining phase question is whether the ledger derives a genuinely different slope germ or non-local even kernel, with its coefficients fixed.

4 Blocking edge records: the two damping limits

The record map adds the records of the two fine halves of each coarse edge. Reading those edge counts as physical records is an identification; converting them to normalized squared-length

perturbations gives the average of the two halves. At a coarse wave θ, one blocking step collects the sixteen fine waves

$$\phi_\eta = \theta/2 + \pi\eta, \quad \eta \in \{0,1\}^4, \qquad w_d(\phi) = \tfrac{1}{2}(1 + e^{i\phi\cdot d}).$$

The normalization that retains a two-derivative long-wave pole is $\lambda = 1/4$, a specified scaling reading. With the Fourier and field rescalings included, the propagator recursion is

$$(\mathscr{B}_\lambda G)_{de}(\theta) = \lambda \sum_\eta w_d(\phi_\eta)\overline{w_e(\phi_\eta)}G_{de}(\phi_\eta). \tag{3}$$

The exact edge-record sum rule is $\sum_\eta |w_d(\phi_\eta)|^2 = 8$. Averaging along m coordinate directions instead gives fiber mass $\mu_m = 16/2^m$, and $\lambda\mu_m = 2^{2-m}$ at $\lambda = 1/4$ (theorems; *Independently verified, with the scope stated below.*). The Gaussian push-forward and the physical normalization in (3) are the stated model reading, not an extra derivation of the carrier.

For this section $A_\kappa(p)$ is the native Hermitian symbol on the eleven non-posting channels at zero strain, and

$$G_\varepsilon(p) = (A_\kappa(p) + i\varepsilon I)^{-1}, \qquad \varepsilon > 0.$$

The damping is a regulator model. We consider its two different placements in the blocking limit.

4.1 Damping fixed on the finest lattice

Theorem 4.1 (Fixed fine-scale damping)**.** Independently verified, with the scope stated below. *For fixed $\kappa > 1/2$ and $\varepsilon > 0$, edge-record blocking at $\lambda = 1/4$ obeys*

$$-\operatorname{Im}\big(y^* G_k(\theta) y\big) \geq 2^k c_\varepsilon \|y\|^2$$

for some $c_\varepsilon > 0$, uniformly in the wave θ and vector y. The corresponding blocked inverse propagator tends uniformly to zero. There is no fixed point among propagators with a uniform strictly positive absorptive quadratic-form bound.

Proof. Hermiticity and the uniform zone bound on the native symbol give $-\operatorname{Im}(y^* G_\varepsilon y) \geq c_\varepsilon\|y\|^2$ at fixed damping. The sum rule in (3) multiplies that quadratic-form bound by $8\lambda = 2$ at each step. The resulting matrix coercivity bounds its inverse by a constant times 2^{-k} and prevents a fixed point in the class. □

This order does not distinguish edge records from ordinary block averages. For hypercube weights the fiber mass is 1, and the blocked diagonal is bounded above by $4^{-k}/\varepsilon$; the damped native propagator again cannot be a fixed point (theorem; *Independently verified, with the scope stated below.*). Fixed fine-scale damping grows like a mass in coarse units. Taking $k \to \infty$ first and then $\varepsilon \to 0$ preserves the zero inverse-propagator limit, but it supplies no interchange of those limits. This result alone does not close the blocked route to a physical action.

4.2 Damping fixed in coarse units

At depth k, put $\varepsilon_k = \varepsilon/4^k$ on the finest lattice. Define

$$G^{\mathrm{edge}}_{k,\varepsilon} = \mathscr{B}^{\,k}_{1/4} G_{\varepsilon_k}, \qquad \langle F\rangle_N = N^{-4} \sum_{\theta \in (2\pi/N)(\mathbb{Z}/N)^4} F(\theta).$$

The side limit is taken first at each fixed k.

Theorem 4.2 (An averaged obstruction at $\kappa = 1$)**.** Proof complete, independent verification pending. *At identified $\kappa = 1$, for every $\varepsilon > 0$, the edge class $d = (0,1,1,1)$ has a constant $c > 0$, independent of k, such that*

$$-\operatorname{Im}\langle G^{\mathrm{edge}}_{k,\varepsilon;dd}\rangle_N \geq c2^k \tag{4}$$

for every k and every sufficiently large N. The threshold in N may depend on k. In the same damping order the hypercube-blocked diagonal satisfies

$$|G^{\mathrm{cube}}_{k,\varepsilon;dd}(\theta)| \leq 1/\varepsilon \tag{5}$$

at every k, every wave and every non-posting class, for every $\kappa > 1/2$.

Proof. The hyperplane $p \cdot d \in 2\pi\mathbb{Z}$ passes through the axis corner $q_* = \pi e_0$. Here is the native Schur reduction. Let $B(q)$ be the full fifteen-class unstrained Hessian at $\kappa = 1$, and $A(q)$ its non-posting restriction. Index an eleven-dimensional slice by the three axes $a = 1, 2, 3$ and the eight residual non-posting classes $D \notin \{e_0 + e_1, e_0 + e_2, e_0 + e_3\}$. Put $z_a = e^{iq_a}$. The columns of T are $-2e_{e_a}$ and the eight residual coordinate vectors; those of Q are the restricted vertex-move vectors

$$\gamma_a(D;q) = 2D_a(e^{iq\cdot D} - 1)$$

and those same residual coordinate vectors. Set $\mathsf{N} = T^*BT$ and $D_q = \operatorname{diag}(z_1 - 1, z_2 - 1, z_3 - 1, 1, \dots, 1)$. The gauge identity gives $Q^*AQ = D_q^*\mathsf{N}D_q$. In slice coordinates define

$$L_a = -\tfrac{1}{2}, \qquad L_D = \tfrac{1}{2}(1 - D_0)\prod_{a=1}^{3}(1 + D_a).$$

Thus $L_{0111} = 4$. Let C include every slice coordinate except axis 1, and put

$$W = C^*\mathsf{N}C, \qquad s = L^*\mathsf{N}L - L^*\mathsf{N}CW^{-1}C^*\mathsf{N}L.$$

The complementary block W is invertible near q_*. Twice differentiating this Schur complement using the native Hessian gives

$$s(q_*) = 0, \qquad \mathrm{d}s(q_*) = 0, \qquad \mathrm{d}^2 s(q_*)[h,h] = \frac{\sqrt{3}}{20} h^{\mathsf{T}} K h, \quad K = \begin{pmatrix} -2 & 9 & 9 & 9 \\ 9 & 52 & -3 & -3 \\ 9 & -3 & 52 & -3 \\ 9 & -3 & -3 & 52 \end{pmatrix}.$$

The quadratic form takes values 330 and -120 on $(0,1,1,-2)$ and $(15,1,1,-2)$, both tangent to the hyperplane. These local data give line segments with opposite endpoint signs and a uniform Lipschitz bound.

The chosen record also has a nonzero overlap with this mode. The Schur test vector is

$$n = L - CW^{-1}C^{*}\mathsf{N}L, \qquad u = QD_q^{-1}n.$$

Choose fixed $\tau, r, \theta_0 > 0$ with $\tau \le 1/20$, $r \le \tau/8$ and $\theta_0 \le \min(r, \tau/60)$. The local tube consists of

$$q(t, w) = (\pi + t, \tau + w_1, \tau + w_2, -2\tau + w_3), \qquad 0 \le t \le 15\tau,$$

where $|w_1|, |w_2| \le r$, $|w_3| \le 3r$ and $|w_1 + w_2 + w_3| \le \theta_0$. The constants can be chosen so that W is invertible, Q, D_q are regular, $\max_x |n_x - L_x| \le 1/8$, the endpoint signs of s persist, and its Lipschitz bound is uniform. On this tube,

$$\|Au\|^2 \le 2|s|^2, \qquad |u_{0111}| \ge 2, \qquad \|u\| \le K_0 = 1400/\tau.$$

For every Hermitian A and $\varepsilon' > 0$, Cauchy–Schwarz applied to $x = (A + i\varepsilon' I)^{-1}e_d$, with $-\operatorname{Im} G_{\varepsilon';dd} = \varepsilon'\|x\|^2$, gives

$$\varepsilon'|u_d|^2 \le (-\operatorname{Im} G_{\varepsilon';dd})\big(\|Au\|^2 + \varepsilon'^2\|u\|^2\big).$$

Hence $|s| \le \varepsilon' K_0/2$ gives $-\operatorname{Im} G_{\varepsilon';dd} \ge 2/(\varepsilon' K_0^2)$. For damping ε_k, a segment of length proportional to ε_k on each sign-changing line satisfies that bound. Grid counting on tori of side $2^k N$, with N sufficiently large at each depth, therefore gives

$$\Big\langle \mathbf{1}_{\{|p\cdot d|_{2\pi} \le \pi/2^k\}}(-\operatorname{Im} G_{\varepsilon_k;dd})\Big\rangle_{2^k N} \ge c_0 2^{-k}.$$

Here $|\cdot|_{2\pi}$ is distance to $2\pi\mathbb{Z}$, and the positive constant is independent of k. The iterated record weights form a Fejér kernel. Its lower bound on this slab multiplies the displayed estimate by $4^k(4/\pi^2)$, giving (4). For hypercubes, the fiber mass is 1 and $\|G_{\varepsilon_k}\| \le 1/\varepsilon_k$; iteration of the diagonal bound gives $4^{-k}/\varepsilon_k = 1/\varepsilon$, proving (5). □

This is a discriminating obstruction to bounded zone-averaged edge-record fluctuations in the stated normalization. It is a lower bound, not an asserted asymptotic equality. It is not a pointwise divergence theorem, not uniform collapse of the blocked inverse propagator, and not a proof against every non-local perfect action. The pointwise coarse-fixed flow, other values of κ for the native Lorentzian symbol, and the full physical action remain open. Any claim of fixed-point exclusion must specify a convergence class in which the divergent zone average is incompatible with the proposed limit.

Exact determinant blocking also retains the determinant of the integrated-out variables. Where each fine determinant has the required strain regularity and is nonzero, regrouping the sixteen fine waves multiplies the remainder per coarse site by 16 (theorem; *Independently verified, with the scope stated below.*). The regularity premise is retained. Dropping the interior determinant would change that statement and the one-loop action being tested.

5 Face records and the finite quadratic construction

The counting factor 2^{2-m} makes two-dimensional averaging marginal. This observation does not establish a face-record repair. One must first show that a face reading defines a map on gauge-equivalent configurations.

Theorem 5.1 (The specified linear face readings)**.** Proof complete, independent verification pending. *Split a coarse Kuhn triangle into its four fine Kuhn triangles. Use the same linear edge reading $L = (\alpha, \beta, \gamma)$ on the four congruent pieces. The unsigned additive sum kills every midpoint vertex move fixing coarse vertices if and only if $L = 0$. Every nonzero such reading fails gauge covariance. The oriented sum, with the middle triangle reversed, factors through the edge-record map.*

Proof. The three corner pieces have the coarse orientation and the middle piece the opposite orientation. In the oriented sum, their interior-edge contributions cancel; the remaining terms are twice the coarse three-edge reading of the edge-record averages. The unsigned sum adds twice the middle-piece reading. Moving vertices of one parity class fixes all coarse vertices but changes this extra term. The three midpoint parity classes force $\alpha = \beta = \gamma = 0$ if every such response vanishes. For any nonzero response the resulting coarse face field is constant within its hinge class, whereas every coarse vertex move has zero class total. It therefore cannot be the face reading of a coarse gauge move. □

The linearized geometric-area reading is a nonzero multiple of the Heron coefficients at a nondegenerate flat triangle, so it is included. This excludes the equal-piece additive linear face construction under dyadic Kuhn refinement at Gaussian order. It does not classify arbitrary nonlinear face variables, additional carriers, or separately derived gauge-fixing prescriptions. Choosing a fine gauge to define a different face map would add model data. These results neither calculate the logarithmic coefficient of a marginal face flow nor construct a face-variable action; those parts of the face-record question remain open.

There is also a regulator-free positive-symbol comparison (theorem; *Proof complete, independent verification pending.*): for Hermitian positive semidefinite massless symbols with a uniform upper bound and invertibility away from reciprocal-lattice zero waves, edge blocking has a 2^k lower bound on fibers avoiding those waves. The native Lorentzian symbol is indefinite, so it is not an instance of that theorem. A positive symbol with a uniform lower gap is a different, massive comparison; that gap must not be described as removing every infrared regulator.

5.1 What exists on the side-4 torus

At $\kappa = 1$ a stationary quadratic blocking of the genuine Regge Hessian from side 4 to side 2 exists (theorem; *Independently verified, with the scope stated below.*). More precisely, for the record map B, there is a linear right inverse u such that

$$Bu = I, \qquad H_f(uU, z) = 0 \quad (z \in \ker B), \qquad H_c(U, V) = H_f(uU, uV).$$

The coarse quadratic form is unique. It retains a zero mode c_0 which is not a constant metric strain plus a vertex move. Existence of this stationary lift is a finite, tree-level result. It neither produces a nonlinear perfect action nor removes that nongeometric mode. Existence at every larger torus side is a separate open question.

At $\kappa = 3/2$ the corresponding side-4 to side-2 refinement step exists and every coarse zero mode in that result is geometric (theorem; *Proof complete, independent verification pending.*). This does not assert proportionality to the coarse Regge Hessian, an all-depth fixed point, or a derivation selecting $\kappa = 3/2$. Neither blocking construction decides the physical value of κ.

6 The action that is still needed

A different order of summation in record time does not alter an integrable, positively regulated finite path sum. The specified native construction retains exact real record-frequency doubler roots on a nonempty open cone of directions for every graviton time; it supplies no replacement phase or new even kernel (theorems at their stated native-model scope [1]). A remedy has to change the relevant Gaussian or interacting structure rather than rename the same sum.

There is a classical reason to pursue the same missing symmetry. The explicit witness is a sampled static cubic with a fixed internal correction. For a nonempty edge class $S \subseteq \{0,1,2,3\}$, put $m = y + \mathbf{1}_S/2$ and

$$\begin{aligned} \xi(m) &= \tfrac{1}{2}(m_0 + m_1 - m_2 - m_3,\ m_0 - m_1 + m_2 - m_3,\ m_0 - m_1 - m_2 + m_3), \\ \Phi(\xi) &= \xi_1^3 - 3\xi_1\xi_2^2, \\ h(y,S) &= -2\Phi(\xi(m))\big(|S| + |S|^2/2\big) + \xi_2(m)P(S). \end{aligned}$$

Here $P = 45/8$ on $\{0\},\{3\},\{0,1,3\},\{0,2,3\}$; $P = -45/8$ on $\{1\},\{2\},\{0,1,2\},\{1,2,3\}$; $P = 45/4$ on $\{1,2\}$; $P = -45/4$ on $\{0,3\}$; and $P = 0$ otherwise. The coordinates ξ_1, ξ_2, ξ_3 are invariant under diagonal record-time translation.

On the side-8 torus at $\kappa = 1$, a local restriction of this field, wrapped to the torus, satisfies the first-order equations on the thirty edges incident to $y = (2,1,-1,0)$. For the vertex move g_y in the zeroth coordinate direction, the Hessian kills g_y, but

$$D^3 S_{\mathrm{R}}(h,h,g_y) = 756\sqrt{3} \neq 0.$$

Consequently no second-order correction can satisfy all thirty second-order equations there (theorem; *Independently verified, with the scope stated below.*). This finite-torus statement requires first-order satisfaction only on those incident edges.

On the infinite lattice $\mathbb{Z}^4$, the corresponding cubic field is a global solution of the linearized equations. At $y = (2,1,-1,0)$ its second-order equations have the same nonzero vertex-projected defect, so no second-order correction cancels it (theorem; *Independently verified, with the scope stated below.*). In particular, the proved family exclusion covers exactly quadratic curves through the flat field with this tangent h. Extending the exclusion to arbitrary C^2 solution families, or identifying this field with every compact-source exterior, requires further results.

Exact vertex symmetry is a structural route to both the quantum and classical consistency requirements, but action symmetry alone does not establish invariance of a determinant with an arbitrary gauge or measure. For the one-loop question the precise sufficient statement is the following existing Ward criterion [1].

Proposition 6.1 (A sufficient determinant symmetry)**.** Independently verified, with the scope stated below. *Let g be a symmetric invertible 4×4 metric and B range over symmetric strains. If D is differentiable near zero, its derivative is differentiable at zero, $D(0) \neq 0$, and*

$$\mathrm{d}D(B)[X^{\mathsf{T}}(g+B) + (g+B)X] = 0 \qquad (\operatorname{tr} X = 0)$$

near $B = 0$, then $\mathscr{R}_H[D] = 0$ for every symmetric g-traceless H. Exact invariance under determinant-one linear vertex maps implies this identity.

Proof. Put $\ell = \mathrm{d}D(0)$ and $Q = \mathrm{d}^2D(0)$. The first-order identity implies $\ell(S) = \frac{1}{4}\operatorname{tr}(g^{-1}S)\ell(g)$ for symmetric S. Differentiate the identity at $B = tH$ with $X = \frac{1}{2}g^{-1}H$. It gives

$$\ell(H) = 0, \qquad Q(H,H) = -\ell(Hg^{-1}H) = -\tfrac{1}{4}\tau_H\ell(g).$$

Substituting the logarithmic derivatives into (1) gives zero. Differentiating an exact determinant-one invariance along its vertex flow gives the Ward identity. □

Target 6.2 (A derived vertex-symmetric action)**.** *Open target. Derive an action from recognition records with every coefficient fixed, the Regge–Einstein long-wave quadratic and cubic limits, and a gauge/measure construction with regular nonzero determinants on every native torus at positive damping and zero ordered remainder on every native transverse-traceless probe. Exact vertex symmetry of the action together with a compatible invariant quantum construction is a sufficient structural objective. The two-probe condition and the corner compatibility condition are intermediate checks, not substitutes for this target. A complete physical construction must also supply consistent record-time dynamics and the required nonlinear classical solutions.*

Holonomy equality, fixed-damping collapse, coarse-fixed averaged growth and finite stationary blocking answer different questions. None is a constructed action meeting this target. The evidence isolates the next derivation: obtain genuinely new one-loop phase data or a different derived carrier/refinement map, then establish its classical limit and Ward response without fitting lattice-scale coefficients.

References

[1] J. Washburn, *Quantum Gravity from the Recognition Ledger: Weighted One-Loop Graviton Responses with Finite Real Parts, Einstein Gravity at Long Wavelengths, and the Massless-Graviton Problem*, version 1.2 (2026), doi:10.5281/zenodo.23839819.

[2] J. Washburn, *Linearized Einstein Gravity from Lorentzian Regge Calculus on the Kuhn Lattice, and the Divergence of Its One-Loop Graviton Limit* (2026), doi:10.5281/zenodo.23764491.