

The Native Lattice Graviton Is Not Massless: Ward Criteria and the Limits of Local Regge Repairs at $\kappa = 1$

Jonathan Washburn

Recognition Physics Institute, Austin, Texas

jon@recognitionphysics.org

Abstract

The native Lorentzian Regge path sum on the four-dimensional Kuhn lattice fails the zero-momentum masslessness test at one loop. At identified $\kappa = 1$, with flat measure and Feynman damping on the eleven non-posting edge channels, its per-site transverse-traceless remainder does not have ordered limit zero. This is a theorem with no remaining mathematical premise; independent verification of its final composition is pending. It does not determine a finite particle mass. We organize possible repairs by an exact identity: the remainder is $i/2$ times the difference between second- and first-order Ward defects. Real weights cannot supply the missing vertex symmetry. Smooth corrections of cubic or higher order in the deficits leave the one-loop remainder unchanged at fixed slope and weight. Area-additive charges reduce to one even-curvature parameter, whose tested values fail numerically. More general local charges retain eight relevant area derivatives when their flat even curvature vanishes. Their corner condition is exact, while cancellation of their integrated remainder and restoration of their classical second-order identity have different evidential status. These results delimit local repairs; they do not prove a universal prohibition on every local charge. A successful repair must derive its coefficients and pass the summed Ward test on every transverse-traceless probe.

1 The question is a zero-momentum Ward test

An interacting lattice action can reproduce Einstein's linear equations at long wavelength and still give its graviton an inadmissible zero-momentum term at one loop. Flat-space vertex moves may be zero modes of the quadratic action without being symmetries of the interacting action. That distinction is decisive on the Lorentzian Kuhn lattice.

We study the Regge phase and the native determinant specified below, following the lattice construction and one-loop analysis in [1, 2]. The central result is a failure of the model's

masslessness condition, without an unresolved estimate near the infrared origin. The second task is to determine what this failure excludes. Three statements must be kept separate: an action has an exact vertex symmetry; its one-loop determinant obeys the associated Ward identity; its summed transverse-traceless remainder tends to zero. The first can motivate the second, and the second is sufficient for the third. The third alone permits cancellations that do not require either stronger statement.

The word "native" fixes the object of the theorem. It means the graviton determinant of the eleven non-posting channels, flat measure, and the regulator $+i\varepsilon I$. It does not include an independently derived ghost or gauge measure, and is not a theorem about every quantization of Regge calculus. Likewise, "not massless" means the failure of a specified ordered limit to be zero. Reading a complex zero-momentum response as a pole mass or decay width requires a propagation law and a normalization. We retain those physical identifications explicitly.

Status convention. Definitions of the lattice and determinant specify a Model. The selection $\kappa = 1$ is an Identification. Results stated as theorems are proved statements; a result whose final independent verification is unfinished carries the sentence "Proof complete, independent verification pending." Values from quadrature or rank computations are marked Numerical, readings of those values are Inference, and unfinished mathematical or physical requirements are marked Open. Numerical evidence is never used here to prove a class-wide exclusion.

2 Lattice, determinant and masslessness condition

2.1 The native lattice

The vertices are $x \in (\mathbb{Z}/N\mathbb{Z})^4$. For each ordering of the four coordinate axes, the successive partial sums of its unit vectors triangulate a cube into a simplex. There are 24 such simplices per cube. Its fifteen edge classes are

$$d \in \{0,1\}^4 \setminus \{0\}.$$

A configuration assigns a real squared length ℓ_e to every edge. The four classes with $|d| = 1$ are the postings. The flat metric and squared edge lengths are

$$g_\kappa = I - \kappa \mathbf{1}\mathbf{1}^\top, \qquad \ell_d^0 = |d|(1 - \kappa|d|), \qquad \det g_\kappa = 1 - 4\kappa. \tag{1}$$

The background is Lorentzian for $\kappa > 1/4$. The hinge geometry used here is nondegenerate for $\kappa > 1/2$. At $\kappa = 1$ the squared lengths are $0, -2, -6, -12$ for classes of size $1, 2, 3, 4$. The postings are null. This last physical reading is an Identification, not a free theorem about the metric of every possible ledger realization.

Let S be the geometrical Lorentzian Regge action with unit overall coefficient, with the sign and area normalization of [1]. A constant symmetric strain B changes the flat metric to $g + B$, where $g = g_1$. The resulting configurations remain flat. On each torus wave k, let $K_N(B,k)$ be the Hessian of S restricted to the eleven classes with $|d| \geq 2$, evaluated on this strained flat family. The regulated wave determinant is

$$D_{N,k,\varepsilon}(B) = \det\big(K_N(B,k) + i\varepsilon I_{11}\big), \qquad \varepsilon > 0. \tag{2}$$

This is the native graviton determinant, with flat measure in the retained squared lengths. Real symmetric Hessians become Hermitian Bloch matrices; positive ε makes their shifted matrices invertible at real waves. The regulator and this choice of determinant are MODEL clauses.

A positive physical phase scale a changes this matrix to $aK_N + i\varepsilon I$ and its determinant to $a^{11}D_{N,k,\varepsilon/a}$. The background-independent factor contributes no strain derivative. Rescaling the positive damping parameter therefore preserves failure of the ordered-zero test for the flat-measure phase aS.

Proof complete, independent verification pending.

2.2 Subtracting the volume response

Let D be a nonzero complex function of symmetric strains, twice differentiable near $B = 0$. A local logarithm may be used; only its derivatives enter. For a symmetric H, put

$$\Sigma_H[D] = \frac{i}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\log D(tH)\Big|_{t=0}, \tag{3}$$

$$\sigma_{\mathrm{dil}}[D] = \frac{i}{2}\frac{\mathrm{d}}{\mathrm{d}t}\log D(tg)\Big|_{t=0}, \qquad \tau_H = \mathrm{tr}(g^{-1}Hg^{-1}H), \tag{4}$$

$$\mathscr{R}_H[D] = \Sigma_H[D] + \frac{\tau_H}{4}\sigma_{\mathrm{dil}}[D]. \tag{5}$$

For $\mathrm{tr}(g^{-1}H) = 0$, this removes the response that depends only on volume. In particular a cosmological term cannot cancel a nonzero remainder. The subtraction coefficient is fixed; it is not a fitted counterterm.

The probe used in the non-masslessness theorem is the posting-null representative

$$H_* = \begin{pmatrix} 0 & 3/2 & -3/2 & 0 \\ 3/2 & 0 & 0 & 0 \\ -3/2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad g^{-1} = I - \frac{1}{3}\mathbf{1}\mathbf{1}^{\top}, \qquad \tau_{H_*} = 6. \tag{6}$$

It has $\mathrm{tr}(g^{-1}H_*) = 0$ and is transverse to the null covector $(1,1,1,0)$ in the sense $H_*g^{-1}(1,1,1,0)^{\top} = 0$. Its diagonal entries vanish, so it does not change the postings. Define the per-site response

$$T_N(\varepsilon) = \frac{1}{N^4}\sum_k \mathscr{R}_{H_*}[D_{N,k,\varepsilon}]. \tag{7}$$

Definition 2.1 (The native one-loop masslessness test)**.** The native model passes the test if there exists $E : (0,\infty) \to \mathbb{C}$ such that

$$\lim_{N\to\infty} T_{N+3}(\varepsilon) = E(\varepsilon) \quad \text{for every } \varepsilon > 0, \qquad \lim_{\varepsilon\to 0^+} E(\varepsilon) = 0. \tag{8}$$

The volume limit is taken first. The shift by 3 simply keeps every torus side in the domain of the lattice construction.

For a repaired model the zero condition is required on every posting-null traceless symmetric probe, rather than just H_*. This five-dimensional space contains two inequivalent components under permutations of the four axes, conventionally denoted E and T_2. A cancellation on a single sum of probes does not establish vanishing on both components. Reducing all probes to one representative in each component additionally requires covariance of the integrated quadratic response under those permutations.

3 The native one-loop graviton is not massless

Theorem 3.1 (Failure of the native masslessness test)**.** *At $\kappa = 1$, the native determinant* (2)*, with flat measure and Feynman damping on all eleven retained channels, fails* (8)*. No additional mathematical premise is required.*

Proof complete, independent verification pending.

Proof. We use the localized analytic estimates for this determinant [1, 2]. With normalized Brillouin measure $\mathrm{d}\mu = (2\pi)^{-4}\mathrm{d}^4q$, write $F_\varepsilon(q) = \Sigma_{H_*}[D_{q,\varepsilon}]$ and $G_\varepsilon(q) = \sigma_{\mathrm{dil}}[D_{q,\varepsilon}]$. At fixed positive damping these are continuous periodic functions, so the volume sums converge to $\mathscr{T}(\varepsilon) = \int F_\varepsilon \mathrm{d}\mu$ and $L(\varepsilon) = \int G_\varepsilon \mathrm{d}\mu$. Native scaling expresses the latter integrand in terms of the eleven eigenvalues $\lambda_j(q)$ of the undamped flat matrix:

$$G_\varepsilon(q) = -\frac{i}{2}\sum_{j=1}^{11}\frac{\lambda_j(q)}{\lambda_j(q) + i\varepsilon}.$$

The matrix is invertible almost everywhere: its determinant is real analytic and not identically zero, as an exact inverse at (π, π, π, π) witnesses. Each summand has modulus at most one, so dominated convergence gives the dilation limit. Since $\tau_{H_*} = 6$, the subtracted combination is

$$E(\varepsilon) = \mathscr{T}(\varepsilon) + \frac{6}{4}L(\varepsilon), \qquad \lim_{\varepsilon\to 0^+} L(\varepsilon) = -\frac{11i}{2}.$$

Thus masslessness would imply $\mathscr{T}(\varepsilon) \to 33i/4$ and in particular $\mathrm{Re}\,\mathscr{T}(\varepsilon) \to 0$. A smooth partition of unity separates the four axis corners, the origin, and the remaining characteristic set of this TT integral.

The native eleven-channel corner Schur complements have quadratic Lorentzian zeros. Constant strain moves the relevant zeros at first order. For the probe H_*, the real corner contributions satisfy $C_c(\varepsilon) \le -r_c \log(1/\varepsilon) + b_c$ with $r_c > 0$ for $c = 1, 2$. The other two corners have uniform upper bounds. Thus the combined corner upper bound tends to $-\infty$ as $\varepsilon \to 0^+$.

To exclude cancellation, the complement must be bounded above. Nondegenerate characteristic points admit local contour bounds. At exceptional points of the full block, hyperbolicity supplies a direction increasing the vanishing eigenvalues together. The remaining native gauge strata require separate exact principal-minor, crossing and dissipative certificates; they are not all covered by the full-block hyperbolicity statement. Together these classifications and bounds give the required upper estimate away from the origin and corners.

The origin requires a separate estimate. There is an open neighbourhood U of zero such that, for every nonnegative, compactly supported C^8 cutoff χ in U, the integral of χ times the real

part of the regulated TT integrand is bounded above for all sufficiently small positive ε. This is the origin estimate used by the partition argument, not an assumption about a single sampled ray.

Its proof resolves the gauge hyperplanes and their intersections. A first reduction covers their generic points, circles and axes. At the four tangency points between a gauge hyperplane and the light cone, a second Schur reduction and rescaling produce a damped four-by-four family. The damping supplies the local dissipative bounds used by the reduction. Bounds away from its zero-damping gauge plane reduce the remaining problem to points on that plane. In normalized coordinates (T, U, V) set $Q = U^2 + UV + V^2$. The plane points are covered by $T = 0$, $T^2 = Q$, $T^2 = 5Q$, and their complement. The equatorial case has $Q > 0$ because the degenerate cap is excluded by the chart's domain. The other strata have explicit local bounds as well. This covers the entire domain used by the reduction and proves the origin estimate.

Choose the two corner cutoffs nonnegative, disjoint and equal to one nearer their corners. The exact partition then gives $\operatorname{Re}\mathscr{T} = C_1 + C_2 + C_{\mathrm{rest}}$, with $C_{\mathrm{rest}}(\varepsilon) \le b$ eventually. Hence

$$\operatorname{Re}\mathscr{T}(\varepsilon) \le -(r_1 + r_2)\log(1/\varepsilon) + b_1 + b_2 + b.$$

The right side tends to $-\infty$, contradicting the consequence $\operatorname{Re}\mathscr{T}(\varepsilon) \to 0$ of (8). This proves the theorem. □

The conclusion is stronger than a finite-grid observation of a nonzero number and narrower than a measured mass. It excludes ordered limit zero for the specified determinant. It neither supplies a finite limiting remainder nor solves a momentum-dependent pole equation. A different action, measure inside the fluctuation matrix, regulator, or derived ghost sector requires its own test.

4 An exact criterion for a repair

The Ward identity organizes the repair problem without presupposing an exact symmetry. Write D for any determinant satisfying the regularity used in (5). A linear vertex map has tangent

$$V_X(B) = X^{\top}(g + B) + (g + B)X.$$

If $\operatorname{tr} X = 0$, the flow e^{sX} preserves volume. Its Ward identity is

$$\mathrm{d}D(B)[V_X(B)] = 0 \qquad (\operatorname{tr} X = 0) \tag{9}$$

at every sufficiently small symmetric strain B.

Theorem 4.1 (The Ward identity removes the remainder)**.** *Let $g = g_\kappa$, $\kappa \neq 1/4$, and $D(0) \neq 0$. Suppose D is differentiable near zero with derivative differentiable at zero, and obeys* (9)*. Then $\mathscr{R}_H[D] = 0$ for every symmetric H with $\operatorname{tr}(g^{-1}H) = 0$.*

Proof. Let $\ell = \mathrm{d}D(0)$ and $Q = \mathrm{d}^2D(0)$. For any symmetric S, take $Y = g^{-1}S/2$ and subtract its scalar part to make it traceless. The identity at zero gives

$$\ell(S) = \frac{1}{4}\operatorname{tr}(g^{-1}S)\ell(g). \tag{10}$$

Differentiate (9) at $B = tH$, with $X = g^{-1}H/2$. Since $\operatorname{tr} X = 0$ and $X^{\top}g + gX = H$, this gives

$$Q(H,H) + \ell(Hg^{-1}H) = 0.$$

Equation (10) yields $\ell(H) = 0$ and $Q(H,H) = -(\tau_H/4)\ell(g)$. Therefore

$$\mathscr{R}_H[D] = \frac{i}{2D(0)}\left(Q(H,H) + \frac{\tau_H}{4}\ell(g)\right) = 0.$$

□

An exact invariance $D(A^{\top}(g+B)A-g) = D(B)$ for matrices A near the identity with $\det A = 1$ gives (9) by differentiation. Every differentiable function of $\det(g+B)$ obeys the identity. These are sufficient conditions. A candidate need not satisfy them wave by wave to pass the summed test.

Theorem 4.2 (The remainder is a difference of Ward defects)**.** *Let $D(0) \neq 0$, let H be symmetric, and choose X with $X^{\top}g + gX = H$. Put*

$$S_0 = X^{\top}H + HX - \frac{\tau_H}{4}g, \tag{11}$$

$$\mathrm{WD}_1 = \frac{\mathrm{d}D(0)[S_0]}{D(0)}, \tag{12}$$

$$\mathrm{WD}_2 = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}D(tH)[X^{\top}(g+tH)+(g+tH)X]}{D(tH)}\bigg|_{t=0}. \tag{13}$$

Then

$$\mathscr{R}_H[D] = \frac{i}{2}(\mathrm{WD}_2 - \mathrm{WD}_1). \tag{14}$$

The same identity holds after a finite wave sum and per-site normalization.

Proof. Set $f(t) = \log D(tH)$. The numerator direction in WD_2 is $H + t(X^{\top}H + HX)$. Thus

$$\mathrm{WD}_2 = f''(0) + \frac{\mathrm{d}D(0)[X^{\top}H + HX]}{D(0)}.$$

Subtracting WD_1 gives $f''(0) + (\tau_H/4)\mathrm{d}D(0)[g]/D(0)$. Multiplication by $i/2$ is (5). Summation is linear. □

For traceless H, $X = g^{-1}H/2$ makes S_0 traceless for g. If the first-order identity holds, WD_1 vanishes. Without that identity it cannot be discarded. In particular, testing just the second-order defect can give the wrong answer for an actual lattice determinant. The necessary and sufficient condition for the defined zero-momentum test is that the ordered limit of the summed *difference* in (14) vanish on every required probe.

5 What weights and higher deficits cannot do

5.1 A real weight cannot create the missing action symmetry

The native flat Hessian annihilates the vertex moves. The interacting action does not possess their full symmetry. Let g_b be the alternating vertex moves generated by $(-1)^{x_0}e_b$. On tori of side divisible by 4, the nonzero cubic pairing detects the combination

$$-4c_0 + 3(c_1 + c_2 + c_3). \tag{15}$$

Theorem 5.1 (No real weight or density supplies that symmetry)**.** *Let $\kappa = 1$, $N \geq 4$, $4 \mid N$, and $a \neq 0$. Let W be any real function of the configuration. A flow with a C^2 generator whose flat value is $\sum_b c_b g_b$, with* (15) *nonzero, cannot preserve the exponent $iaS - W$ on a neighbourhood of flat space. The conclusion also holds with a real density Jacobian.*

Proof. Invariance of the complex exponent forces invariance of its imaginary part aS. The cubic vertex obstruction excludes the stated generator for S [1, Sec. 6]. A real Jacobian only changes the real part and does not alter the obstruction. □

This excludes a mechanism for obtaining exact symmetry. It does not prove that every real weight leaves the summed remainder nonzero. A weight inside the Gaussian precision changes that matrix and may change both parts of its remainder.

There is also a useful distinction between a weight inside the operator and a real scalar normalization of the determinant. A real, nonzero factor $w(B)$, twice differentiable along H and differentiable along g, contributes a purely imaginary remainder, because the derivatives of $\log|w|$ are real. Consequently $\operatorname{Re}\mathscr{R}_H[Dw] = \operatorname{Re}\mathscr{R}_H[D]$. Similarly, a nonsingular Hermitian one-loop matrix without a regulator has a real determinant and a purely imaginary remainder. Neither statement licenses removing the regulator before controlling the singular waves.

5.2 Cubic and higher deficit terms

Theorem 5.2 (Higher deficits preserve the obstruction)**.** *Let $\kappa = 1$, $N \geq 3$, $c \neq 0$, and let every Γ_h be C^3 near flat space. For*

$$S' = cS + \sum_h \delta_h^3 \Gamma_h \tag{16}$$

the flat Hessian is cD^2S. Its cubic vertex with a vertex-move leg equals that of cS. If $4 \mid N$, the flows excluded in Theorem 5.1 are excluded for S' as well.

Proof. At a flat configuration, $\delta_h = 0$. The first two derivatives of $\delta_h^3\Gamma_h$ vanish. Its third derivative is

$$6\Gamma_h \, \mathrm{d}\delta_h[u] \, \mathrm{d}\delta_h[v] \, \mathrm{d}\delta_h[w].$$

Vertex moves leave deficits unchanged to first order, so this expression is zero when any argument is a vertex move. The Regge cubic obstruction is therefore unchanged apart from the nonzero factor c. □

Theorem 5.3 (The whole strained one-loop family is unchanged)**.** *Compare* (16) *with cS, using the same weight, retained channels, and regulator. Their Gaussian matrices agree along every sufficiently small constant strain. Their wave determinants, remainders on every probe, and regular summed defect differences agree before either limit is taken.*

Proof complete, independent verification pending.

Proof. Every nearby constant strain is another flat lattice, not merely a tangent to one. Thus every deficit remains zero along this family. At each member of the family, every term in $D^2(\delta_h^3\Gamma_h)$ retains a factor of δ_h, so the Hessian correction vanishes. The finite hinge sum, Bloch transform, channel restriction and common weight and regulator preserve equality. Taking determinant derivatives proves the assertion. □

The comparison is at the *same* slope and weight. Changing the scale of the phase with a fixed weight is a different operator. The theorem covers smooth terms of fourth and higher deficit order, and the familiar hinge charges $cA\delta + \delta^3\gamma(A,\delta)$. It does not cover a quadratic deficit term, a different area slope, or an insufficiently smooth correction. Native posting phases proportional to S, a nonzero fixed-coupling deficit-linear loop phase, and the average over the 24 simplex-order phases all remain within the excluded same-slope family when their defining identifications make them proportional to Regge's phase [2].

6 Local charges: reductions, choices and numerical exclusions

6.1 Only the slope and even curvature enter

Take a C^3 local charge $C(A,\delta)$ that vanishes at zero deficit, and absorb its common action unit into C. Write

$$\sigma(A) = \partial_\delta C(A,0), \qquad e(A) = \partial_\delta^2 C(A,0). \tag{17}$$

Along a strained flat family, the Hessian is

$$D^2 \sum_h C(A_h,\delta_h) = \sum_h \big[\sigma(A_h) D^2\delta_h \tag{18}$$

$$+ \sigma'(A_h)(\mathrm{d}A_h \otimes \mathrm{d}\delta_h + \mathrm{d}\delta_h \otimes \mathrm{d}A_h) + e(A_h)\mathrm{d}\delta_h \otimes \mathrm{d}\delta_h\big]. \tag{19}$$

Proof complete, independent verification pending.

The chain rule proves the formula: the pure-area derivatives vanish because $C(A,0) = 0$, while the displayed terms are exactly the remaining second derivatives. Thus equal slope and even-curvature functions give equal flat-family Gaussian operators with a common weight and regulator. This is a reduction of the one-loop problem; it does not derive either function from the records.

For differentiating these flat-family matrices twice in strain, the required derivatives of e must exist. The general-charge finite-jet result below assumes $C \in C^3(\mathbb{R}^2)$ and $e \in C^3(\mathbb{R})$. The stronger regularity of e is part of the result; it does not follow merely by calling a two-variable charge C^3.

6.2 Area-additive and ratio charges

If a charge adds over pieces of a hinge at every fixed deficit, and is continuous in area, then

$$C(A, \delta) = Af(\delta).$$

For $f'(0) > 0$ its bare one-loop problem reduces, up to rescaling the regulator, to

$$\beta = \frac{f''(0)}{f'(0)}. \tag{20}$$

This reduction is a theorem. At fixed positive regulator the response still sees the overall scale through that rescaling; the ordered-zero condition is invariant under this positive regulator rescaling. More generally, positive regulator rescaling preserves both the existence and the value of any finite complex ordered limit.

Proof complete, independent verification pending.

Indeed, if the volume-limit function is $F(\varepsilon)$, rescaling replaces it by $F(\varepsilon/c)$; the inverse substitutions $\eta = \varepsilon/c$ and $\varepsilon = c\eta$, with $c > 0$, preserve the approach to zero from above. A common nontrivial weight cannot simply be rescaled away with the phase.

Numerical evidence. The published calculations tested the bare β family from -10 to 1000, at $\kappa = 1$ and $3/2$, on side-16 midpoint grids at regulators 0.05 and 0.025. No tested value made both components vanish. At $\kappa = 1$, the tested E response had modulus at least 4.98, and its imaginary part exceeded 3.88 in the densely sampled band near the smallest values [2]. These are finite computations, without interval enclosures. They do not exclude every real β or certify an ordered-limit lower bound. A constant-amplitude even charge was tested over a comparable amplitude range and likewise gave nonzero responses; that statement also remains NUMERICAL.

An even charge depending on area only through its ratio to a reference area admits an exact reduction of its relevant derivatives. Its reported failure on the E probe, with modulus above 6 after satisfying the corner condition, is NUMERICAL. The algebraic reduction and the numerical evaluation of the reduced integral are distinct claims. An even term homogeneous of degree one under every positive simultaneous rescaling of area and reference area fails the corner condition exactly; this is a sharper structural exclusion than a scan.

6.3 The eight numbers that the even freedom retains

At $\kappa = 1$ the four flat hinge areas are

$$(A_1, A_2, A_3, A_4) = \left(\frac{1}{2}, 1, \frac{3}{2}, \sqrt{3}\right). \tag{21}$$

Consider a fixed positive Regge slope $\sigma(A) = cA$, $c > 0$, and even curvature with $e(A_o) = 0$ at these areas. Then the flat Hessian is c times Regge's. The first two strain derivatives of its even contribution depend only on

$$p_o = e'(A_o), \qquad q_o = e''(A_o), \qquad o = 1, 2, 3, 4. \tag{22}$$

Proof complete, independent verification pending.

This finite-jet reduction holds at finite torus side and regulator before discussing a limit. It does not say that those eight numbers have been selected by the ledger.

A record-compatible realization is an even term

$$e(A)\,\frac{1-\cos(k\delta)}{k^2}, \tag{23}$$

where the posting units are integer multiples of $2\pi/k$. It vanishes at every posting record. With $e(A_o)=0$ it does not change the flat quadratic form; its long-wave cubic correction drops out of the classical limit under the established boundedness conditions. Thus the classical and record clauses alone leave the area derivatives free. Smooth interpolation of prescribed finite jets is a construction, not a physical selection rule.

Proof complete, independent verification pending.

At unit posting scale and the corresponding unit Regge slope the exact corner condition is

$$p_1 - p_3 = -\frac{1}{3\sqrt{3}}. \tag{24}$$

On every admitted even torus $N \geq 4$, for a Regge-slope charge with even curvature vanishing at (21), the strain derivative in the T_2 direction of the quadratic form on the doubler vanishes at all four axis corners if and only if (24) holds. The values p_2, p_4 do not enter this condition.

Proof complete, independent verification pending.

Interpreting a nonzero corner pairing as a surviving logarithm uses the corner asymptotic mechanism (Inference). The corner condition alone cancels that pairing; it does not prove that the remainder has ordered limit zero.

Numerical tuning is not a derived repair. After imposing (24), the bare integrated TT test produced four real linear conditions on the q_o, with a numerically nonsingular coefficient matrix. At $p=(-1/(3\sqrt{3}),0,0,0)$ the reported solution was approximately

$$q = (-15.1,\ 4.3,\ -3.6,\ 0.3).$$

These values are Numerical, as are the invertibility of that integrated coefficient matrix and convergence of finite-regulator fits. They exhibit possible tuning at one point, rather than a proof that all eight-number choices fail or a proof of a massless ordered limit. With the derived weight, the reported tuned second derivatives are of order 10^4. The construction remains a choice unless a physical rule determines these numbers independently of the desired cancellation.

7 The classical second-order test is independent

Restoring a zero-momentum one-loop response is not the same as repairing the action's classical vertex identity. The latter has an explicit obstruction. Here is the first-order field used to exhibit it. For an edge (y, S) let $m = y + \mathbf{1}_S/2$ be its midpoint, $s = |S|$, and put

$$\xi(m) = \frac{1}{2}(m_0 + m_1 - m_2 - m_3,\ m_0 - m_1 + m_2 - m_3,\ m_0 - m_1 - m_2 + m_3).$$

Write $\Phi(\xi) = \xi_1^3 - 3\xi_1\xi_2^2$. The squared-length field is

$$h(y, S) = -2\Phi(\xi(m))\left(s + \frac{s^2}{2}\right) + \xi_2(m)P(S), \tag{25}$$

where P is $45/8$ on $\{0\},\{3\},\{0,1,3\},\{0,2,3\}$; $-45/8$ on $\{1\},\{2\},\{0,1,2\},\{1,2,3\}$; $45/4$ on $\{1,2\}$; $-45/4$ on $\{0,3\}$; and zero otherwise. The second term is the slaved non-metric correction. It is essential: the sampled metric harmonic alone is not this exact linearized solution.

The static field (25) satisfies the first-order equations everywhere on the infinite lattice. For the compact move of the record $y = (2, 1, -1, 0)$ in direction e_0, let g_y be the induced first-order squared-edge change. The notation $D^3S(h, h, g_y)$ here means the finite local contraction on the hinges meeting that move, not an infinite sum of the polynomial field's action. Its value is

$$D^3S(h, h, g_y) = 756\sqrt{3} \neq 0. \tag{26}$$

The corresponding side-8 torus calculation gives the same local obstruction, with first-order equations verified at the thirty edges incident to that record. These are independently verified theorems; the infinite-lattice result additionally establishes the first-order equations everywhere.

The mechanism is direct. If a second-order correction d existed, its row equations would give

$$D^2S(d, \delta_e) + D^3S(h, h, \delta_e) = 0$$

at the incident edges. Pair with g_y. The linearized Bianchi identity and symmetry of D^2S kill the first term for every d, while the second equals (26). Hence no such correction exists for this first-order field. The conclusion does not exclude every alternative first-order field, every nonregular family, or every modified action.

For a general local Regge-slope charge the classical vertex test reads the eight numbers $e(A_o), e'(A_o)$; with $e(A_o) = 0$, it reads only the four slopes $p_o = e'(A_o)$. It never reads the $q_o = e''(A_o)$ that remain available to tune the one-loop response. The general classical finite-jet reduction has proof complete with independent verification pending. The reported attempt to satisfy the second-order Ward test within the zero-flat-curvature class fails numerically. It is a Numerical obstruction, not a theorem excluding every even local charge.

An exact algebraic certificate can isolate what is missing from this numerical exclusion. At unit posting scale put $u_o = e'(A_o)$ and $v = (\sqrt{3}u_1, \sqrt{3}u_2, \sqrt{3}u_3, u_4)$, with components numbered $0, 1, 2, 3$ in the following rows. Four reconstructed rows are

$$r_0 = 14 + 16v_0 - 128v_1 - 192v_2, \tag{27}$$

$$r_1 = 141 + 96v_0 + 348v_1 + 384v_2, \tag{28}$$

$$r_3 = -243 - 72v_0 - 36v_1, \tag{29}$$

$$r_6 = -36v_0 - 72v_1. \tag{30}$$

Proposition 7.1 (The reconstructed rows are inconsistent)**.** *No real v makes r_0, r_1, r_3 and r_6 all zero.*

Proof complete, independent verification pending.

Proof. Expansion gives $54r_0 + 27r_1 + 41r_3 + 14r_6 = -5400$. Four zero rows would give $0 = -5400$. □

The Open step is geometric: prove that these exact reconstructed coefficients equal the four canonical local cubic Ward probes for every admissible charge. The rows were obtained from numerical data with coefficient rounding. Their exact inconsistency does not prove that correspondence, so the class-wide exclusion remains numerical. A named mathematical target is precisely that equality of probe functions, rather than the desired exclusion assumed as a premise.

Charges with nonzero $e(A_o)$ change the flat operator itself and are outside that exclusion. The reported leading, fifth-order-in-momenta Ward system has a numerical two-dimensional solution family in this larger class. An exact real solution certificate and the higher-order conditions remain Open. Passing that leading classical system would not by itself settle the summed quantum Ward test. The exact obstruction (26), the corner condition (24), the two different finite-jet reductions, and the numerical exclusions therefore carry separate statuses.

8 What is excluded and what remains to derive

The exclusions have different force. The native flat-measure determinant fails the ordered masslessness test by theorem. Real weights and densities cannot create the stated exact vertex symmetry. Cubic and higher deficit terms leave the relevant one-loop family unchanged at fixed slope and weight. Odd charges with Regge slope have no even curvature and remain in that family at one loop. Area-additive charges reduce to one parameter but are excluded only at the computed values; the ratio class has an exact reduction followed by a numerical exclusion. A degree-one homogeneous even term fails the corner condition exactly. More general even charges retain the finite data (22), and classical compatibility is not a derivation of those data.

The derived measure provides an important example of the distinction. Its cycle-count and geometric-amplitude readings are Identification clauses. Its positive semidefinite deficit weight has a proved kernel identity: away from the reciprocal lattice its kernel consists exactly of vertex moves. The restriction to eleven non-posting channels is positive definite when every coordinate wave phase differs from one; the coordinate planes require separate control [2]. This weight does not create an exact Regge vertex symmetry. With the derived dual-cycle weight and phase coefficient $a = \ln(2)/(2\pi)$ at unit action scale relative to that fixed weight, the reported weighted eleven-channel remainder is

$$\mathscr{R}_{H_*} \simeq (0.1130 \pm 0.0002) + (24.656 \pm 0.002)i \qquad \text{(Numerical)}.$$

This is a grid result at zero regulator away from the singular planes, not an interval-certified ordered-limit theorem for that operator. Dividing by a real normalization per background

can change its imaginary part, while its real part remains unchanged. Finiteness of weighted fifteen-channel responses and masslessness of this eleven-channel remainder are different questions.

Target 8.1 (A derived action passing the one-loop test)**.** At the stated $\kappa = 1$, derive a single phase and measure from the records, including every coefficient, on all admitted tori $N \geq 3$, such that:

(i) a local charge, when used, is C^3 and reproduces every posting record; the phase is stationary on the flat lattice, its flat Hessian vanishes in either slot on every first-order deficit-preserving direction, and its long-wave quadratic and cubic vertices equal Regge's with a common positive scale;

(ii) its regulated one-loop determinants are regular on every admitted torus for every $\varepsilon > 0$;

(iii) on every posting-null traceless symmetric probe, the summed defect difference (14) has ordered limit zero;

(iv) its local coefficients, if any, are derived independently of the desired zero remainder.

This is an Open target. An exact symmetry of the determinant would supply (iii), but (iii) permits a weaker summed cancellation. A local charge outside the excluded readings has not been ruled out universally. A nonlocal action can change the slope map or even kernel by coupling different hinges; nonlocality confined to higher deficit terms cannot help if the whole strained Gaussian family stays unchanged. A successful candidate must pass both the quantum test and the appropriate classical test rather than infer one from the other.

9 Physical interpretation of the obstruction

The theorem is a constraint on an actual proposed gravitational path sum, at a stated order and with its physical identifications exposed. It is not a statement that an observed graviton has a measured nonzero mass. The unrepaired model lacks the vanishing zero-momentum response required by its massless interpretation.

For scale, the native numerical response was approximately $0.75 + 6.71i$, $0.62 + 6.93i$ and $0.45 + 7.03i$ at damping 0.1, 0.05 and 0.025, respectively, with the volume sums converged at those fixed dampings [2]. These numbers do not establish the theorem; the corner and complement estimates do. In particular their order-one imaginary parts do not replace the limiting question.

The conditional mass-scale diagnostic identifies the spacing with $b = (2/\sqrt{3})\lambda_{\mathrm{rec}}$ and uses a response-to-mass map proportional to $(\ell_P/b)^2$. With the computed unrepaired remainders in the specified one-loop conventions, it gives scales of order 3×10^{28} eV for the native phase and $5{\times}10^{28}$ eV for the weighted model [2]. These are Numerical diagnostics under Identification clauses. A complex response need not be a real positive pole mass; its interpretation requires the record-time evolution and momentum dependence of the self-energy.

The GWTC-3 bound is $m_g \leq 2.42{\times}10^{-23}$ eV$/c^2$ at 90% credibility [4]. Comparing that bound to the quoted diagnostic only makes sense under the same mass interpretation and normalizations.

The diagnostic shows the scale of what an unrepaired model would have to remove under those readings. The unconditional mathematical conclusion is instead Theorem 3.1: its native one-loop TT response does not have ordered limit zero. The next physical result must come from a derived action and measure that satisfy Target 8.1.

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