Newton's Constant and the Recognition Ledger: Einstein's Equations, the Graviton, and the One Number That Fixes G

Jonathan Washburn

Recognition Physics Institute, Austin, Texas

jon@recognitionphysics.org

doi: 10.5281/zenodo.23839820 (version 1.2) all versions: 10.5281/zenodo.23806705

version 1.1: 10.5281/zenodo.23813520; Pith preprint 10.67347/pith.preprint.2026.000023

version 1.0: 10.5281/zenodo.23806706; Pith preprint 10.67347/pith.preprint.2026.000019

Abstract

In units where the speed of light and Planck's constant are fixed, Newton's constant is one pure number: for the electron, $\alpha_G = Gm_e^2/(\hbar c) = 1.7518 \times 10^{-45}$. We ask how far the recognition ledger's lattice path sum goes toward deriving it. At the identified value $\kappa = 1$ of the lattice's one geometric parameter, its long-wavelength limit is Einstein gravity at linear order and, with matter, at second order, and the lattice form of Newton's law holds for exactly one tree-level coupling, $G_{\text{tree}} = 1/(8\pi s)$, where s is the scale of the Regge action. Under the ledger's phase premise the Planck length of that coupling is a fixed fraction of the record length, $\ell_P = \lambda_{\text{rec}}/(2\sqrt{\ln 2}) \approx 0.6006\,\lambda_{\text{rec}}$, and if light crosses one record length per record rank the lattice spacing is $b \approx 1.92\,\ell_P$. In SI units $G_{\text{tree}} = \pi^2\hbar c/(\ln 2\, m_e^2 N_e^2)$, with N_e the number of ledger ticks in one Compton period of the electron. A laboratory measures the long-wave coupling ZG_{tree}, with a positive factor Z that the quantum path sum fixes and that is not yet computed, so the measured G fixes $N_e/\sqrt{Z} = 9.0156 \times 10^{22}$. The present premises leave N_e free, every candidate on record misses that value by many orders of magnitude, and deriving N_e is the paper's precise open target. A massless graviton and the nonlinear equations are open too: in a stated one-loop model, the path sum as it stands is proved not to have a massless graviton, and read through two named maps its remainder corresponds to a mass scale fifty-one orders of magnitude above the measured bound, a conditional diagnostic. Measured values enter only as comparisons. Where a proof is complete but its independent verification is not, the text says so.

1 What it would take to predict Newton's constant

Take two electrons. Their electric repulsion has the strength $\alpha \approx 1/137$, a pure number that does not depend on units. Their gravitational attraction has a strength of the same kind,

$$\alpha_G = \frac{G m_e^2}{\hbar c} = \left(\frac{m_e}{m_P}\right)^2 = 1.7518 \times 10^{-45},$$

where $m_P = \sqrt{\hbar c/G}$ is the Planck mass [1]. Since 2019 the SI fixes c and h by definition and defines the kilogram through h [2], and the electron's mass in kilograms is known with a relative standard uncertainty of about 3×10^{-10} [3]. A laboratory measurement of G is therefore a measurement of α_G, and a theory that predicts G predicts this one number; the step to cubic metres per kilogram per second squared is arithmetic. Today the number comes only from experiment. G is the least precisely known fundamental constant, at a relative uncertainty of 2.2×10^{-5}, and the best measurements disagree with one another by more than their quoted uncertainties (Section 6).

This paper asks how far the recognition ledger's quantum gravity goes toward deriving α_G. In Recognition Science events are records kept on a ledger: four counters advance one step at a time, each step a *posting*, and the lattice of records they generate carries an order before it carries a geometry [4]. A real-time path sum over the geometries this lattice can carry, with the Lorentzian Regge action [5], is the candidate quantum theory of gravity studied in [6, 7]. A theory of gravity without free parameters has to do four things.

1. *Einstein's equations.* Its long-wavelength limit is Einstein gravity with matter.
2. *A massless graviton.* Its quantum corrections do not give the graviton a mass.
3. *A fixed coupling.* Newton's constant is fixed by the theory's own scales, with nothing tuned.
4. *One derived number.* Those scales are tied to a mass or length we can measure by a number the theory derives.

The first three put G into the theory's own units. The fourth turns it into a prediction.

What is established. The lattice path sum meets the first requirement at linear order and at second order with matter. Its coupling is fixed at tree level: the lattice form of Newton's law holds for exactly one coupling, $G_{\mathrm{tree}} = 1/(8\pi s)$, where s is the scale of the Regge action [6, Sec. 3.5]. The ledger's phase premise ties s to the scale of the path sum's phase, and then the Planck length of that coupling, $\ell_P = \sqrt{G_{\mathrm{tree}}\hbar/c^3}$, is a fixed fraction of the record length λ_{rec},

$$\ell_P = \frac{\lambda_{\mathrm{rec}}}{2\sqrt{\ln 2}} \approx 0.6006\,\lambda_{\mathrm{rec}}$$

(Section 3). If light crosses one record length per record rank, the lattice spacing at $\kappa = 1$ is $b = (2/\sqrt{3})\,\lambda_{\mathrm{rec}} \approx 1.92\,\ell_P$ (Section 4). Under these premises nothing about gravity's strength is left to choose: every gravitational length is a pure multiple of λ_{rec}, and the one factor still to be computed, the Z below, is fixed by the quantum path sum rather than by a choice.

Table 1. The steps from the ledger to the value of Newton's constant and their status. "Proved in" a cited paper means proved there and recalled, with its premises, in the section named; "proved here" means proved in this paper; "pending" marks a step whose proof is complete and whose independent verification is not yet finished; "identification" marks a physical premise that is not a consequence of the definitions; "open" marks a precise statement that is neither proved nor refuted.

step	status	where
Einstein's equations	proved in [6, 7] at linear order, with Newton's potential for compact bodies, and at second order with matter, at identified $\kappa = 1$; the metric part of the long-wave field equation is Einstein's, every nonlinear term included; with worldline matter the solutions lie near a continuum field that solves the full Einstein equation with the matter's own stress at first order and at every nonzero wave of the second order, and at the zero wave (pending); open: the nonlinear equations, for which the action as it stands meets a second-order obstruction one order past Einstein's	Section 2
massless graviton	open: the doublers are gapped by a derived measure; for the path sum as it stands the graviton is not massless in the one-loop model (proved in [7]); the one-loop remainder is nonzero for every candidate computed, and read through two named maps it corresponds to a mass scale near 10^{28} eV/c^2, fifty-one orders of magnitude above the measured bound (numerical; a conditional diagnostic); the readings of a triangle's charge that the ledger supplies fix its even part at zero or at a multiple of the area (pending), and neither removes the remainder (the first proved in the one-loop model, the second as computed)	Section 2
fixed coupling	proved in [6]: $G_{\text{tree}} = 1/(8\pi s)$ for every $\kappa > 1/2$; under the phase premise (identification), $G_{\text{tree}}\hbar = \lambda_{\text{rec}}^2/(4 \ln 2)$	Section 3
lattice spacing	proved here under two identifications: $b = \lambda_{\text{rec}}/\sqrt{\kappa - 1/4}$, so $b \approx 1.92\, \ell_P$ at $\kappa = 1$	Section 4
measured coupling	open: a measurement gives the long-wave coupling $G_{\text{long}} = Z G_{\text{tree}}$ of the quantum path sum; power counting allows $Z - 1$ of order $G_{\text{tree}}\hbar/b^2 \approx 0.27$	Section 2
derived number	open: N_e, the electron's Compton period in ledger ticks; G_{tree} in SI units is equivalent to it (proved here), and the measured G fixes $N_e/\sqrt{Z}$	Section 5

What is open. The fourth requirement is not met. The value of G_{tree} in SI units needs λ_{rec} in metres, and that is one pure number: the number N_e of ledger ticks in one Compton period of the electron, in terms of which $G_{\text{tree}} = \pi^2 \hbar c/(\ln 2\, m_e^2 N_e^2)$. A laboratory does not measure G_{tree}. It measures the coupling at long wavelengths, which for the quantum path sum is $G_{\text{long}} = Z G_{\text{tree}}$, with a positive factor Z that equals 1 at tree level; at a spacing near the Planck length the loop corrections to it need not be small (Section 2). The measured G therefore puts $N_e/\sqrt{Z}$, not N_e itself, at 9.0156×10^{22}. The framework's present premises leave N_e free

and every candidate on record misses that value by many orders of magnitude; Section 5 states the derivation of N_e as a target and says what a derived value would have to reach. The second requirement is open as well, and the first is met only in part. The graviton is not yet shown to be massless at one loop: at $\kappa = 1$ the lattice has massless doublers at the corners of its Brillouin zone, a measure derived from the ledger gaps them, and every candidate action computed so far leaves a nonzero remainder in the graviton's self-energy [7]; for the path sum as it stands the remainder is proved not to vanish, in the stated one-loop model of [7]. Read through the spacing of Section 4 and a map from the remainder to a mass term, that remainder corresponds to a one-loop mass scale near $10^{28}\,\mathrm{eV}/c^2$, about fifty-one orders of magnitude above the bound from gravitational waves [8]. That is a conditional diagnostic, not a prediction: under these readings the path sum without a repaired action conflicts with measurement, and removing the conflict is what an action derived from the ledger has to do (Section 2). Einstein's equations are recovered at linear and second order, not nonlinearly. Table 1 lists the steps.

Relation to other work. Planck combined G, $\hbar$ and c into natural units of length, time and mass [1]; the smallness of α_G in those units is the observation behind Dirac's large-number hypothesis [9]. Sakharov read G as an induced coupling, fixed by the vacuum fluctuations of matter fields up to a cutoff [10], and Jacobson derived Einstein's equations from the Clausius relation with an entropy proportional to horizon area, which ties G to the entropy per unit area [11]. In lattice approaches to quantum gravity, Newton's constant is a bare coupling whose value is set at a critical point [12]. Here the coupling is fixed by the scale of the action and tied to the scale of the phase by a premise of the ledger; what remains is a single number linking the ledger's tick to the atomic clocks by which we measure, and the factor by which quantum corrections carry the tree-level coupling to the long-wave one.

The sections can be read separately. Section 2 recalls what the lattice path sum establishes about Einstein's equations and the graviton; Section 3 follows the chain from the ledger's units to the Planck length; Section 4 gives the lattice spacing; Section 5 states the step to SI units; Section 6 gives the measured values a prediction would be compared with; and Section 7 lists what remains.

2 Einstein's equations and the graviton

Status: Einstein's equations and the graviton

Proved in [6, 7]. For every $\kappa > 1/2$: the linearized Einstein equation in de Donder gauge as the long-wave limit of the linear lattice field, with $G_{\mathrm{tree}} = 1/(8\pi s)$. At identified $\kappa = 1$: exact solutions of the Regge equations with a resting body, whose weak-field limit is that field and, for a compact body, Newton's potential; solutions at fixed small strength with an explicit second-order limit, with a model medium, a posting gas or worldline matter. With worldline matter, for a body whose first- and second-order waves avoid six blind lines, those solutions lie within a cubic error of a continuum field that solves the full Einstein equation with the matter's own stress at first order and at every nonzero wave of the second order; the uniform part of the second order is balanced by a constant. Massless corner doublers occur exactly at $\kappa = 1$; a measure weight derived from the ledger's cost of curvature gaps all four of them and gives the real part of the ordered

one-loop anisotropy of the graviton self-energy a finite limit. For the path sum as it stands, with the flat measure, the one-loop remainder of the model determinant (graviton and scalar loops, a unit-weight gauge term, no ghost line, the Feynman regulator) does not vanish in the ordered limit: the model's graviton is not massless. On a sampled smooth field the metric part of the lattice field equation, ten of its fifteen combinations of edge classes, is at second order in the spacing $-\frac{1}{2}$ times the Einstein density, every nonlinear term included, at every background joined to the flat metric by Lorentzian metrics with nondegenerate flat simplices: every long-wave vertex of the action is the Einstein–Hilbert one in that part. The condition that a linearized vacuum solution must meet to extend to second order fails for the action at $\kappa = 1$, one order in the derivatives past Einstein's, for a sampled cubic field, on a torus and on the infinite lattice, and for the cubic Taylor fields of a point mass's potential. Blocking the Gaussian path sum with the ledger's refinement map, with the damping held fixed on the fine lattice, gives no fixed point with an Einstein–Hilbert long-wave part at any $\kappa > 1/2$; in that order ordinary block averages have none either, so that result does not decide the route.

Proof complete, independent verification pending (in [7]). The one-loop remainder of a local charge on the triangles depends on it only through its slope and its even curvature, and the readings of a charge that the ledger supplies fix the even curvature at zero or at a multiple of the area. A phase carried by the holonomy of postings, a non-local candidate, acts at one loop like the Regge action. At $\kappa = 1$, with the damping held fixed on the coarse lattice, the zone-averaged blocked path sum has no fixed point with an Einstein–Hilbert long-wave part either. The constant that balances the uniform part of the second order with worldline matter is the matter's own uniform stress, so nothing there is left free.

Numerical. The one-loop remainder of the path sum as it stands, read through two named maps as a graviton mass scale: near 10^{28} eV/c^2, fifty-one orders of magnitude above the measured bound, a conditional diagnostic. A term even in the deficit angles cancels the remainder of the bare phase, at the one point found, when eight lattice-scale numbers are tuned for it; an even curvature proportional to the area leaves it nonzero at every coefficient computed. Read as the far field of a point mass of mass M, the second-order defect is a force on the vertex placements of order $M^2 r^{-7}$.

Identifications. $\kappa = 1$; for second order with worldline matter, the least-work law of the worldlines, the ratios of the reference occupation of the edge classes and the split between body and carrier, with point-particle postings and a reference background that does not gravitate as models; for the blocked route, that the ledger's records are counts on edges; for the measure, that the cost of a hinge record reads its area and that the weight of a geometry is the Gibbs weight of its cost; for the mass scale, the spacing of Section 4 and the map from the remainder to a mass term; for the speed, that the dressed zero of the propagator is the observed wave.

Open. The nonlinear Einstein equations as the long-wave limit; an action derived from the ledger whose one-loop graviton is massless, that is, whose summed Ward defect vanishes, with a one-loop light cone Lorentz invariant to about twelve digits, as GW170817 requires at a spacing of two Planck lengths or less; the blocked route at other values of κ; a ghost or gauge measure derived from the ledger, which could change every one-loop statement above; the long-wave coupling of the quantum path sum, the factor Z in $G_{\text{long}} = ZG_{\text{tree}}$.

What would settle it. The targets of [7]: a convergence theorem for lattice solutions at finite strength, and a ledger action with the classical limit and a vanishing Ward defect, or a proof that none exists.

We recall the lattice and what its path sum is known to do; the proofs are in [6, 7].

The lattice. Vertices are points of $\mathbb{Z}^4$ or of the side-N torus, with coordinates x_a that count the postings on four counters. Four counters advanced one step at a time from $(0,0,0,0)$ to $(1,1,1,1)$ have 24 arrival orders, and these are the 24 simplices of Kuhn's triangulation of the unit cube [13, 14]. Every edge joins x to $x+d$ for one of fifteen classes $d \in \{0,1\}^4 \setminus \{0\}$, and a configuration is a squared length on every edge. The flat lattice has the metric

$$g_\kappa = I - \kappa\,\mathbf{1}\mathbf{1}^\top, \qquad g_\kappa(d,d) = |d|\,(1-\kappa|d|),$$

in record coordinates, Lorentzian for $\kappa > 1/4$ with the time direction $\mathbf{1}$. At $\kappa = 1$ every posting is null and every longer edge is timelike. That value is an identification: it says that the cone of the lattice interval is the cone along which postings run, equivalently that the interval reads the posting order on some pair of positive duration [6, Thm. 2.2]. The ledger's foundation as stated does not decide it, and a derivation of $\kappa = 1$ would derive the doublers below [7].

The action and its two scales. The classical action is sS, where $S = \sum_h A_h \delta_h$ is the Lorentzian Regge action, a sum over triangles h of area times deficit angle, and s is the *action scale*: a source τ on the edges solves $s\,H_0 h = -\tau$ at linear order, with H_0 the flat Hessian. The path sum weighs a configuration by e^{jasS}, with the *phase scale* a (at unit action scale this is the weight e^{jaS} of [6]). Reading $\hbar = 1/a$ makes the weight the familiar $e^{jS_{\mathrm{cl}}/\hbar}$; that reading and $c = 1$ are definitions [6, Sec. 3.5]. These two scales are all the lattice has: there is no cosmological term, no matter coupling to adjust and no second gravitational constant.

Einstein gravity at long wavelengths. For $\kappa > 1/2$ and a conserved source family off the light cone, the linear lattice equations have the de Donder solution at small wave numbers u, and

$$q_\kappa(u)\,h_{\mu\nu}(u) \longrightarrow 16\pi G_{\mathrm{tree}}\Big(T_{\mu\nu}(0) - \tfrac{1}{2}g_{\mu\nu}T(0)\Big) \qquad (u \to 0), \qquad G_{\mathrm{tree}} = \frac{1}{8\pi s},$$

which is the linearized Einstein equation [6, Thm. 4.6]. We write G_{tree} for this coupling, the coefficient of the tree-level exchange; the end of this section says how it is related to the coupling a laboratory measures. At identified $\kappa = 1$, with a resting body and a model medium, the full Regge equations on all fifteen edge classes have exact solutions for small coupling, their weak-field long-wave limit is that field, and for a compact body of mass M the time-time component gives Newton's potential $-G_{\mathrm{tree}}M/R$ outside the body with an explicit finite-size remainder [6, Thms. 4.8–4.10]. At fixed small strength the long-wave equations have solutions uniformly in the torus size, with an explicit second-order limit [6, Thms. 4.12–4.16], and the medium can be replaced by a posting gas or by worldline matter that conserves postings, under named identifications [6, App. E], [7]. With worldline matter the second-order statement is also one about continuum fields: on every large torus the coupled lattice problem has a solution, unique in a ball of size of order ε, within $K\varepsilon^3$ of the edge readings of a continuum field $\varepsilon H_1 + \varepsilon^2 H_2$, where H_1 solves the linearized Einstein equation sourced by the dust of the body's resting postings; and, for a body whose first- and second-order waves avoid six blind lines, $\varepsilon H_1 + \varepsilon^2 H_2$ solves the full Einstein equation with the matter's own stress at first order and at every nonzero wave of the second order. The proof rests on two identities, both bilinear in two plane waves, and on the step that carries them to the field of the body: the image of the gravitational cubic vertex is the quadratic part of the Einstein density, and the contracted Bianchi identity holds at second order, from which the load of the matter's support follows

[7]. The uniform part of the second-order load is balanced by a constant, and that constant is the matter's own uniform stress: for the coupled solution, the uniform stress of its own support, divided by the square of the strength, tends to the continuum virial of the body, so this wave is checked as well and nothing is left free (proof complete, verification pending). Its premises are point-particle postings and a reference background that does not gravitate (MODEL), and the least-work law of the worldlines, the ratios of the reference occupation of the edge classes and the split between body and carrier (IDENTIFICATION). At tree level every non-gauge long-wave mode propagates on the light cone up to a relative correction of order $(kb)^2$ in the physical wave number k and spacing b, and the static Newton kernel is correct to second order [6, Thms. 5.1 and 5.4]. Beyond second order the vertices of the action reduce to one linear identity [7]: if, at every constant background metric near the flat one, the linear long-wave residual of the action is $-\frac{1}{2}$ times the linearized Einstein density at that background, then every long-wave vertex of the action is the Einstein–Hilbert one. The identity holds, so on a sampled smooth field the metric part of the field equation, ten of its fifteen combinations of edge classes, is at second order in the spacing exactly $-\frac{1}{2}$ times the Einstein density, every nonlinear term included, at every constant background joined to the flat metric by a segment of Lorentzian metrics with nondegenerate flat simplices [7, Thm. 4.2]. The other five combinations are not covered, and the statement concerns the field equation on sampled fields, not its solutions. The nonlinear Einstein equations need lattice solutions at finite strength whose long-wave limit solves $G_{\mu\nu} = 8\pi G_{\text{tree}}\, T_{\mu\nu}$, and matter with equations of its own, and on the way the action as it stands meets an obstruction one order past Einstein's [7, 15]. A linearized vacuum solution extends to second order only if its quadratic source is orthogonal, at every record, to the move of that record's vertex, which is a zero mode of the flat lattice. For the action at $\kappa = 1$ this Ward condition fails one order in the derivatives past Einstein's [7, Thm. 4.3]: on the side-8 torus, and on the infinite lattice as well, a weak static field with a cubic potential, which solves the first-order equations at every edge through one record, pairs with the move of that record to $756\sqrt{3}$, so no second-order correction exists there. The cubic Taylor fields of a point mass's potential about records on an axis, a face diagonal and a body diagonal fail in the same way. Read as a far field, the defect is a force on the vertex placements of order $M^2 r^{-7}$ (NUMERICAL), so a static vacuum solution with a regular far field is not expected to extend beyond that order unless the action is exactly symmetric under vertex moves, the symmetry whose one-loop form a massless graviton needs (INFERENCE).

The graviton at one loop. The identified lattice is not yet a quantum theory with a massless graviton. The one-loop statements below concern the model determinant of [7], assembled from graviton and scalar loops with a unit-weight gauge term, no ghost line and the Feynman regulator (MODEL): they are diagnostics of that model, which a ghost or gauge measure derived from the ledger could change. At $\kappa = 1$, and only there in the family $\kappa > 1/2$, the gauge-fixed flat operator has exact massless doublers at the four axis corners of the Brillouin zone [7], and with a chosen flat measure they make the zero-momentum anisotropy of the one-loop graviton self-energy diverge like $\log(1/\varepsilon)$ as the regulator ε is removed after the volume [6, Thms. 6.7 and 6.8]. A measure weight built from the ledger's cost of the curvature around each hinge, under the two identifications in the status box, gaps all four doublers, vanishes to fourth order on long waves, and gives the real part of that anisotropy a finite limit [7]. Whether the graviton stays massless is decided by a Ward identity: if the one-loop determinant is invariant under

volume-preserving linear moves of the vertices, the zero-momentum transverse-traceless remainder vanishes. No real measure weight or density restores that symmetry, no phase of the form $cS + \sum_h \delta_h^3 \Gamma_h$ has it, and the remainder is nonzero for every candidate computed to convergence [7].

For the path sum as it stands, with the flat measure, the conclusion does not rest on computation alone. At $\kappa = 1$ with the Feynman regulator, the per-site remainder of the model determinant does not have ordered limit zero: the model's graviton is not massless at one loop [7, Thm. 3.3]. The cause is the doublers again: near the axis corners the real part of the remainder falls like $-r \log(1/\varepsilon)$ with $r > 0$, while the rest of the zone contributes a real part that is bounded above. Near the origin that bound reduces to local estimates at the points of four planes of waves, and they hold everywhere: at the generic points, on the circles and axes where the planes meet, and, after a second reduction, at the four points where the light cone touches the planes [7].

A term added to each triangle's phase, even in its deficit angle and vanishing at the four triangle areas of the flat lattice so that the flat quadratic action stays Regge's, can cancel the remainder of the bare phase, whose doublers are still massless, at the one point found so far, and only when its first and second derivatives at those four areas, eight lattice-scale numbers, are chosen for the purpose (NUMERICAL) [7]. That is a tuning of exactly the kind a theory without free parameters has to exclude, until the ledger is shown to supply the eight numbers. The ledger's own readings of a charge on the triangles do not supply them. The one-loop remainder of any local charge, on every probe and at every regulator, depends on the charge only through two functions of a triangle's area: its slope and its even curvature, the second derivative in the deficit at zero deficit. A charge with the Regge slope that is odd under reversing the postings, as the ledger's records are, has even curvature zero and exactly the remainder of the Regge action, so it is not massless; a charge that adds over the pieces of a triangle has even curvature proportional to the area, and at every coefficient computed it leaves a nonzero remainder (NUMERICAL) [7]. An even part that removes the remainder would need a new premise that singles out the four flat triangle areas, and choosing one would be a fit. An action derived from the ledger whose summed Ward defect vanishes is the decisive open step of the quantum theory. What remains open is a phase outside these local classes. Neither of the two non-local routes examined so far supplies it at $\kappa = 1$ (proofs complete, verification pending) [7]. At one loop any phase acts only through its slope and its even part at zero deficit, so a phase that is odd under reversing the postings and has the Regge slope is massless exactly when the path sum as it stands is, whatever its non-local terms; the phase carried by the holonomy of postings around hinge cycles is of that kind, and under the ledger's bridge from loops to phases (MODEL) it is the Regge phase itself. Non-locality can help only by coupling different hinges. The second route blocks the Gaussian path sum with the ledger's own refinement map, which reads a coarse record from the two halves of an edge when the ledger's records are read as counts on edges (IDENTIFICATION). With the damping held fixed on the fine lattice the blocked path sum has no fixed point with an Einstein–Hilbert long-wave part, at any $\kappa > 1/2$, but that does not close the route: in that order the damping grows like a mass on the coarse lattice, and ordinary block averages over whole cells have no damped fixed point either. The order that can tell them apart holds the damping fixed on the coarse lattice, and at $\kappa = 1$ it decides the route: the doubler keeps an on-shell weight near a plane of waves through its corner, so the zone average of the blocked record fluctuation grows

like 2^k, while for block averages over whole cells it stays bounded. At $\kappa = 1$ the blocked path sum therefore has no fixed point with an Einstein–Hilbert long-wave part in either order of limits. The proof uses the doubler, so it is specific to $\kappa = 1$; other values of κ and the pointwise form of the statement are open.

The unrepaired path sum also gives a conditional diagnostic. Read through two identifications, the spacing $b = (2/\sqrt{3})\,\lambda_{\mathrm{rec}}$ of Section 4 and a map that turns the zero-momentum remainder into a mass term, linear in the remainder and proportional to $(\ell_P/b)^2$, the computed remainders correspond to a one-loop graviton mass scale of about $3 \times 10^{28}\ \mathrm{eV}/c^2$, or $5 \times 10^{28}\ \mathrm{eV}/c^2$ with the derived measure weight, most of it from their imaginary parts (NUMERICAL) [7]. Whether that scale is a mass or a width depends on the evolution of the path sum in record time, which is open; either way it exceeds the bound $m_g \le 2.42 \times 10^{-23}\ \mathrm{eV}/c^2$, at 90% credibility, from the propagation of gravitational waves [8] by about fifty-one orders of magnitude. The statement is conditional four times over: on the two maps, on the computed values, on the action being the unrepaired one, and on the one-loop model, with no ghost line and a unit-weight gauge term. It is not a prediction of the theory; it says that the path sum without a repaired action conflicts with measurement under these readings, and so measures what that action has to do.

A repaired action meets a second measurement, of the graviton's speed rather than its mass. At one loop the speed is fixed by the difference between the spatial and the temporal coefficients of the self-energy. Read with the identifications that the dressed zero of the propagator is the observed wave and that its phase speed is the speed of arrival, at a spacing of at most two Planck lengths any difference above 10^{-12}, for temporal coefficients up to 10 in size, contradicts the near-simultaneous arrival of the gravitational and electromagnetic signals of GW170817 [16, 7]. The lattice has a preferred frame, so nothing in its symmetry guarantees a one-loop light cone that is Lorentz invariant to twelve digits (INFERENCE); the repaired action has to supply one.

Tree level and the measured coupling. The coupling $G_{\mathrm{tree}} = 1/(8\pi s)$ is the coefficient of the tree-level exchange. The G measured at laboratory distances is the long-wave coupling of the full path sum, and power counting says how far apart the two can be. The one-loop effective action of lattice fields with spacing b contains a term proportional to the curvature with a coefficient of order $\hbar/b^2$; it shifts $1/G$ by a relative amount of order $G_{\mathrm{tree}}\hbar/b^2$, and Sakharov's induced gravity is this term [10]. For the lattice of Section 4, $G_{\mathrm{tree}}\hbar/b^2 = (\ell_P/b)^2 = 3/(16\ln 2) \approx 0.27$, so the shift need not be small (INFERENCE, from power counting). The coupling that a measurement determines is therefore

$$G_{\mathrm{long}} = Z G_{\mathrm{tree}} = \frac{Z}{8\pi s},$$

with a factor Z that the quantum path sum fixes: once that path sum has a massless graviton, G_{long} is read off the coefficient of its long-wave static exchange, as G_{tree} is read off the tree-level one. Z is positive because the measured coupling is attractive, and it equals 1 at tree level. Computing Z needs the quantum theory of [7] to exist with a massless graviton. Until it is computed, every measured value of G in this paper is a value of G_{long}: through the relations of Sections 3 and 5 it fixes G_{tree} only up to the factor $1/Z$, the record length and the Planck length of G_{tree} only up to $1/\sqrt{Z}$, and the number N_e of Section 5 only up to $\sqrt{Z}$. The long-distance quantum corrections of the effective theory are of relative order $G\hbar/(c^3 r^2)$ [17, 18], and they

play no role at laboratory distances: a one-loop correction $\xi(\ell_P/r)^2$ to Newton's potential stays below the sensitivity of short-range tests of gravity at every distance they reach, for every coefficient $|\xi| \le 10^{40}$ [7]. So the question of this paper has two parts: what fixes s, and what fixes Z.

3 From the ledger's units to the Planck length

Status: the coupling in the ledger's units

Definitions. The ledger's units: one record length λ_{rec} per tick, so $c = 1$; the phase scale a of the path sum read as $\hbar = 1/a$.

Proved in [6, Sec. 3.5]. For every $\kappa > 1/2$ and $s > 0$ the lattice Newton clause holds for exactly one tree-level coupling, $G_{\mathrm{tree}} = 1/(8\pi s)$.

Identification. The phase premise $as = \ln 2/(2\pi\lambda_{\mathrm{rec}}^2)$: a full turn around one face of area λ_{rec}^2, weighed in imaginary time, halves the weight.

Consequence. $G_{\mathrm{tree}}\hbar = \lambda_{\mathrm{rec}}^2/(4\ln 2)$, and the Planck length of G_{tree} is $\ell_P = \lambda_{\mathrm{rec}}/(2\sqrt{\ln 2}) \approx 0.6006\,\lambda_{\mathrm{rec}}$; that of the measured coupling $G_{\mathrm{long}} = ZG_{\mathrm{tree}}$ is $\sqrt{Z}\,\ell_P$.

Open. A derivation of the phase premise from the ledger that does not assume one bit per face.

The lattice carries no length unit of its own. Squared lengths, waves and Brillouin-zone integrals are in lattice units, and the path sum has exactly two scales, the phase scale a and the action scale s (Section 2). Everything gravitational about the theory's units comes from these two numbers and from how the ledger reads them.

Three relations. The ledger's own units take the record length λ_{rec} and the tick τ_0, with $c = \lambda_{\mathrm{rec}}/\tau_0$: one record length per tick. Reading the phase scale as $\hbar = 1/a$ is a definition, as $c = 1$ is. The third relation is not a definition. On the lattice, Newton's law is the statement that every static mode of every solution for a point mass M has $h_{tt}\,q(u) = 8\pi GM$, and this clause holds for exactly one value of the coupling,

$$G_{\mathrm{tree}} = \frac{1}{8\pi s} \qquad (\kappa > 1/2,\ s > 0),$$

because the linear field equation $s\,H_0 h = -\tau$ fixes the response to a source up to the factor $1/s$ [6, Sec. 3.5]. It is the tree-level coupling of Section 2. Together the three give

$$G_{\mathrm{tree}}\hbar = \frac{1}{8\pi as},$$

so the Planck area in lattice units is fixed by one product, as, the weight's phase per unit of Regge action. Classical physics does not fix it: the classical equations see s alone, through the response $1/s$ to a source, and the product as enters only through the quantum weight. In lattice units $1/(8\pi as) = G_{\mathrm{tree}}\hbar/b^2 = (\ell_P/b)^2$: fixing as is the same as fixing the size of the lattice in Planck units.

The phase premise. The ledger fixes the product through a reading of its smallest record. Take one face of the lattice, a triangle of area A, and turn once around it: a deficit angle of 2π.

Its Regge action is $sA \cdot 2\pi$, and its weight continued to imaginary time is $e^{-2\pi asA}$. The *phase premise* reads a face of area λ_{rec}^2 as holding one bit, so that a full turn halves the weight:

$$e^{-2\pi as\lambda_{\mathrm{rec}}^2} = \frac{1}{2}, \qquad \text{that is,} \qquad as = \frac{\ln 2}{2\pi\lambda_{\mathrm{rec}}^2}$$

The formula appears in [6, Sec. 3.5] as a model premise that no result there uses; reading a face as one bit is the ledger's premise. It is an identification, not a consequence of the definitions. With it,

$$G_{\mathrm{tree}}\hbar = \frac{\lambda_{\mathrm{rec}}^2}{4\ln 2}, \qquad \ell_P = \sqrt{G_{\mathrm{tree}}\hbar} = \frac{\lambda_{\mathrm{rec}}}{2\sqrt{\ln 2}} \approx 0.6006\,\lambda_{\mathrm{rec}}, \qquad \lambda_{\mathrm{rec}} \approx 1.665\,\ell_P.$$

Here and below ℓ_P is the Planck length of the tree-level coupling; the Planck length of the coupling a laboratory measures is $\sqrt{Z}\ell_P$ (Section 2).

Table 2 collects the chain, and Fig. 1 shows where each link comes from. Each line is either a definition, a theorem with its conditions, an identification, or a consequence of the lines above it.

Table 2. The chain from the ledger's units to the Planck length.

relation	status
$c = \lambda_{\mathrm{rec}}/\tau_0 = 1$	definition of the ledger's units
$\hbar = 1/a$	definition: the phase scale of the path sum
$G_{\mathrm{tree}} = 1/(8\pi s)$	theorem: the lattice Newton clause, every $\kappa > 1/2$ [6, Sec. 3.5]
$as = \ln 2/(2\pi\lambda_{\mathrm{rec}}^2)$	identification: one bit per face of area λ_{rec}^2
$G_{\mathrm{tree}}\hbar = \lambda_{\mathrm{rec}}^2/(4\ln 2)$	consequence of the four lines above
$\ell_P = \lambda_{\mathrm{rec}}/(2\sqrt{\ln 2})$	consequence, for the Planck length of G_{tree}

What the premise fixes, and what it does not. The phase premise fixes a ratio of lengths, $\ell_P/\lambda_{\mathrm{rec}}$. It does not fix G_{tree} in a unit system that defines its length independently of λ_{rec}: with c and $\hbar$ fixed, $G_{\mathrm{tree}} = \ell_P^2 c^3/\hbar$ is known once ℓ_P is known in that system's unit of length, and that needs λ_{rec} in the same unit. In the SI this is the question of Section 5. The premise is also a premise about entropy. With $G_{\mathrm{tree}} = 1/(8\pi s)$ and $\hbar = 1/a$, the Bekenstein–Hawking entropy per unit area of a horizon, read at tree level, is $1/(4G_{\mathrm{tree}}\hbar) = 2\pi as$ [19, 20], and the premise sets it to $\ln 2/\lambda_{\mathrm{rec}}^2$: one bit per record area. A count of a horizon's records that assumed one bit per λ_{rec}^2 would therefore return the quarter of the Bekenstein–Hawking law by construction. The companion paper on black holes [15] poses the count that would supply this number independently. A horizon of the lattice hides the postings that cross it, and under the phase premise, $\kappa = 1$ and one record length per rank (Section 4) neither a count that gives each crossing a bit nor a count of independent records gives the quarter: the coefficient depends on the orientation of the horizon and on none of the horizons computed is it $1/4$. If a count that correlates the records gives one bit per λ_{rec}^2, the phase premise becomes a consequence of the ledger instead of a reading of it.

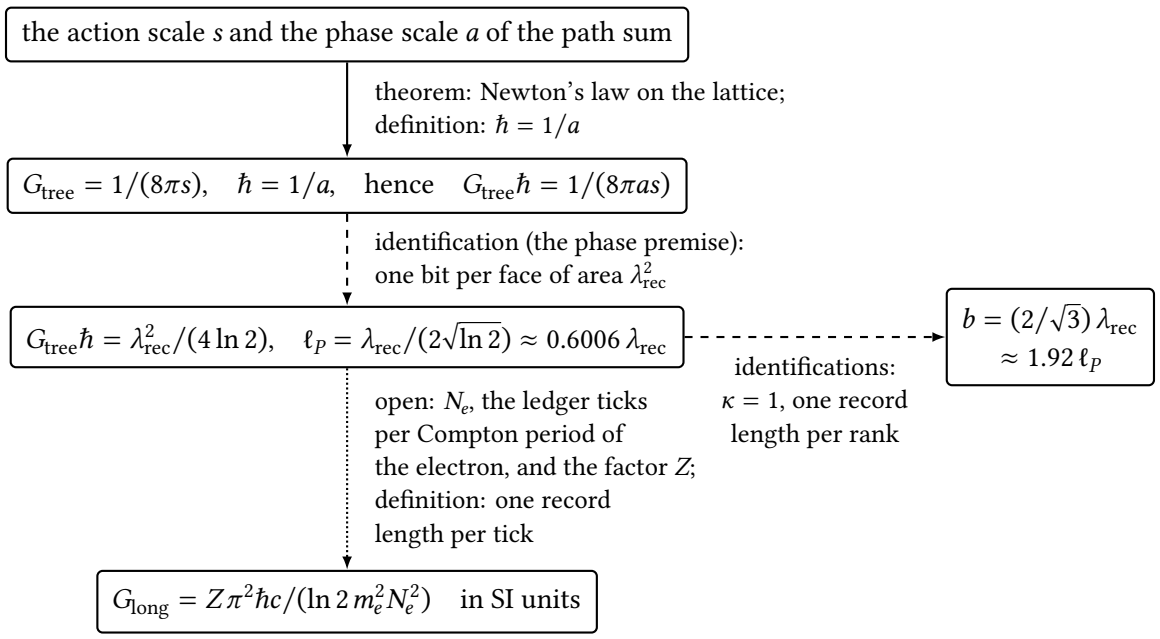


Figure 1. From the lattice's two scales to the value of G. Solid arrows are theorems or definitions, dashed arrows identifications, the dotted arrow the open step. At tree level every gravitational length is a fixed multiple of the record length λ_{rec}; the value of G in SI units needs one more number, N_e, and the factor Z that carries the tree-level coupling to the long-wave one.

4 The lattice spacing

Status: the lattice spacing

Proved, for every $\kappa > 1/4$: the cone of the lattice interval crosses a spatial distance $b\sqrt{\kappa - 1/4}$ per record rank. Under the identification below, $b = \lambda_{\text{rec}}/\sqrt{\kappa - 1/4}$ and $(\ell_P/b)^2 = (\kappa - \frac{1}{4})/(4\ln 2)$; at $\kappa = 1$, $b = (2/\sqrt{3})\,\lambda_{\text{rec}}$ and $b/\ell_P = \sqrt{16\ln 2/3} \approx 1.923$. The three readings "$\kappa = 1$", "$b = \lambda_{\text{rec}}$" and "light crosses one record length per rank" cannot all hold: any two fix the third, and $b = \lambda_{\text{rec}}$ with the third gives $\kappa = 5/4$.

Identifications. Light runs on the cone of the lattice interval and one record rank lasts one tick, so light crosses one record length per rank; $\kappa = 1$; the phase premise of Section 3.

Open. A derivation of either identification from the ledger.

The spacing b is the physical length of the lattice's unit step, and it sets the size of every lattice-scale effect: the corrections to propagation of relative order $(kb)^2$ [6, Thm. 5.1], the scale at which the doublers of Section 2 live, and the loop factor Z of Section 2. It is fixed by asking how far light goes in one record rank.

How far the cone reaches per rank. In record coordinates the time direction is $\mathbf{1}$ and the record rank of a point is $\sum_a x_a$. The vectors orthogonal to the time leg for $g_\kappa = I - \kappa\mathbf{1}\mathbf{1}^\top$ are those with $\sum_a v_a = 0$, and on them g_κ is the Euclidean form. A null vector $\mathbf{1} + v$ with $\sum_a v_a = 0$ advances the rank by 4 and has $|v|^2 = 16\kappa - 4$, so per rank the cone crosses a spatial distance

$$\frac{|v|}{4}\,b = b\sqrt{\kappa - \tfrac{1}{4}}.$$

At $\kappa = 1$ this is $(\sqrt{3}/2)\,b$, the spatial step of one posting: postings run on the cone.

One record length per rank. If light runs on the cone of the lattice interval and one record rank lasts one tick $\tau_0 = \lambda_{\mathrm{rec}}/c$, light crosses one record length per rank, and

$$b = \frac{\lambda_{\mathrm{rec}}}{\sqrt{\kappa - 1/4}}, \qquad \left(\frac{\ell_P}{b}\right)^2 = \frac{\kappa - 1/4}{4\ln 2},$$

using $\ell_P^2 = \lambda_{\mathrm{rec}}^2/(4\ln 2)$ from Section 3. At $\kappa = 1$,

$$b = \frac{2}{\sqrt{3}}\,\lambda_{\mathrm{rec}} \approx 1.155\,\lambda_{\mathrm{rec}}, \qquad \left(\frac{\ell_P}{b}\right)^2 = \frac{3}{16\ln 2} \approx 0.2705, \qquad \frac{b}{\ell_P} = \sqrt{\frac{16\ln 2}{3}} \approx 1.923.$$

The identification is physical: it says that the cone of the interval is the cone of light and that the tick is the duration of one rank. Without it the lattice fixes $b\sqrt{\kappa - 1/4}$ per rank and nothing in metres.

Three readings that cannot all hold. A tempting alternative is to set the spacing equal to the record length, $b = \lambda_{\mathrm{rec}}$. Together with one record length per rank this forces $\kappa - \frac{1}{4} = 1$, that is $\kappa = 5/4$, and at $\kappa = 1$ it would need a photon tick of $2/\sqrt{3}$ ranks, not a whole number. So "$\kappa = 1$", "$b = \lambda_{\mathrm{rec}}$" and "one record length per rank" are jointly inconsistent, and any two of them fix the third. The two readings $b = \lambda_{\mathrm{rec}} \approx 1.67\,\ell_P$ and $b \approx 1.92\,\ell_P$ differ by 15%; both put the lattice at the Planck scale. A horizon normal to a cube axis tells them apart: with one bit for each posting that crosses it, its inward crossings, counted at the ranks where it holds records, give exactly the Bekenstein–Hawking quarter at $b = \lambda_{\mathrm{rec}}$ and 3/16 at $b \approx 1.92\,\ell_P$, while at $b = \lambda_{\mathrm{rec}}$ the other horizons of the lattice give 0.144 and 0.433 [15]. That is a consistency condition between the phase premise and the spacing, not a reason to choose either reading.

What the spacing means for observation. At tree level the lattice has no observable signature for spacings up to 10^6 Planck lengths [6, Sec. 5]: for a gravitational wave at 100 Hz, $kb \approx 6.5 \times 10^{-41}$ at $b = 1.92\,\ell_P$. A spacing near the Planck length is invisible at tree level and makes the loop parameter of the path sum large: $1/(as) = 8\pi G_{\mathrm{tree}}\hbar/b^2 = 3\pi/(2\ln 2) \approx 6.8$ in lattice units at $\kappa = 1$, which is the reason the factor Z of Section 2 has to be computed rather than assumed to be 1. It also sets the scale of the one-loop graviton mass: read through this spacing and a map from the remainder to a mass term, the remainder of the unrepaired path sum corresponds to a mass scale near $10^{28}\,\mathrm{eV}/c^2$, a conditional diagnostic (Section 2).

5 From the ledger's units to the SI: one number

Status: the value of G in SI units

Proved. With c and h exact, every SI value of the tree-level coupling is the same datum as one pure number N_e, the number of ledger ticks in one Compton period of the electron: $G_{\mathrm{tree}} = \pi^2\hbar c/(\ln 2\, m_e^2 N_e^2)$, under the phase premise of Section 3. The coupling a measurement determines is $G_{\mathrm{long}} = ZG_{\mathrm{tree}}$. The framework's present premises leave N_e free, and every candidate

on record misses the measured value by many orders of magnitude.

Comparison. With the CODATA values $G = 6.674\,30(15) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ and $m_e = 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg}$, and h and c exact, $N_e/\sqrt{Z} = \pi/\sqrt{\ln 2\,\alpha_G} = 9.015\,58(10) \times 10^{22}$; two standard uncertainties, rounded to five figures, give the window $(9.0154,\ 9.0158) \times 10^{22}$.

Open. Target 5.1: a derivation of N_e from the ledger. The factor Z of Section 2.

Not a prediction. Taking the measured G as the tree-level coupling ($Z = 1$) gives $\lambda_{\mathrm{rec}} = 2.691 \times 10^{-35}\,\mathrm{m}$, $b = 3.108 \times 10^{-35}\,\mathrm{m}$ and a tick of $8.98 \times 10^{-44}\,\mathrm{s}$; for another Z each of these is divided by $\sqrt{Z}$. They are readings of the measured value, and they are used nowhere as inputs.

The relations of Sections 3 and 4 put every gravitational length into the ledger's units. To compare with a laboratory we need the ledger's units in the laboratory's. This section says exactly what that takes, and the answer is one number and the factor Z of Section 2.

What the SI fixes. The 2019 SI fixes seven constants by definition: the caesium hyperfine frequency $\Delta\nu_{\mathrm{Cs}}$, the speed of light c, Planck's constant h, the elementary charge, Boltzmann's constant, Avogadro's number and the luminous efficacy [2]. The second is the duration of a fixed number of caesium periods, the metre is the distance light travels in a fixed fraction of a second, and the kilogram follows from h. Lengths and masses are thereby tied to atomic physics. Newton's constant is not among the seven. With c and $\hbar$ exact, $G_{\mathrm{tree}} = \ell_P^2 c^3/\hbar$, so a value of G_{tree} in SI units is a value of ℓ_P in metres, and by Section 3 a value of the record length, or of the tick $\tau_0 = \lambda_{\mathrm{rec}}/c$, in SI units. Under the identifications of Section 3 the tick is $2\sqrt{\ln 2}$ Planck times of the tree-level coupling.

The number. A mass is a clock: a particle of mass m has the Compton period $h/(mc^2)$. The electron's is $T_e = h/(m_e c^2) = 8.09 \times 10^{-21}\,\mathrm{s}$, and the electron's mass is known with a relative standard uncertainty of about 3×10^{-10} [3]. Let N_e be the number of ledger ticks in one Compton period, $N_e = T_e/\tau_0$. Then $\lambda_{\mathrm{rec}} = h/(m_e c N_e)$, and the Planck area of Section 3 gives

$$G_{\mathrm{tree}} = \frac{\pi^2\,\hbar c}{\ln 2\; m_e^2 N_e^2}, \qquad G_{\mathrm{long}} = Z G_{\mathrm{tree}}, \qquad \alpha_G = \frac{G_{\mathrm{long}} m_e^2}{\hbar c} = \frac{Z\pi^2}{\ln 2\; N_e^2}.$$

The map from N_e to the SI value of G_{tree} is one-to-one, so a value of G_{tree} and a value of N_e are the same datum. A measurement determines G_{long}. With the CODATA values $G = 6.674\,30(15) \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ and $m_e = 9.109\,383\,7139(28) \times 10^{-31}\,\mathrm{kg}$ [3], and h and c exact, $\alpha_G = 1.751\,809 \times 10^{-45}$ and

$$\frac{N_e}{\sqrt{Z}} = \frac{\pi}{\sqrt{\ln 2\;\alpha_G}} = 9.015\,58(10) \times 10^{22},$$

with the standard uncertainty in parentheses: half the relative uncertainty of G, 1.1×10^{-5}, to which the electron's mass adds 3×10^{-10}. Two standard uncertainties, with the end points rounded to five figures, give the window

$$N_e/\sqrt{Z} \in (9.0154,\ 9.0158) \times 10^{22}$$

(comparison), and the measured values put N_e itself in that window only at tree level, $Z = 1$. The prefactor $\pi^2/\ln 2$ rests on the phase premise, one bit per face of area λ_{rec}^2, and on the

ledger's units, one record length per tick. A reading that put the bit on a lattice triangle of side b instead of λ_{rec} would change G_{tree} at fixed N_e by the factor $4/3$.

How gravity sees the electron. The number has a direct physical reading. A source whose quantum phase turns once every n ticks has, on the lattice, the gravitational radius $G_{\mathrm{tree}}M/c^2 = \pi\lambda_{\mathrm{rec}}/(2\ln 2\, n)$, whatever the value of the quantum of action, with $G_{\mathrm{tree}} = 1/(8\pi s)$ at identified $\kappa = 1$, the phase premise, and the source's mass read from its phase rate $\omega = 2\pi/n$ as $M = \hbar\omega$. The ledger's gravity sees a particle only through how many ticks its phase takes to turn once. For the electron that radius is 6.76×10^{-58} m at the measured G; the smallness of gravity between electrons is the largeness of N_e.

The framework does not yet fix it. The framework's account of particle masses gives masses in its own units, through a ladder whose unit relative to the tick is a premise that is not yet derived. With every other premise held fixed, each positive value of G_{tree} is realized by some choice of that unit, so the present premises are compatible with every value of N_e. The candidates on record all miss the measured value, by margins that no factor Z of order one could close. A mass of L coherence quanta, $L\hbar/\tau_0$ in energy, has a Compton period of $2\pi/L$ ticks. The framework's ladder records four forms of the electron's mass in coherence quanta, $2^{-22}\varphi^{56}(1 + 1332/\varphi)$, $2^{-22}\varphi^{64}$, φ^3 and $2^{-22}\varphi^{51}$, with φ the golden ratio; each exceeds 4, so each gives $N_e < 2$. Taking as the ladder's unit the one-loop scale at which the framework's strong coupling at the ledger scale, $\alpha_3^{-1} = 3\pi\varphi^5/2$ (below), becomes strong, the factor $\exp(-2\pi\alpha_3^{-1}/7) = \exp(-3\pi^2\varphi^5/7)$, with the first of those forms gives $N_e < 10^{14}$, and G_{tree} larger than 10^7 in SI units. Laws in which N_e grows as a power γ of the number of ticks since the earliest records make α_G drift at the rate $-2\gamma/t$. Lunar laser ranging gives $\dot{G}/G = (7.1 \pm 7.6) \times 10^{-14}$ per year [21], so $|\dot{G}/G| < 3 \times 10^{-13}$ per year at three standard deviations; at the present age of 13.79 billion years this gives $|\gamma| \le 0.0021$, and with at most e^{140} ticks since the earliest records such a count can supply at most a factor $e^{0.294} \approx 1.34$ of N_e. The two laws of this kind that the framework has scored, with exponents $1/3$ and an effective $1/4$, break the ranging bound by factors above 100.

Target 5.1 (The electron's period in ticks)**.** Derive from the ledger, with every premise named, the number N_e of ledger ticks in one Compton period of the electron, or an equivalent premise fixing the unit of the framework's mass ladder against the tick.

The target is not trivially satisfiable. The values 0 and 1, everything below 2, the framework's present identification of the ladder unit, which gives the four forms above, and the strong-coupling unit all miss the measured value of $N_e/\sqrt{Z}$ for every Z above 10^{-17}. Every inverse whole power of the golden ratio in place of the unit misses the window at tree level, but not for every Z of order one: with the form φ^3, the unit φ^{-109} lands in the window at $Z \approx 0.981$. A candidate of that kind can be tested only once Z is computed, and a power chosen to land in the window would be a fit, not a derivation. The target is not met by an anchor that is itself fixed by the measured G, since that reads the answer from the input: an SI bridge that fixes the tick from the measured G returns that G.

What a derived value would give. G_{long} scales as Z/N_e^2; a relative error $\varepsilon \le 1/10$ in N_e gives at most 3ε in G_{long}. To match the CODATA uncertainty of G a derived N_e has to be good

to about 10^{-5}, and Z to about 2×10^{-5}; to choose between the laboratory measurements of Section 6, which correspond to $N_e/\sqrt{Z}$ between 9.0147 and 9.0172×10^{22}, both have to be good to about 10^{-4}. Through m_e the SI value would carry only 6×10^{-10} of measured uncertainty, more than four orders of magnitude below the CODATA uncertainty of G.

Two other routes. The vacuum energy of the companion paper [22] gives a second route, with the expansion of the universe in place of the electron as the measured unit. With the measured Hubble rate H_0 and density fraction Ω_Λ, a value of $X = \rho_\Lambda c^2 \lambda_{\mathrm{rec}}^4/(\hbar c)$, the vacuum energy density in units of $\hbar c/\lambda_{\mathrm{rec}}^4$, fixes $G = \pi X c^5/\big(6(\ln 2)^2 \hbar H_0^2 \Omega_\Lambda\big)$ at tree level; the Planck 2018 values, with the CODATA G, put X between 8.4 and 9.0×10^{-123} (with the long-wave factor they fix XZ^2), and a derived X would give G at the 2% precision of $H_0^2\Omega_\Lambda$, before the 17% by which the tension between measurements of H_0 moves it. A third route runs through the proton. The framework reads the strong coupling at the ledger scale $\hbar c/\lambda_{\mathrm{rec}}$ as $\alpha_3^{-1} = 3\pi\varphi^5/2 \approx 52.26$; the measured coupling $\alpha_s(M_Z) = 0.1180$ [23], run up to that scale with the Standard Model, gives about 51.9 at one loop and 52.45 at two loops (an estimate by hand, not a computation; top threshold at 172.7 GeV, without electroweak and Yukawa terms at two loops). Since the proton's mass is a fixed multiple of the scale at which the strong coupling becomes strong, that reading would fix $G_{\mathrm{tree}} m_p^2/(\hbar c)$, and the comparison at $Z = 1$ puts G within about a factor of two of the measured value (INFERENCE). It is not a derivation: the scheme constant of the reading, the premise that anchors it, and the ratio of the proton's mass to the strong scale are not derived, and the scale depends exponentially on the scheme constant.

6 What a prediction would be compared with

No value of G is derived in this paper, so there is nothing yet to compare. This section records what a derived value would meet. The numbers below are measurements; none of them enters any relation above.

The CODATA 2018 and 2022 adjustments recommend the same value,

$$G = 6.674\,30(15) \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}},$$

a relative standard uncertainty of 2.2×10^{-5} [24, 3]. That uncertainty is larger than the uncertainty of most individual measurements, because the measurements disagree. Table 3 lists several of the most precise ones. They span 6.6719 to 6.6755 in units of $10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, a relative spread of about 5×10^{-4}, many times the uncertainties their authors quote.

A derived value of N_e (Target 5.1), with the factor Z, would give G to about 6×10^{-10} plus the derivation's own uncertainty. If that value fell inside the spread of Table 3 it would single out the measurements it agrees with; if it fell outside, the theory as stated, with its identifications, would be in conflict with every measurement. Both outcomes are informative, and neither is available until the target is met.

Table 3. Measurements of Newton's constant, in units of $10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, with one-standard-deviation uncertainties in the last digits. The values are input data of the CODATA 2018 and 2022 adjustments, copied with every digit of the 2022 report's table of input data for G (Table XXX), not rounded; that table includes corrections the authors reported after publication [3]. Each determines the long-wave coupling $G_{\mathrm{long}} = ZG_{\mathrm{tree}}$ of Section 2, so these are comparisons for G_{long}.

measurement	method	G
Gundlach and Merkowitz [25]	torsion balance, angular acceleration feedback	6.674 255(92)
Quinn *et al.* [26]	torsion balance, servo and Cavendish modes	6.675 54(16)
Rosi *et al.* [27]	atom interferometry	6.671 91(99)
Newman *et al.* [28]	cryogenic torsion pendulum	6.674 35(13)
Li *et al.* [29]	time-of-swing torsion pendulum	6.674 184(78)
Li *et al.* [29]	angular acceleration feedback	6.674 484(77)
Parks and Faller [30]	simple pendulum, laser interferometry	6.672 60(25)
CODATA 2018 and 2022 [24, 3]	adjustment of all data	6.674 30(15)

7 What remains

A theory of gravity without free parameters would have none of the premises below except definitions. The paper has shown where each one enters; here they are in order of consequence for the value of G.

1. *The number N_e* (Target 5.1). Without it there is no value of G in SI units: the electron's Compton period counted in ledger ticks, or equivalently the unit of the framework's mass ladder against the tick.
2. *The long-wave coupling Z.* The measured G is the long-wave coupling of the quantum path sum, $G_{\mathrm{long}} = Z/(8\pi s)$. At a spacing near the Planck length the loop parameter is of order one, so Z has to be computed; that needs the quantum theory of [7] with a massless graviton. Until Z is known, the measured G fixes $N_e/\sqrt{Z}$, not N_e.
3. *The phase premise.* One bit per face of area λ_{rec}^2 fixes *as* and hence $\ell_P/\lambda_{\mathrm{rec}}$. A derivation that does not assume it, for instance a count of a horizon's records [15], would make it a consequence.
4. *The value $\kappa = 1$ and one record length per rank.* Together they fix b/λ_{rec}; $\kappa = 1$ is also the value at which the doublers appear, so its derivation and the doublers' removal belong together [7].
5. *A massless graviton and the nonlinear equations.* The first two requirements of Section 1, open in [7]. Read through the identified spacing and a map from the remainder to a mass term, the unrepaired path sum has a one-loop graviton mass scale fifty-one orders of magnitude above the measured bound, a conditional diagnostic (Section 2). The remainder fixes the square of the mass, so it has to fall by more than a hundred orders of magnitude: in practice it has to vanish, which is the Ward identity of Section 2. The readings of a

Table 4. The premises of each relation in the paper.

relation	premises
linearized Einstein equation, $G_{\text{tree}} = 1/(8\pi s)$	every $\kappa > 1/2$; a conserved source off the light cone [6, Thm. 4.6]
Newton's potential for a compact body	$\kappa = 1$ (identification); model medium; odd sides not divisible by 105 [6, Thm. 4.10]
second order as a continuum field, with the full equation	$\kappa > 1/2$; worldline matter with point-particle postings and a reference background that does not gravitate (models); the least-work law, the ratios of the reference occupation of the edge classes and the split between body and carrier (identifications); waves off six blind lines; the uniform part of the second order balanced by the matter's own uniform stress (proof complete, verification pending) [7]
the metric part of the long-wave field equation is Einstein's	$\kappa > 1/2$; a background joined to the flat metric by Lorentzian metrics with nondegenerate flat simplices; sampled smooth fields; the field equation, not its solutions [7, Thm. 4.2]
second-order obstruction past Einstein's order	$\kappa = 1$ (identification); the action as it stands; proved for a sampled cubic field on the side-8 torus and on the infinite lattice, and for a point mass's cubic Taylor fields; the point mass's far field numerical; the consequence for regular far fields an inference [7, Thm. 4.3], [15]
$G_{\text{tree}}\hbar = \lambda_{\text{rec}}^2/(4\ln 2)$	definitions $c = 1$, $\hbar = 1/a$; the phase premise (identification)
$b = \lambda_{\text{rec}}/\sqrt{\kappa - 1/4}$; $b \approx 1.92\,\ell_P$	one record length per rank (identification); the phase premise; $\kappa = 1$ for the number
$G_{\text{long}} = ZG_{\text{tree}}$	the quantum path sum (open); every measured value of G is a value of G_{long}
one-loop mass scale near 10^{28} eV/c^2, a conditional diagnostic	the spacing $b \approx 1.92\,\ell_P$ and the map from the remainder to a mass term (identifications); the computed remainders (numerical); the unrepaired action; the one-loop model [7]
no massless graviton for the path sum as it stands	$\kappa = 1$ (identification); the Feynman regulator; the flat measure; the one-loop model: graviton and scalar loops, a unit-weight gauge term, no ghost line [7, Thm. 3.3]
no massless even part among the ledger's readings	a local charge with three continuous derivatives that vanishes at zero deficit; oddness under reversing the postings, or additivity over the pieces of a triangle (readings); the one-loop model; the computed remainders (numerical) for the second; proof complete, verification pending [7]
GW170817 needs a one-loop light cone Lorentz invariant to 10^{-12}	a spacing of at most two Planck lengths; the dressed zero is the observed wave and its phase speed the speed of arrival (identifications) [7]
$G_{\text{tree}} = \pi^2\hbar c/(\ln 2\, m_e^2 N_e^2)$	the phase premise (identification); one record length per tick (the ledger's units)
N_e, hence G_{long} in SI units	Target 5.1 and the factor Z (open)

triangle's charge that the ledger supplies do not achieve it: their even part is zero, which keeps the Regge remainder, or proportional to the area, which leaves a remainder at every coefficient computed, and an even part chosen to remove it would be a fit (Section 2). At the identified spacing the same action also has to give the graviton a one-loop light cone that is Lorentz invariant to about twelve digits, or, read with the identifications of Section 2, the arrival of GW170817 rules it out. For the nonlinear equations the action as it stands meets a second-order obstruction one order past Einstein's; an action exactly symmetric under vertex moves would avoid it, and the same symmetry at one loop is what a massless graviton needs (Section 2).

What is established is the shape of the answer. The ledger's lattice gives gravity one coupling, fixes it by the scale of its action, and ties that scale to the scale of its phase through a reading of its smallest record. If the reading holds, Newton's constant at tree level is a pure number in the ledger's units, $G_{\mathrm{tree}}\hbar = \lambda_{\mathrm{rec}}^2/(4\ln 2)$. One number, the electron's period in ticks, stands between that and a value in cubic metres per kilogram per second squared, and one factor, Z, stands between the tree-level coupling and the measured one.

References

[1] M. Planck, Über irreversible Strahlungsvorgänge, *Sitzungsber. Königl. Preuss. Akad. Wiss. Berlin*, 440–480 (1899).

[2] Bureau International des Poids et Mesures (BIPM), *The International System of Units (SI)*, SI Brochure, 9th edition (BIPM, Sèvres, 2019), ISBN 978-92-822-2272-0, https://www.bipm.org/en/si-brochure-9.

[3] P. J. Mohr, D. B. Newell, B. N. Taylor and E. Tiesinga, CODATA recommended values of the fundamental physical constants: 2022, *Rev. Mod. Phys.* **97**, 025002 (2025).

[4] J. Washburn, The record cell of the four-dimensional lattice: time, Lorentzian signature and the Regge action near flatness, in *The Formation of the Universe from Recognition: The Integrated Manuscript and Its Focused Papers*, Zenodo (2026), doi:10.5281/zenodo.23738724.

[5] T. Regge, General relativity without coordinates, *Nuovo Cimento* **19**, 558–571 (1961).

[6] J. Washburn, Linearized Einstein gravity from Lorentzian Regge calculus on the Kuhn lattice, and the divergence of its one-loop graviton limit, Zenodo (2026), doi:10.5281/zenodo.23764491; Pith preprint doi:10.67347/pith.preprint.2026.000017.

[7] J. Washburn, Quantum gravity from the recognition ledger: weighted one-loop graviton responses with finite real parts, Einstein gravity at long wavelengths, and the massless-graviton problem, Zenodo (2026), doi:10.5281/zenodo.23806703; Pith preprint doi:10.67347/pith.preprint.2026.000018.

[8] R. Abbott *et al.* (LIGO Scientific, Virgo and KAGRA Collaborations), Tests of general relativity with GWTC-3, *Phys. Rev. D* **112**, 084080 (2025), doi:10.1103/PhysRevD.112.084080; arXiv:2112.06861.

[9] P. A. M. Dirac, The cosmological constants, *Nature* **139**, 323 (1937).

[10] A. D. Sakharov, Vacuum quantum fluctuations in curved space and the theory of gravitation, *Dokl. Akad. Nauk SSSR* **177**, 70–71 (1967) [*Sov. Phys. Dokl.* **12**, 1040–1041 (1968)].

[11] T. Jacobson, Thermodynamics of spacetime: the Einstein equation of state, *Phys. Rev. Lett.* **75**, 1260–1263 (1995).

[12] H. W. Hamber, *Quantum Gravitation: The Feynman Path Integral Approach*, Springer, Berlin (2009).

[13] H. Freudenthal, Simplizialzerlegungen von beschränkter Flachheit, *Ann. of Math.* **43**, 580–582 (1942).

[14] H. W. Kuhn, Some combinatorial lemmas in topology, *IBM J. Res. Dev.* **4**, 518–524 (1960).

[15] J. Washburn, The recognition ledger of a black hole: horizon records, entropy counts and lattice Hawking radiation, Zenodo (2026), doi:10.5281/zenodo.23806711; Pith preprint doi:10.67347/pith.preprint.2026.000020.

[16] B. P. Abbott *et al.* (LIGO Scientific, Virgo, Fermi GBM and INTEGRAL Collaborations), Gravitational waves and gamma-rays from a binary neutron star merger: GW170817 and GRB 170817A, *Astrophys. J. Lett.* **848**, L13 (2017), doi:10.3847/2041-8213/aa920c.

[17] J. F. Donoghue, General relativity as an effective field theory: the leading quantum corrections, *Phys. Rev. D* **50**, 3874–3888 (1994).

[18] N. E. J. Bjerrum-Bohr, J. F. Donoghue and B. R. Holstein, Quantum gravitational corrections to the nonrelativistic scattering potential of two masses, *Phys. Rev. D* **67**, 084033 (2003).

[19] J. D. Bekenstein, Black holes and entropy, *Phys. Rev. D* **7**, 2333–2346 (1973).

[20] S. W. Hawking, Particle creation by black holes, *Commun. Math. Phys.* **43**, 199–220 (1975).

[21] F. Hofmann and J. Müller, Relativistic tests with lunar laser ranging, *Class. Quantum Grav.* **35**, 035015 (2018).

[22] J. Washburn, The cosmological constant on the recognition lattice: vacuum energy, record creation and the expansion of the universe, Zenodo (2026), doi:10.5281/zenodo.23806713; Pith preprint doi:10.67347/pith.preprint.2026.000021.

[23] R. L. Workman *et al.* (Particle Data Group), Review of particle physics, *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022).

[24] E. Tiesinga, P. J. Mohr, D. B. Newell and B. N. Taylor, CODATA recommended values of the fundamental physical constants: 2018, *Rev. Mod. Phys.* **93**, 025010 (2021).

[25] J. H. Gundlach and S. M. Merkowitz, Measurement of Newton's constant using a torsion balance with angular acceleration feedback, *Phys. Rev. Lett.* **85**, 2869–2872 (2000).

[26] T. Quinn, H. Parks, C. Speake and R. Davis, Improved determination of G using two methods, *Phys. Rev. Lett.* **111**, 101102 (2013); Erratum, *Phys. Rev. Lett.* **113**, 039901 (2014).

[27] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli and G. M. Tino, Precision measurement of the Newtonian gravitational constant using cold atoms, *Nature* **510**, 518–521 (2014).

[28] R. Newman, M. Bantel, E. Berg and W. Cross, A measurement of G with a cryogenic torsion pendulum, *Phil. Trans. R. Soc. A* **372**, 20140025 (2014).

[29] Q. Li *et al.*, Measurements of the gravitational constant using two independent methods, *Nature* **560**, 582–588 (2018).

[30] H. V. Parks and J. E. Faller, Simple pendulum determination of the gravitational constant, *Phys. Rev. Lett.* **105**, 110801 (2010); Erratum, *Phys. Rev. Lett.* **122**, 199901 (2019).