Energy-Bearing Radiation from Native Charged Matter

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Abstract

How can a local charged source create a field excitation that carries energy and changes subsequent matter motion? We derive this mechanism for the native one-electron source on the infinite cubic lattice, with all four eight-state registers and the complete transverse bosonic field. The source's charged hopping creates a nonzero one-photon energy component from a localized vacuum input at every sufficiently small positive time. An exact partition of the original Hamiltonian shows that the gained free photon energy equals the decrease in the combined raw interaction, clock, Coulomb and hopping energies. The same hopping reads displaced field arms through a different matter current. The derivations retain the original operator domain and every photon sector. They identify the physical coupling available for a recognition boundary and the further construction needed to turn a field-sensitive response into a retained posting event.

1 A charged step changes the field

A physical boundary needs a way for an interaction to leave information that can affect a later interaction. The charged source studied here supplies a concrete part of that mechanism. Moving its negative charge across an edge acts on two objects together: the native register and the transverse field. The field receives a Weyl displacement. Reverse motion acts with the inverse displacement. The same hopping can read the displaced field through a later matter current.

For background on charged matter coupled to a quantized radiation field, see Spohn [1, Chapter 17].

There is also an energy question. A nonzero field amplitude by itself does not specify where its energy came from. We therefore derive the field onset and its energy account on the same complete source. The energy account uses the actual terms in the Hamiltonian, including the raw field prices. Their sum decreases by exactly the increase in free photon energy.

The source has a fixed positive unit charge at the origin and a moving negative unit charge. Set lattice spacing and $\hbar$ to one. Its inputs are $a > 0$, $C_E > 0$, hopping g, clock coefficient ϵ, and an initial state. The energy partition uses $g, \epsilon \geq 0$; strict funded emission

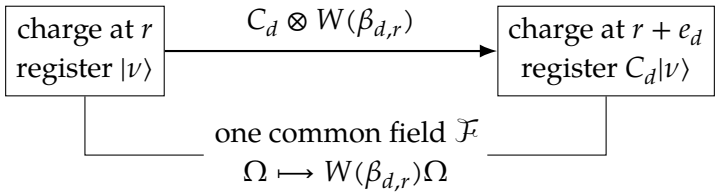

Figure 1: The charged step acts on the register and field together. Each of the six incident field prices acts on this same field space.

uses $g > 0$. Selecting the nucleus, units, coefficients and initial preparation remains part of the boundary construction.

2 The complete source

2.1 Field and register

Let $\mathbb{T}^3 = [-\pi, \pi]^3$ with normalized Lebesgue measure $\mathrm{d}\mu(k) = (2\pi)^{-3}\,\mathrm{d}^3k$. Define

$$\lambda(k) = 2\sum_{j=0}^{2}(1 - \cos k_j), \qquad \omega(k) = 2C_E \operatorname{arsinh}\left(\frac{\sqrt{\lambda(k)}}{2}\right). \tag{1}$$

Write $q_j(k) = 1 - \mathrm{e}^{\mathrm{i}k_j}$. The one-photon space consists of $L^2(\mathbb{T}^3; \mathbb{C}^3)$ fields satisfying $\sum_j q_j(k)v_j(k) = 0$ almost everywhere. Let $P(k)$ be the orthogonal projection onto this two-dimensional kernel, with its evident definition at $q(k) = 0$. The full field is the symmetric Fock space

$$\mathcal{F} = \bigoplus_{n=0}^{\infty} \mathcal{H}_n, \qquad \mathcal{H}_n = \operatorname{Sym}^n \mathcal{H}_1, \qquad B = \mathrm{d}\Gamma(\omega). \tag{2}$$

Thus B acts in sector n by multiplication by $\sum_{j=1}^{n} \omega(k_j)$. Its domain is

$$\mathcal{D}(B) = \left\{ f \in \mathcal{F} : \sum_{n=0}^{\infty} \|B_n f_n\|^2 < \infty \right\}. \tag{3}$$

Although ω is bounded on the Brillouin zone, B retains arbitrarily large photon number. The vacuum Ω has unit norm and $B\Omega = 0$.

The native label is $\nu \in N = \{0, \dots, 7\}^4$. The internal space and complete source space are

$$\mathcal{K} = \mathbb{C}^{4096} \otimes \mathcal{F}, \qquad \mathcal{H} = \ell^2(\mathbb{Z}^3; \mathcal{K}). \tag{4}$$

All norms and inner products below are those of these spaces; the inner product is conjugate linear in its first argument.

For completeness, the native crossing is $C_d = S_d F_3$, $d = 0, 1, 2$. Here

$$S = \operatorname{diag}(1, -1, -1, 1, -1, 1, 1, -1),$$

and F exchanges $0 \leftrightarrow 1$, $2 \leftrightarrow 3$, $4 \leftrightarrow 5$, and $6 \leftrightarrow 7$. A subscript places the matrix on the indicated register and the identity on the others. Hence $C_d^* = C_d$ and $C_d^* C_d = \mathrm{I}$.

The single-register clock A is the Hermitian circulant matrix

$$A_{ij} = \frac{8\mathbf{1}_{i=j} - 1}{2} + \frac{\mathrm{i}}{2}(u_{i-j} + \sqrt{2}\, v_{i-j}), \tag{5}$$

where subtraction is modulo eight and

$$u = (0,1,1,-1,0,1,-1,-1), \qquad v = (0,1,0,1,0,-1,0,-1).$$

Its Fourier eigenvectors have components $\mathrm{e}^{-2\pi \mathrm{i} m j/8}$ and eigenvalues $m = 0, \dots, 7$, as substitution in the finite geometric sum shows. In particular $A \geq 0$. The complete native clock is $A_\epsilon = \epsilon \sum_{j=0}^{3} A_j$.

2.2 Charged profiles and the six raw prices

For the positive edge from r to $r + e_d$, put

$$\beta_{d,r}(k) = \mathrm{i}\left(\frac{a}{2C_E \sqrt{\lambda(k)}\sqrt{1 + \lambda(k)/4}} \right)^{1/2} \mathrm{e}^{\mathrm{i} r \cdot k} P(k) e_d, \tag{6}$$

$$h_{d,r}(k) = \mathrm{i}\left(\frac{\sqrt{\lambda(k)}\sqrt{1 + \lambda(k)/4}}{4a \operatorname{arsinh}(\sqrt{\lambda(k)}/2)} \right)^{1/2} \mathrm{e}^{\mathrm{i} r \cdot k} P(k) e_d. \tag{7}$$

Values at $k = 0$ may be set to zero, since this point has measure zero. These are full one-photon vectors. Near zero, the squared radial coefficient of β is of order $|k|^{-1}$, which is integrable in three dimensions; the coefficient of h is bounded. No neighborhood of zero is removed. Both vectors are transverse by construction. Multiplication of (7) by $\sqrt{\omega}$ gives the electric read coefficient

$$\sqrt{\omega}\, h_{d,r}(k) = \mathrm{i}\left(\frac{C_E \sqrt{\lambda(k)}\sqrt{1 + \lambda(k)/4}}{2a} \right)^{1/2} \mathrm{e}^{\mathrm{i} r \cdot k} P(k) e_d.$$

The charge at site r is $\rho_r = \delta_0 - \delta_r$, with Fourier transform $\hat{\rho}_r(k) = 1 - \mathrm{e}^{\mathrm{i} r \cdot k}$. Its longitudinal energy is

$$V(r) = \frac{a}{2} \int_{\mathbb{T}^3} \frac{|1 - \mathrm{e}^{\mathrm{i} r \cdot k}|^2}{\lambda(k)} \, \mathrm{d}\mu(k) \geq 0. \tag{8}$$

The positive edge load at the current charge position is

$$\ell_{r,d,+} = \operatorname{Re} \int_{\mathbb{T}^3} \frac{\overline{\mathrm{e}^{\mathrm{i} r \cdot k} q_d(k)}\, (1 - \mathrm{e}^{\mathrm{i} r \cdot k})}{\lambda(k)} \, \mathrm{d}\mu(k). \tag{9}$$

For the negative incident edge use $h_{r,d,-} = -h_{d,r-e_d}$ and

$$\ell_{r,d,-} = -\operatorname{Re} \int_{\mathbb{T}^3} \frac{\overline{\mathrm{e}^{\mathrm{i}(r-e_d)\cdot k} q_d(k)}\, (1 - \mathrm{e}^{\mathrm{i} r \cdot k})}{\lambda(k)} \, \mathrm{d}\mu(k).$$

The positive incident read is $h_{r,d,+} = h_{d,r}$. These six loads come from the same charge and the same lattice divergence.

The raw scalar law and raw field operator are

$$x(s) = \frac{s + \sqrt{s^2 + 4}}{2}, \quad f(s) = \frac{x(s) + x(s+1)}{2}, \quad R_{h,\ell} = f(\ell + Q(\sqrt{\omega}\, h)), \tag{10}$$

where $Q(z) = a^\dagger(z) + a(z)$ and the last expression uses spectral calculus. Equivalently, in the Gaussian coordinate of the mode $\sqrt{\omega}\, h$, this is multiplication by $f(\ell + \|\sqrt{\omega}\, h\|\, s)$, with s a standard real Gaussian. If $\sqrt{\omega}\, h = 0$, it is $f(\ell)\mathrm{I}$. Every $R_{h,\ell}$ acts on the common $\mathcal{F}$ in (2).

2.3 The Hamiltonian and its original domain

The site action on $\mathcal{K}$ is

$$S_r = \mathrm{I}_N \otimes \left(B + g \sum_{d,\sigma=\pm} R_{h_{r,d,\sigma}, \ell_{r,d,\sigma}} \right) + A_\epsilon \otimes \mathrm{I} + V(r)\mathrm{I}. \tag{11}$$

Its domain is the finite product of $\mathcal{D}(B)$ over N. The charged edge unitary is

$$U_{d,r} = C_d \otimes W(\beta_{d,r}), \qquad W(z) = \exp(a^\dagger(z) - a(z)). \tag{12}$$

The full hopping and Hamiltonian are

$$(K\psi)_r = -g \sum_{d=0}^{2} \left(U_{d,r-e_d} \psi_{r-e_d} + U_{d,r}^{-1} \psi_{r+e_d} \right), \tag{13}$$

$$(H\psi)_r = S_r \psi_r + (K\psi)_r, \tag{14}$$

$$\mathcal{D}(H) = \left\{ \psi \in \mathcal{H} : \psi_r \in \mathcal{D}(S_r) \text{ for every } r, \sum_r \|S_r \psi_r\|^2 < \infty \right\}. \tag{15}$$

The label r is the moving charge coordinate. The field is the global field within each component of its joint state, just as in a particle-field tensor product. There is no independent field for each incident price.

Lemma 1. *For $a > 0$, $C_E > 0$ and real g, ϵ, the operator H is self-adjoint on* (15)*. For every $\psi \in \mathcal{D}(H)$, its unitary evolution $\psi(t) = \mathrm{e}^{-\mathrm{i}tH}\psi$ preserves that domain and* $\mathrm{Re}\langle \psi(t), H\psi(t)\rangle = \mathrm{Re}\langle \psi, H\psi\rangle$.

Proof. The scalar f is nonnegative and one-Lipschitz: differentiation gives $0 < x'(s) < 1$, and f' is the average of two such derivatives. Thus spectral calculus and the weighted annihilation estimate give, for $v \in \mathcal{D}(B)$,

$$\begin{aligned} \|R_{h,\ell} v\| &\le f(\ell)\|v\| + \|Q(\sqrt{\omega} h) v\| \\ &\le 2\|h\|\|B^{1/2} v\| + (\|\sqrt{\omega} h\| + f(\ell))\|v\| \\ &\le \eta \|Bv\| + \left(\frac{\|h\|^2}{\eta} + \|\sqrt{\omega} h\| + f(\ell) \right) \|v\| \quad (\eta > 0). \end{aligned}$$

The middle bound follows by contracting one symmetric Fock coordinate and applying Cauchy–Schwarz with weight ω; creation contributes the additional $\|\sqrt{\omega}h\|\|v\|$. The last bound uses $\|B^{1/2}v\|^2 \le \|Bv\|\|v\|$ and $2ab \le \eta a^2 + \eta^{-1}b^2$.

Each finite sum of raw prices therefore has relative B bound zero and is symmetric on $\mathcal{D}(B)$. Choose its relative bound less than one. The self-adjoint perturbation theorem [2, Chapter V], obtained by inverting its resolvent by a Neumann series in both half planes, gives the site operator on that same domain. The clock and $V(r)$ are bounded symmetric operators at each site. The direct sum of these site operators is self-adjoint precisely on (15): its resolvent acts component by component, with the common bound $|\operatorname{Im} z|^{-1}$.

Each of the six edge shifts in (13) is unitary. Consequently $\|K\| \le 6|g|$, and reversing an edge gives its adjoint. Bounded symmetric perturbation now gives H. Finally spectral calculus gives $He^{-\mathrm{i}tH}\psi = e^{-\mathrm{i}tH}H\psi$ on $\mathcal{D}(H)$; unitarity gives the claimed domain preservation and energy identity. □

3 The emitted energy has an exact source

We now keep $g \ge 0$ and $\epsilon \ge 0$. For $\psi \in \mathcal{D}(H)$, write $\psi_{r,\nu} \in \mathcal{D}(B)$ for its field component. Define the actual energy terms

$$\mathcal{E}_{\mathrm{ph}}(\psi) = \sum_{r,\nu} \langle \psi_{r,\nu}, B\psi_{r,\nu} \rangle, \tag{16}$$

$$\mathcal{E}_{\mathrm{raw}}(\psi) = g \sum_{r,\nu,d,\sigma} \langle \psi_{r,\nu}, R_{h_{r,d,\sigma},\ell_{r,d,\sigma}} \psi_{r,\nu} \rangle, \tag{17}$$

$$\mathcal{E}_{\mathrm{nat}}(\psi) = \sum_{r} \langle \psi_r, (A_\epsilon \otimes \mathrm{I} + V(r)\mathrm{I})\psi_r \rangle, \tag{18}$$

$$\mathcal{E}_{\mathrm{hop}}(\psi) = \operatorname{Re}\langle \psi, K\psi \rangle, \tag{19}$$

$$\mathcal{E}_{\mathrm{s}}(\psi) = \mathcal{E}_{\mathrm{raw}}(\psi) + \mathcal{E}_{\mathrm{nat}}(\psi) + \mathcal{E}_{\mathrm{hop}}(\psi). \tag{20}$$

The first three expressions are real and nonnegative. The hopping term can have either sign. Equation (20) specifies the energy held in the three indicated source terms by their own operators.

Theorem 2 (Exact energy exchange)**.** *Let $a > 0, C_E > 0, g \ge 0$ and $\epsilon \ge 0$. For every $\psi \in \mathcal{D}(H)$, all the sums above converge. At every real time,*

$$\boxed{\mathcal{E}_{\mathrm{ph}}(\psi(t)) - \mathcal{E}_{\mathrm{ph}}(\psi) = \mathcal{E}_{\mathrm{s}}(\psi) - \mathcal{E}_{\mathrm{s}}(\psi(t)).} \tag{21}$$

Proof. At each site, expanding (11) gives

$$e_r := \operatorname{Re}\langle \psi_r, S_r\psi_r \rangle = e_{\mathrm{ph},r} + e_{\mathrm{raw},r} + e_{\mathrm{nat},r}, \qquad e_{\mathrm{ph},r}, e_{\mathrm{raw},r}, e_{\mathrm{nat},r} \ge 0.$$

The original domain is enough to sum this equality. Indeed,

$$\sum_r |\langle \psi_r, S_r\psi_r \rangle| \le \left(\sum_r \|\psi_r\|^2 \right)^{1/2} \left(\sum_r \|S_r\psi_r\|^2 \right)^{1/2} < \infty. \tag{22}$$

Each nonnegative component is bounded above by e_r, so each is summable. The native sum is finite. Summing the site identity and adding the bounded hopping term yields

$$\mathrm{Re}\langle \psi, H\psi \rangle = \mathcal{E}_{\mathrm{ph}}(\psi) + \mathcal{E}_{\mathrm{s}}(\psi). \tag{23}$$

Apply this same identity to $\psi(t)$, which remains in $\mathcal{D}(H)$ by Lemma 1, and subtract the conserved total energies. □

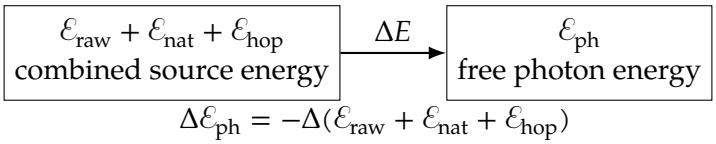

Figure 2: The exact exchange concerns the combined source terms. Its identity allows individual terms to increase while their sum decreases.

The argument does not interchange an uncontrolled infinite sum with time differentiation. It partitions the energy separately at the initial and evolved domain vectors and uses conservation between them. This matters for an infinite lattice with an unbounded photon-number index.

4 Radiation forms from the localized vacuum

Let $v_0 = |0,0,0,0\rangle \otimes \Omega$ and let x_r be the vector with component v_0 at r and zero elsewhere. It lies in $\mathcal{D}(H)$: the vacuum lies in $\mathcal{D}(B)$, each raw price has the estimate used in Lemma 1, and only one site action occurs in the domain sum. Its free photon energy is exactly zero.

For a site s and native label ν, let P_1 extract the one-photon sector and define the energy read

$$L_{s,\nu}\psi = \sqrt{\omega}\, P_1 \psi_{s,\nu}, \qquad q_{s,\nu}(\psi) = \|L_{s,\nu}\psi\|^2. \tag{24}$$

This is a bounded linear read on the complete state space because ω is bounded in the one-photon sector. For a domain vector, positivity of all photon sectors gives

$$0 \le q_{s,\nu}(\psi) \le \mathcal{E}_{\mathrm{ph}}(\psi). \tag{25}$$

Theorem 3 (Energy-bearing radiation onset)**.** *Let $a > 0$, $C_E > 0$, $g > 0$ and $\epsilon \ge 0$. For every r and direction d, there are a native label ν and $t_* > 0$ such that, for $0 < t < t_*$,*

$$0 < q_{r+e_d,\nu}(\mathrm{e}^{-\mathrm{i}tH}x_r) \le \mathcal{E}_{\mathrm{s}}(x_r) - \mathcal{E}_{\mathrm{s}}(\mathrm{e}^{-\mathrm{i}tH}x_r). \tag{26}$$

In particular the combined source energy lies strictly below its initial value throughout this interval, by exactly the gained total free photon energy.

Proof. The neighboring component is initially zero. Since the site action is diagonal in r, its initial derivative comes entirely from the charged hop:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{e}^{-\mathrm{i}tH}x_r)_{r+e_d}\Big|_{t=0} = \mathrm{i}g(C_d \otimes W(\beta_{d,r}))v_0. \tag{27}$$

To see its one-photon content explicitly, the coherent-vector expansion [3] is

$$W(z)\Omega = \mathrm{e}^{-\|z\|^2/2}\bigoplus_{n=0}^{\infty}\frac{z^{\otimes n}}{\sqrt{n!}}, \qquad P_1 W(z)\Omega = \mathrm{e}^{-\|z\|^2/2}z. \tag{28}$$

Because C_d is unitary, its column at the zero label has a nonzero entry $(C_d)_{\nu 0}$. The profile $\beta_{d,r}$ is nonzero: its radial coefficient is positive almost everywhere and $P(k)e_d$ is nonzero on a set of positive measure. Also $\omega > 0$ almost everywhere for $C_E > 0$. Hence the initial derivative of the read (24) is the nonzero vector

$$\mathrm{i}g(C_d)_{\nu 0}\mathrm{e}^{-\|\beta_{d,r}\|^2/2}\sqrt{\omega}\,\beta_{d,r}. \tag{29}$$

Boundedness of $L_{r+e_d,\nu}$ lets it pass through the strong derivative of the trajectory. Its value at zero is zero, so the read divided by t tends to (29). Its norm is therefore positive for every sufficiently small positive t. This proves positive q without a spectral gap or a photon-number cutoff.

Finally $\mathcal{E}_{\mathrm{ph}}(x_r) = 0$. Theorem 2 gives

$$\mathcal{E}_{\mathrm{ph}}(\mathrm{e}^{-\mathrm{i}tH}x_r) = \mathcal{E}_{\mathrm{s}}(x_r) - \mathcal{E}_{\mathrm{s}}(\mathrm{e}^{-\mathrm{i}tH}x_r).$$

Combine this equality with (25). Strict positivity of q then implies that the combined source energy is strictly below its initial value. □

The initial field is a vacuum, while the initial state of the complete interacting source carries its raw, clock, Coulomb and hopping energies. Theorem 3 identifies the account that funds the emitted field. It gives a small positive-time interval; it makes no monotonicity claim for the individual energy terms or for the entire later trajectory.

5 The same coupling reads the displaced field

Radiation can influence later matter motion through the existing charged hopping. Define its oriented matter current by

$$J_{r,d}(\psi) = 2g\ \mathrm{Im}\langle\psi_r, U_{d,r}^{-1}\psi_{r+e_d}\rangle. \tag{30}$$

For $\psi \in \mathcal{D}(H)$, differentiating the site norm and using symmetry of S_r gives the exact continuity equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\psi(t)_r\|^2 = \sum_{d=0}^{2}\left(J_{r-e_d,d}(\psi(t)) - J_{r,d}(\psi(t))\right). \tag{31}$$

The site terms cancel because their expectation is real. The remaining inner products are the six terms of (13), paired by edge orientation.

For the localized input x_r, the edge current is initially zero. In its derivative, every term containing the initial neighboring component vanishes. Equation (27) therefore gives

$$J_{r,d}(x_r) = 0, \qquad \frac{\mathrm{d}}{\mathrm{d}t} J_{r,d}(\mathrm{e}^{-\mathrm{i}tH} x_r)\Big|_{t=0} = 2g^2. \tag{32}$$

Thus the current is positive at every sufficiently small positive time for $g \neq 0$. The emerging neighboring arm has the definite relative phase i fixed by the source evolution.

To isolate how this coupling reads the field, compare the two normalized domain vectors

$$\begin{aligned} y_b &= \frac{1}{\sqrt{2}} \left(|r\rangle v_0 + \mathrm{i}|r + e_d\rangle w_b \right), \quad b = 0, 1, \\ w_1 &= (C_d \otimes W(\beta_{d,r}))v_0, \qquad w_0 = (C_d \otimes \mathrm{I})v_0. \end{aligned} \tag{33}$$

They have identical initial matter densities. Each arm has unit norm, but the two arms differ because $\beta_{d,r} \neq 0$. Write

$$m = 1 - \mathrm{Re}\langle w_1, w_0 \rangle > 0. \tag{34}$$

Positivity follows directly from $\|w_1 - w_0\|^2 = 2 - 2\,\mathrm{Re}\langle w_1, w_0\rangle$ and the nonzero one-photon component of w_1.

Theorem 4 (Passive field-sensitive matter response)**.** *For the preparations* (33)*, evolved by the same H,*

$$J_{r,d}(y_b) = g\,\mathrm{Re}\langle w_1, w_b \rangle, \tag{35}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\|(\mathrm{e}^{-\mathrm{i}tH} y_1)_r\|^2 - \|(\mathrm{e}^{-\mathrm{i}tH} y_0)_r\|^2 \right)\Big|_{t=0} = -gm. \tag{36}$$

For $g \neq 0$, the two origin matter densities therefore differ at every sufficiently small positive time.

Proof. Insert the two nonzero components of y_b in (30) and use $\langle v_0, U_{d,r}^{-1} w_b \rangle = \langle w_1, w_b \rangle$. The factor $1/2$ from the two amplitudes cancels the factor two in the current, and $\mathrm{Im}(\mathrm{i}z) = \mathrm{Re}\, z$. This gives (35). Every other current incident to r is initially zero, because its other endpoint is unoccupied. Equation (31) therefore gives derivative $-g\,\mathrm{Re}\langle w_1, w_b\rangle$ for the origin density. Subtract the two derivatives and use $\langle w_1, w_1 \rangle = 1$. The density difference starts at zero and has nonzero derivative, proving the last assertion. □

6 From a response to a boundary event

The charged source now gives a connected physical account: a native charged step changes the field, a positive field-energy component forms, the energy comes from explicit source

Generated by the source: $x_r(t)_{r+e_d} = \mathrm{i}gt\, w_1 + o(t)$	The emerging arm and phase follow from H.
Prepared for the comparison: $y_b = 2^{-1/2}(\lvert r\rangle v_0 + \mathrm{i}\lvert r + e_d\rangle w_b)$	Balanced amplitudes and the comparison input are preparations.

Figure 3: The generated arm supplies the physical field dependence. A balanced readiness state has an additional preparation requirement.

terms, and those same charged steps respond to a displaced field. A retained boundary must build on these objects and their transformations.

An enacted reader must provide an actual input and a physical output that persists long enough to be read again. For the balanced comparison above, that includes a supplier that prepares the two-site amplitudes and their relative phase, with its energy included in the joint evolution. The autonomous initial derivative supplies an emerging arm; it does not supply the whole balanced preparation at a chosen later time. A different reader may use the actual unequal arms directly, but it still needs a physical interaction and a retained output.

The event connection has a further concrete requirement. A posting changes the retained record; its tick and information count belong to that same change. The field-sensitive density in (36) must therefore be connected to an actual retained record transformation, including which occurrence is posted and when. A two-input density contrast supplies a physical dependence on the field. Constructing the retained output, its timing and its attachment to that posting remains the next step.

That construction must also account for the boundary's continued resources, its preparation and maintenance, and composition with other boundaries. Keeping the original source, every field sector and the energy account in the construction prevents a reader from silently acquiring a new coupling or an unaccounted source of work. The result here supplies the radiation and read interaction on which that construction can proceed.

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