The Forced Core of Physical Reality

A Parameter-Free Foundation for a Theory of Everything

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Abstract

What structure must physical comparison, composition, and retention preserve? We characterize a parameter-free core through explicit operations and exclusion proofs. Finite records support canonical arithmetic. Conservation and register symmetries force quadratic signal energy, and an internal phase reference forces the normalized reciprocal cost $J(x) = (x + x^{-1})/2 - 1$. Independently, a composition law and unit local curvature characterize the same function; continuity and reciprocity follow. For a hierarchy preserving all positive signed comparisons, binary generation of the first new level and closure of the seed sum force the golden ratio. Reversibility on two independent integer registers supplies another derivation of the unequal join, so its positive comparison cost need not be assumed. For spatial retention, globally defined continuous clock-compatible transport, preserving the source reference norm and enacting a uniformly repeated cycle along a closed probe, produces a nonzero additive path reading. The resulting homology class selects dimension three in a sphere, or in Euclidean space when the completed circle leaves its complement path connected. A concrete realization satisfies the transport requirements. The proofs establish uniqueness across realizations of the stated operations and locate the physical identifications needed to apply that uniqueness to observed space and dynamics.

Keywords: Recognition Science; parameter-free framework; physical foundations; reciprocal comparison cost; golden scaling; spatial dimension

1 Introduction

Physical reality supports distinguishable occurrences, comparisons between them, and records that remain available to later events. Recognition Science begins with these

operations and asks which mathematical structures they force. The central result is a parameter-free structural core: canonical arithmetic for finite records, a reciprocal comparison cost, golden scaling for a recognizing hierarchy, and three dimensions for retained loop linking. These structures provide the foundation for a theory of everything by fixing relations on which a physical account of states, interactions, and dynamics can be built.

Consider comparing a present event with a record of an earlier event. The comparison requires distinguishable representations and information about both terms. It is also an occurrence beyond the cutoff of the earlier record. Section 2 gives these facts an exact operational form. Iterating the record construction then supplies arithmetic whose relations agree across independently defined representations (Washburn and Zlatanović 2026a).

A quantitative comparison must also assign magnitudes, compose readings, and preserve a result. Those operations connect the arithmetic to physical structure. For the comparator described below, the dimensionless cost is forced to have the form

$$J(x) = \frac{1}{2}\left(x + \frac{1}{x}\right) - 1 = \frac{(x-1)^2}{2x}, \qquad x > 0. \tag{1}$$

The measurement derivation starts with an unspecified continuous reporting function. Conservation and register symmetries fix quadratic signal energy; a linear comparator and an internal phase reference fix the normalized report. A second theorem selects the same J from composition and local calibration alone. An additive update preserving all positive signed comparisons has one common scale factor. Binary generation and seed-sum closure fix that scale to $\varphi = (1+\sqrt{5})/2$; a reversible two-register census supplies another route. For space, transporting the comparison source around a uniformly repeated cycle produces a nonzero path reading. Its existence throughout a circle complement in S^D forces $D = 3$.

Here "forced" has its mathematical meaning: every realization satisfying a theorem's operational conditions has the stated structure. "Parameter-free" means that no fitted parameter determines the normalized cost shape, hierarchy ratio, or selected dimension. The comparator's positive reference unit κ changes its numerical report F while leaving $(\kappa/2)F = J$ fixed. Similarly, the initial hierarchy level sets an overall scale while leaving the ratio fixed. The distinction between structural relations and dimensional calibration is made explicit in Section 7.

The deductions use different aspects of recognition. Arithmetic supplies the comparator's operands. The zero of its cost distinguishes equal readings from unequal ones. The hierarchy establishes the inequality by identifying a composition rule or a reversible census operation. The spatial argument uses the full comparison signal as a phase reference and derives a reading on paths from its transport. A chronological register alone supplies no such spatial operation. Figure 1 shows the branches and the operations connecting them.

The exclusion arguments give the values their significance. The identity $2J(x) = x + x^{-1} - 2$ alone would not determine an instrument's report. The equation $r^2 = r + 1$ alone would not explain why a hierarchy obeys it. The proofs below derive these relations from the specified operations and exhibit a spatial realization of retained

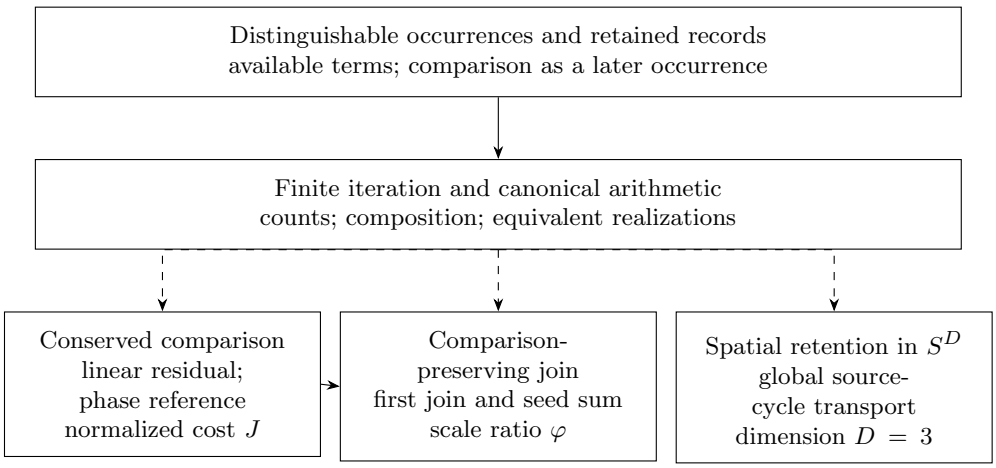


Fig. 1 Arithmetic supplies operands and level counts. The dashed arrows indicate where a branch requires additional operations. The horizontal arrow gives one route to unequal join operands, through positive comparison cost. The join and seed-sum rules establish inequality independently. The spatial branch derives a nonzero path reading from transport throughout the complement of a circle.

linking. Countermodels show exactly which requirement is lost when a competing structure is substituted.

Reciprocal-cost uniqueness, conserved-ledger coherence, and the arithmetic of distinction have complementary treatments in Washburn and Zlatanović (2026b); Pardo-Guerra et al. (2026); Washburn and Zlatanović (2026a). The contribution here is their connection to a common physical question: what must comparison, composition, and retention preserve? The resulting forcing arguments identify a fixed architecture together with the operational conditions under which it governs a realization.

2 Operational Records and Finite Arithmetic

2.1 Represented Questions and Operational Preconditions

Take records to be finite binary words. A history is a finite sequence of three kinds of action: posting a record, retaining the current records while advancing the stage, and comparing two indexed records. Posting appends a word. Retaining leaves the records unchanged. Comparing two present records returns exactly one of the outcomes "equal" and "different", according to whether their binary words agree. If either indexed record is absent, it returns no comparison outcome.

An operational world specifies a prefix-closed collection of permitted histories, containing the empty history. An event consists of a past history h and its next action; its stage is $|h|+1$. A coding c assigns a nonempty record to each sentence and satisfies

$$c(P) \neq c(\neg P). \tag{2}$$

A represented act of entertaining P is a permitted comparison of these two codes while both are present. This definition specifies representation and an action. It does not

require P to be true. For example, codes for a false sentence and its negation can be posted and compared in precisely the same way.

Proposition 2.1 (Operational grounding)**.** *If a world permits a represented asking act, then:*

*(i) some permitted comparison distinguishes two records (*Dist*);*
*(ii) a successful comparison occurs, and every successful comparison has both represented terms present (*Weak*);*
*(iii) if a permitted asking event entertains the claim that a snapshot r is complete, and r precedes that event, the event itself falls outside r (*Iter*).*

Proof By (2), comparing $c(P)$ with $c(\neg P)$ returns the outcome "different", which proves (i). The interpreter produces an outcome only from two present records, which proves (ii). For an asking event with past h and earlier snapshot r as a prefix of h, all events in r have stage at most $|r| \leq |h|$, while the asking event has stage $|h| + 1$. It therefore lies outside the earlier snapshot r, proving (iii). □

The physical identification is that an actual question has distinguishable representations and performs a comparison of them. Under that identification, the occurrence supplies a witness to these structural facts. The argument uses neither assent to the sentence being considered nor a claim that entertaining an arbitrary sentence makes it true.

The proposition fixes the structural content of a represented event: distinct terms, their joint availability, and the later comparison. Quantitative and spatial realizations add the operations through which these records acquire magnitudes, compose into levels, and persist under motion.

The temporal content of Iter is limited to the later event that is actually permitted. A world may permit exactly the prefixes of one finite asking history and permit no further action. Such a world still satisfies the proposition. An available future event, an endless sequence of events, and a completed infinite totality are distinct claims.

2.2 The Finite-Record Arithmetic Construction

The finite arithmetic starts with a freely generated formal syntax, whose successor is an operation on expressions rather than an assumed indefinitely repeatable physical process:

$$0, \quad \delta 0, \quad \delta^2 0, \quad \ldots. \tag{3}$$

Here 0 marks the empty construction and δ appends one operation. The word "free" means that no equations identify different construction lengths. The carrier is the inductively generated set of these finite expressions. Consequently 0 is not a successor, successor is injective, and structural induction applies to every member of the carrier. These properties are proved for the constructed object.

For an object X with a specified point x_0 and operation $d : X \to X$, the interpretation of (3) is fixed recursively:

$$f(0) = x_0, \qquad f(\delta n) = d(f(n)). \tag{4}$$

Any other interpretation respecting the same point and operation agrees with f at 0, and agreement at n implies agreement at δn. Induction gives uniqueness. This is the initiality of the free orbit. This universal property places no injectivity requirement on an arbitrary target X. In particular, the ability to pose one question does not establish that every finite successor operation can be physically realized.

The arithmetic can be realized directly as natural counts, or as finite lists over a one-element set. In the list realization, zero is the empty list, successor adds one element, addition is concatenation, and multiplication repeats concatenation. The length map satisfies

$$\begin{aligned} \operatorname{len}([]) &= 0, & \operatorname{len}(\delta u) &= \operatorname{len}(u) + 1, \\ \operatorname{len}(u + v) &= \operatorname{len}(u) + \operatorname{len}(v), & \operatorname{len}(u \cdot v) &= \operatorname{len}(u)\operatorname{len}(v). \end{aligned} \tag{5}$$

Every such list is the repetition of its sole element exactly as many times as its length. Replication and length are inverse maps.

The arithmetic signature has variables, zero, successor, addition, and multiplication. Its first-order formulas are built from equality and falsehood by conjunction, disjunction, implication, and universal and existential quantification; negation abbreviates implication to falsehood.

Theorem 2.2 (Arithmetic independent of the chosen realization)**.** *For the arithmetic signature defined above, length and replication identify the independently defined natural-count and one-element-list interpretations. They preserve term evaluation and preserve and reflect satisfaction of every formula under corresponding environments. In particular, a formula is valid in one interpretation exactly when it is valid in the other.*

Proof The identities (5) follow from the definitions and induction on the lists. Induction on terms then proves that length commutes with evaluation. For atomic equality, the result follows from this commutation and injectivity of length. For each logical connective, the induction hypotheses transport its constituent truth conditions. In the quantifier cases, every natural count has a list representative by replication, and every list has a natural count by length. These maps transport both witnesses and universally quantified values. Formula induction therefore gives preservation and reflection of satisfaction, and the validity statement follows by quantifying over environments. □

The construction fixes arithmetic relations across these representations; they do not depend on the labels used for the records. Signed counts and ratios are subsequently obtained by quotient constructions over the orbit arithmetic (Washburn and Zlatanović 2026a). The inductive syntax, definitions, equality, recursion, and quotient reasoning belong to the mathematical background. The resulting arithmetic is canonical for this finite construction: changing between these realizations preserves its relations and the truth of its formulas.

A physical record system realizes this arithmetic by implementing distinguishable construction stages, extension, combination, and repetition. Its natural counts supply

the positive integer operands of the comparator. Its signed counts supply comparisons between collections. The next step is to determine how an apparatus assigns a quantitative reading to those operations.

3 Conserved Comparison and the Forced Reciprocal Cost

A comparison measures a residual against a reference generated by the same apparatus. Equal operands leave no residual; unequal operands can leave a measurable difference. The quadratic energy entering this measurement can itself be derived from conservation.

3.1 Quadratic Energy from Conservation

Let $m \geq 1$ and let $N : \mathbb{C}^m \to [0, \infty)$ be definite, subadditive, and homogeneous under real rescaling:

$$N(x) = 0 \iff x = 0, \qquad N(x + y) \leq N(x) + N(y), \qquad N(ax) = |a|N(x) \quad (a \in \mathbb{R}).$$

Assume that N is invariant under permutations of the registers and independent unit-phase changes in each register. An initially unspecified power N^q, with $q > 0$, is conserved by the sum-and-difference operation with a common gain $\lambda > 0$:

$$N\big(\lambda(x + y)\big)^q + N\big(\lambda(x - y)\big)^q = N(x)^q + N(y)^q \qquad (x, y \in \mathbb{C}^m). \tag{6}$$

Proposition 3.1 (Conservation forces quadratic energy)**.** *Under these conditions,*

$$q = 2, \qquad \lambda = \frac{1}{\sqrt{2}}, \qquad N(x)^2 = c \sum_{j=1}^{m} |x_j|^2 \quad \textit{for a single } c > 0. \tag{7}$$

Proof Fix $x \neq 0$. Substituting $y = 0$ and $y = -x$ in (6) and cancelling $N(x)^q > 0$ gives

$$2\lambda^q = 1, \qquad (2\lambda)^q = 2.$$

Thus $2^q = 4$, so $q = 2$ and $\lambda = 1/\sqrt{2}$. Equation (6) becomes the parallelogram identity

$$N(x + y)^2 + N(x - y)^2 = 2N(x)^2 + 2N(y)^2.$$

If x and y have disjoint coordinate supports, independent phase changes of -1 on the support of y take $x+y$ to $x-y$. Their sizes are equal, and the identity gives $N(x+y)^2 = N(x)^2+N(y)^2$. Decompose an arbitrary vector into its coordinate vectors and apply this equality successively. Phase invariance and real homogeneity give $N(x_j e_j) = |x_j|N(e_j)$, where e_j is the jth unit vector. Permutation invariance makes all $N(e_j)^2$ the same positive number c. Summing proves (7). □

The exponent and relative register weights are therefore fixed by conservation and symmetry. The remaining factor c sets an overall energy unit. It cancels when the residual and reference are read by the same detector. The number of signal registers is distinct from the spatial dimension in Section 5.

3.2 The Residual and Its Internal Reference

Consider a finite complex inner-product signal space with squared-norm energy, as supplied by Proposition 3.1 after choosing its energy unit. Its operand pulses are u, v, and its complex-linear residual operation is T. The apparatus has opposite, nonzero operand responses,

$$Tv = -Tu, \qquad k = \|Tu\|^2 > 0. \tag{8}$$

A native report reads the residual against a reference generated by this same apparatus. Equation (8) specifies its cancellation response; the phase-reference program below specifies its normalization.

Encoding positive real amplitudes a, b with relative phase z gives $au+zbv$. Linearity and (8) give the energy reading

$$E(a, b; z) = \|T(au + zbv)\|^2 = k|a - zb|^2. \tag{9}$$

In particular, the same-phase residual has energy $k(a - b)^2$. A quarter phase gives $k(a^2 + b^2)$. The native reference subtracts the same-phase reading from the quarter-phase reading. At positive reference unit κ, its value is

$$R_\kappa(a, b) = \frac{\kappa}{2}\big(E(a, b; i) - E(a, b; 1)\big) = \kappa kab. \tag{10}$$

The meter reports residual energy divided by this positive reference. The common gain k cancels. Thus the instrument measures a relative comparison, independent of its overall energy scale.

Theorem 3.2 (Uniqueness of a continuous native report)**.** *Fix the comparator* (8)–(10) *and* $\kappa > 0$*. Let* $F : (0, \infty) \to \mathbb{R}$ *be continuous. Suppose that for every pair of positive natural operands* a, b *it literally reports this meter:*

$$F(a/b) = \frac{E(a, b; 1)}{R_\kappa(a, b)}. \tag{11}$$

Then for every $x > 0$*,*

$$\frac{\kappa}{2}F(x) = J(x). \tag{12}$$

For the convention $\kappa = 2$*, the unique report is* $F = J$*; other positive calibrations give the proportional reading* $(2/\kappa)J$*.*

Proof For positive natural a, b, substitution gives

$$F(a/b) = \frac{k(a - b)^2}{\kappa kab} = \frac{1}{\kappa}\left(\frac{a}{b} + \frac{b}{a} - 2\right) = \frac{2}{\kappa}J(a/b).$$

Positive rational ratios are dense in $(0, \infty)$. Both F and $(2/\kappa)J$ are continuous there, so their agreement on those ratios extends to every positive real argument. □

The report F starts arbitrary. The residual and its internal reference determine its values on positive rational ratios, and continuity fixes the entire positive domain. The common detector gain has disappeared from the result.

Continuity also has an exact role. Reporting on positive natural operands determines a function on positive rational ratios. An arbitrary change at one irrational argument leaves those reports unchanged. Continuity rules out that change. If the report is instead the literal instrument response on all positive real amplitudes, equations (9) and (10) give the expression directly, and its continuity follows from that expression.

The selected function has the mathematical properties

$$J(x) \geq 0, \qquad J(x) = 0 \iff x = 1, \qquad J(x^{-1}) = J(x). \tag{13}$$

The first two follow from the square in (1); the third follows by exchanging the operands. The coefficient in (12) is set by the reference unit κ, not by the residual energy alone. For example, at $\kappa = 2$ the quarter-phase reference is $R_2 = 2kab$, whereas the full phase-swing reference would be

$$R_{\text{full}}(a, b) = E(a, b; -1) - E(a, b; 1) = 4kab.$$

Reporting $E(a, b; 1)/R_{\text{full}}$ would give $J(a/b)/2$ rather than $J(a/b)$. Both references use the same detector and amplitudes, but they select different normalized readings. Similarly, the smooth reciprocal function $(\log x)^2/2$ agrees with J to quadratic order at $x = 1$ but does not match this instrument's readings away from equilibrium. Theorem 3.2 is a uniqueness statement for the declared measurement protocol.

3.3 Operational Origin of Cancellation and Cross Reference

The cancellation and reference conditions can also be approached from the observed behavior of an apparatus. Consider an eight-register realization with the conserved energy of Proposition 3.1, a complex-linear residual operation T, and a reference formed by finite real linear combinations of its unit-phase energy records. Write

$$E(a, b; z) = \|T(au + zbv)\|^2, \qquad R(a, b) = \sum_{j=1}^{n} \alpha_j E(a, b; z_j), \qquad |z_j| = 1.$$

Suppose the balanced reference is positive and the balanced report vanishes: $R(1, 1) > 0$ and $E(1, 1; 1)/R(1, 1) = 0$. Then $\|Tu + Tv\|^2 = 0$, which forces $Tv = -Tu$. If $Tu = 0$, every phase record would vanish, contradicting the positive reference. Hence $k = \|Tu\|^2 > 0$.

The reference therefore has the form

$$R(a, b) = k\big[A(a^2 + b^2) + Bab\big], \qquad A = \sum_j \alpha_j, \quad B = -2\sum_j \alpha_j \operatorname{Re} z_j.$$

If the reference vanishes when the second operand is absent, $R(1,0) = 0$, then $A = 0$. Positivity at $(1,1)$ gives $B > 0$. Writing $B = \kappa$ yields $R(a,b) = \kappa kab$, the cross reference used in Theorem 3.2. A contrast of two phase readings supplies the empty-input condition directly: when $b = 0$, changing the relative phase cannot change the input or its energy. The quarter-phase contrast in (10) is one such implementation.

These deductions identify the operations behind the comparator specification. Conservation and register symmetry fix the energy law; balanced-zero reporting fixes cancellation; an empty-input reference removes the same-operand contribution. The positive reference unit remains a normalization convention.

3.4 The Same Cost from Composition

The normalized native report $C = (\kappa/2)F$ also has a characterization that does not specify an apparatus. Multiplication and division of positive ratios describe the two orientations of composition. Require the corresponding costs to satisfy

$$C(xy) + C(x/y) = 2C(x)C(y) + 2C(x) + 2C(y), \qquad x, y > 0, \tag{14}$$

with unit local curvature in the logarithmic ratio,

$$\lim_{t\to 0,\ t\neq 0} \frac{2C(e^t)}{t^2} = 1. \tag{15}$$

This is a global composition requirement and a local calibration, respectively (Washburn and Zlatanović 2026b; Aczél and Dhombres 1989).

Theorem 3.3 (Composition Characterizes the Normalized Cost)**.** *A real-valued function on the positive reals satisfies (14) and (15) if and only if it equals J. Continuity, reciprocity, nonnegativity, and $C(1) = 0$ follow from these two conditions.*

Proof Setting $y = 1$ in (14) gives $C(1)(C(x) + 1) = 0$. The alternative $C \equiv -1$ contradicts (15), so $C(1) = 0$. Put $H(t) = 1 + C(e^t)$. Then

$$H(t+u) + H(t-u) = 2H(t)H(u), \qquad H(0) = 1.$$

Calibration gives $H(t) \to 1$ as $t \to 0$. The same identity gives evenness and, by applying it to the sum and difference of $t + u$ and $t - u$,

$$[H(t+u) - H(t-u)]^2 = 4[H(t)^2 - 1][H(u)^2 - 1].$$

For fixed t, the right side tends to zero as $u \to 0$, while the sum $H(t+u) + H(t-u)$ tends to $2H(t)$. Thus H is continuous everywhere.

Let $\Phi(t) = \int_0^t H(s)\,ds$. Choose $\delta > 0$ with $\Phi(\delta) \neq 0$, possible because $H(0) = 1$. Integrating the sum identity with respect to u from 0 to δ gives

$$H(t) = \frac{\Phi(t+\delta) - \Phi(t-\delta)}{2\Phi(\delta)}.$$

This representation successively supplies derivatives of every order. Evenness gives $H'(0) = 0$, and (15) gives $H''(0) = 1$. Differentiating the sum identity twice in u at zero yields $H''(t) = H(t)$. These initial values determine $H(t) = \cosh t$, hence $C(x) = \cosh(\log x) - 1 = J(x)$. Conversely, substitution and the expansion of $\cosh t$ at zero verify both conditions for J. □

The two characterizations select the same invariant. The comparator determines its normalized report from conserved measurement and an internal reference. Composition and local calibration determine the same function without naming that instrument. A different normalized cost must therefore change a stated operation or one of these two conditions. Local quadratic agreement alone is insufficient: $(\log x)^2/2$ has the required local curvature but fails the composition law. The physical identification of that law concerns the available composition of comparisons; its functional form is explicit in the theorem.

4 Comparison-Preserving Updates and Golden Scaling

A recognizing hierarchy composes earlier levels while preserving their comparisons. Requiring one additive update to preserve every positive signed comparison forces a common scale factor. Its value can be identified by measuring the cost of the first join or by determining which sums the apparatus realizes.

Let G be an additive commutative group of native readings. Let $L : G \to \mathbb{R}$ be additive and let $A : G \to G$ be an additive update. Differences in G represent signed comparisons, including integer combinations of readings. Assume

$$L(x) > 0 \quad \Longrightarrow \quad L(Ax) > 0 \qquad \text{for every } x \in G. \tag{16}$$

This condition applies to the whole comparison group, rather than just to the positive levels on one observed trajectory.

Lemma 4.1 (One scale factor from preserved comparisons)**.** *Let $L, M : G \to \mathbb{R}$ be additive. Suppose that $L(e) > 0$ for some $e \in G$ and that $L(x) > 0$ implies $M(x) > 0$ for every $x \in G$. Then*

$$M(x) = \frac{M(e)}{L(e)} L(x) \quad \textit{for every } x \in G, \qquad \frac{M(e)}{L(e)} > 0. \tag{17}$$

Proof The positivity condition gives $M(e) > 0$. Suppose first that $L(x)/L(e) < M(x)/M(e)$. Choose a rational p/q strictly between them, where $p \in \mathbb{Z}$ and $q \in \mathbb{N}$ is positive. Additivity gives

$$L(pe - qx) = pL(e) - qL(x) > 0, \qquad M(pe - qx) = pM(e) - qM(x) < 0,$$

a contradiction. If the two ratios have the opposite order, use $qx - pe$ instead. Thus the ratios agree for every x, which is (17). □

Apply the lemma to $M = L \circ A$. The update has a single positive scale factor r, even though no scalar action, vector-space basis, continuity, or inverse for A was supplied. This is the mechanism that removes nonuniform alternatives.

To connect the readings to physical states, let S have successor $U : S \to S$ and positive level $\ell : S \to (0, \infty)$. A reading $R : S \to G$ obeys

$$L(R(s)) = \ell(s), \qquad R(U(s)) = A(R(s)). \tag{18}$$

For a base state s_*, write $s_n = \ell(U^n(s_*))$. The lemma gives $\ell(U(s)) = r\ell(s)$ for every state and hence $s_n = r^n s_0$.

4.1 Composition Selects the First Unequal Join

The comparison-preserving update fixes one positive scale factor. Two composition requirements determine its value. The first new level is made from exactly two available seed contributions, with repetition allowed. Combining one contribution of each seed size also produces a reading somewhere on the same orbit:

$$s_2 = s_a + s_b, \qquad a, b \in \mathbb{N}, \quad a \le b < 2, \tag{19}$$

$$s_m = s_0 + s_1, \qquad m \in \mathbb{N}. \tag{20}$$

The generating pair and the landing index m are unspecified. These are the two local composition requirements isolated in Washburn (2026, Proposition 2).

Theorem 4.2 (Golden Scaling from Composition)**.** *Assume* (16)*,* (18)*,* (19)*, and* (20)*. Then* $(a, b) = (0, 1)$*,* $m = 2$*, and*

$$\ell(U(s)) = \varphi\ell(s), \qquad s_n = \varphi^n s_0, \qquad \varphi = \frac{1+\sqrt{5}}{2}. \tag{21}$$

The first join therefore compares unequal readings and has positive canonical comparison cost.

Proof Lemma 4.1, applied with $e = R(s_*)$, gives a common $r > 0$ and $s_n = r^n s_0$. If $r \le 1$, both available seed readings are at least s_2. Their sum is then at least $2s_2 > s_2$, contrary to (19). Hence $r > 1$.

The sequence is strictly increasing. Since $s_m = s_0 + s_1 > s_1$, we have $m \ge 2$. Dividing the two composition equalities by $s_0 > 0$ gives

$$r^2 = r^a + r^b, \qquad r^m = 1 + r.$$

There are three possible ordered generating pairs. If $(a, b) = (0, 0)$, then $r^2 = 2$. Since $r > 1$,

$$r^2 = 2 < 1 + r < 2r = r^3,$$

so $1 + r$ cannot be a power of r with a natural exponent. If $(a, b) = (1, 1)$, then $r^2 = 2r$, hence $r = 2$. But

$$r < 1 + r < r^2,$$

which again excludes every natural landing index. Thus $(a, b) = (0, 1)$ and $r^2 = 1 + r$.

The unique positive root is $r = \varphi$. Strict increase gives $m = 2$. The common scaling law applies to every state represented by (18), yielding (21). Finally $s_1/s_0 = \varphi > 1$, so (13) gives $J(s_1/s_0) > 0$. □

The theorem derives the costly unequal join from composition. A physical application must identify the operation producing level two and the operation whose seed-sum reading lies on the same orbit. Exactly two contributions are required: the doubling

readings $1, 2, 4$ satisfy $4 = 2 + 1 + 1$, but this uses three contributions. Distinguishing their provenance and multiplicity is therefore part of identifying the join.

Positive comparison cost provides a shorter alternative when it is already established operationally. If

$$s_2 = s_a + s_b, \qquad a \le b < 2, \qquad J(s_b/s_a) > 0, \tag{22}$$

then $a = b$ is impossible because $J(1) = 0$. Thus $(a, b) = (0, 1)$, and the common scaling law immediately gives $r^2 = 1+r$. This alternative does not require the separate seed-sum closure condition.

4.2 Reversibility Selects the Mixed Census Join

A second selection mechanism applies when the native readings count two independent contributions. Here the join is an equality of integer census states, before assigning their real-valued sizes.

Proposition 4.3 (Reversible Binary Census)**.** *Let* $e_0 = (1, 0)$ *and* $e_1 = (0, 1)$ *generate* $\mathbb{Z}^2$*. Suppose* A *is an additive automorphism with* $Ae_0 = e_1$ *and*

$$Ae_1 \in \{2e_0,\ e_0 + e_1,\ 2e_1\}.$$

Then

$$Ae_1 = e_0 + e_1, \qquad A(u, v) = (v, u + v), \qquad A^2 = A + I.$$

Proof Additivity gives $A(u, v) = uAe_0 + vAe_1$. If $Ae_1 = 2e_0$, every image has even first coordinate. If $Ae_1 = 2e_1$, every image has first coordinate zero. Neither possibility admits a preimage of e_0, contradicting surjectivity. Hence $Ae_1 = e_0 + e_1$. The coordinate formula follows by substitution, and applying it twice gives $A^2x = Ax + x$. □

If an additive real reading L has $L(e_0) > 0$ and preserves all positive signed comparisons under this update, Lemma 4.1 gives $L(Ax) = rL(x)$ with $r > 0$. Reading $A^2e_0 = Ae_0 + e_0$ gives $r^2 = r + 1$, hence $r = \varphi$. This route replaces seed-sum closure with reversible evolution on two independent integer registers. Reversibility here means that every census state has a predecessor; injectivity on the forward orbit alone does not supply it. The native event is still binary. A conserved-ledger treatment gives a related scaling theorem under its own composition condition (Pardo-Guerra et al. 2026).

For an explicit choice of readings, use the signed census group $G = \mathbb{Z}^2$ with

$$A(a, b) = (b, a + b), \qquad L(a, b) = a + \varphi b.$$

The identity $\varphi^2 = \varphi + 1$ gives $L(Ax) = \varphi L(x)$ for every signed census x, so every positive comparison is preserved. Starting from $(1, 0)$ gives readings $1, \varphi, 1 + \varphi$, and the third is the first costly join of the preceding two. Integer census entries and their real-valued readings therefore instantiate the same hierarchy.

Exact hierarchy levels are positive real readings. Integer counts obeying the Fibonacci recurrence remain integers, and their consecutive ratios approach φ. They are distinct from the exact real-valued hierarchy (21). Confusing these descriptions would make an irrational ratio of integer counts appear to be asserted at a finite stage.

5 Spatial Linking and Dimensional Selection

What does it take to remember which way a completed act ran? A register that stores the order of its entries can tell a run from its reversal. A pair of counters that records only how often a binary coordinate rose and fell cannot: on a closed run, every rise is matched by a fall. An ordered register preserves information that those totals lose (Pardo-Guerra and Washburn 2026b). Neither device needs a second loop in physical space. The dimension argument concerns a different record, one that can be read by taking a probe around the trace of the act itself.

Suppose a completed act traces an embedded circle K in a space M. A probe can follow paths in $X = M \setminus K$. The first question is whether joining two probe paths adds their readings and whether a path can move with its endpoints fixed without changing its reading. The second is whether some closed probe has a nonzero reading. These questions make sense before specifying any homology group.

Proposition 5.1 (What an additive path reading detects)**.** *Let q assign a real number to every path in X. Suppose that $q(p * p') = q(p) + q(p')$ whenever paths can be concatenated, and that homotopies in X fixing both endpoints preserve the reading. On closed paths the reading is the evaluation of a homomorphism $H_1(X;\mathbb{Z}) \to \mathbb{R}$. Consequently a closed probe L for which $q(L) \neq 0$ has $[L] \neq 0$.*

Proof Extend q linearly to integral singular one-chains by evaluating each oriented edge in its given direction. In any singular triangle the path along edges 01 and 12 deforms, with endpoints fixed, to edge 02 inside the triangle. Hence $q(01) + q(12) - q(02) = 0$. This is the reading of the oriented boundary. By linearity every two-chain boundary reads zero. On one-cycles the reading therefore descends to the quotient by boundaries, which is $H_1(X;\mathbb{Z})$. □

A signed reading that distinguishes the two senses of a closed probe supplies a nonzero value: the path followed by its reversal contracts, so the two readings are negatives. This is a way to establish a nonzero class without postulating one. It requires the reading to extend to paths throughout the complement, including the deformations used in the proof. Recording the probe's history only on a prescribed track would not establish that property. One completed pair with this behavior suffices for the dimension theorem. An argument that every kind of physical memory uses such a pair would be a stronger claim.

5.1 A Source Cycle Supplies the Retained Reading

We can derive the nonzero reading from a finite comparison source. Consider eight registers with orthonormal Fourier modes

$$e_k(j) = \frac{\omega^{jk}}{\sqrt{8}}, \qquad \omega = e^{-\pi i/4}, \qquad j,k = 0,\dots,7.$$

Write $\widehat{v}_k = \langle e_k, v\rangle$. Let $E \subset \{0,\dots,7\}$ contain the odd indices and exclude zero. The source update is

$$Ue_k = \begin{cases} \omega^k e_k, & k \in E, \\ 0, & k \notin E. \end{cases}$$

Eight specifies the source apparatus here. This register count is supplied independently of the spatial dimension being determined.

For positive integers a, b, the adjacent comparison pulse and its committed state are

$$h = \frac{1}{\sqrt{2}}(a, -a, 0, 0, -b, b, 0, 0), \qquad f = Uh.$$

Direct Fourier expansion gives

$$\widehat{f}_1 = \omega\,\frac{1-\overline{\omega}}{\sqrt{2}\sqrt{8}}\,(a+b) \neq 0. \tag{23}$$

This calculation retains the complete committed signal for the positive comparison operands. Its first coefficient supplies a reference with which to read its phase.

Let $X = M \setminus K$. Suppose each position $x \in X$ supplies a complex-linear operation T_x on the registers, with

$$\begin{aligned} &x \longmapsto T_x v \text{ continuous for every } v, \\ &T_x U = U T_x, \qquad \|T_x e_1\| = 1. \end{aligned} \tag{24}$$

The last condition follows from losslessness of the operation; only this particular norm preservation is needed. Commutation says that moving the comparison and advancing its source update agree in either order.

For $z \in S^1$, define the phase action on the entire signal by $R_z e_k = z^k e_k$. A continuous source tick is any path $\tau : [0,1] \to S^1$ with $\tau(0) = 1$ and $\tau(1) = \omega$. Repeat the same tick eight times, advancing its starting phase each time:

$$\alpha\left(\frac{j+t}{8}\right) = \omega^j \tau(t), \quad 0 \le t \le 1, \quad j = 0,\dots,7. \tag{25}$$

Adjacent pieces agree, and $\alpha(0) = \alpha(1) = 1$.

Proposition 5.2 (A Source Cycle Forces a Nonzero Spatial Reading)**.** *Suppose* (24) *holds throughout* X*. If a closed probe* $\gamma : [0,1] \to X$ *enacts the complete source cycle,*

$$T_{\gamma(t)} f = R_{\alpha(t)} f \qquad \textit{for every } t \in [0,1], \tag{26}$$

then an additive, endpoint-homotopy-invariant reading on paths in X has a nonzero value on γ. Consequently $[\gamma] \neq 0$ in $H_1(X;\mathbb{Z})$.

Proof The eigenvalue ω of U occurs only on e_1. Commutation therefore gives $T_x e_1 = c(x)e_1$. For $k \neq 1$, the eigenvalue of e_k is either $\omega^k \neq \omega$ or zero, so $T_x e_k$ has no e_1 coefficient. Expanding an arbitrary signal in the Fourier basis yields

$$\widehat{T_x v}_1 = c(x)\widehat{v}_1.$$

The norm condition gives $|c(x)| = 1$, and continuity makes $c : X \to S^1$ continuous. By (23),

$$c(x) = \frac{\widehat{T_x f}_1}{\widehat{f}_1}.$$

Thus the phase is read from the original comparison signal. Taking its first coefficient in (26) gives $c(\gamma(t)) = \alpha(t)$.

Lift the tick to a real angle θ with $\tau(t) = e^{i\theta(t)}$ and $\theta(0) = 0$. Its endpoint has the form

$$\theta(1) = -\frac{\pi}{4} + 2\pi m, \qquad m \in \mathbb{Z}.$$

Each rotated copy has this same angle displacement. The full cycle therefore has displacement $8\theta(1) = 2\pi(-1 + 8m)$, which is nonzero.

For any path p in X, lift $c \circ p$ to a real angle and let $q(p)$ be its angle displacement divided by 2π. Changing the lift adds a constant multiple of 2π, leaving $q(p)$ unchanged. Concatenating paths adds displacements, and a homotopy fixing the endpoints preserves them. Hence $q(\gamma) = -1 + 8m \neq 0$. Proposition 5.1 gives $[\gamma] \neq 0$. □

The enactment condition concerns the whole cycle, including what happens between sampled states. Uniform repetition supplies its nonzero winding without choosing a principal angular branch. The domain condition concerns every position in the complement, including the deformations used to compare probe paths. These are the operational requirements from which the homological conclusion follows. To apply it to observed space, X must be the region in which the physical source and probe move, and T_x must describe their operation throughout that region.

5.2 The sphere complement

First take $M = S^D$. This includes Euclidean space only after a point at infinity has been added. Taking homology before or after that operation can give different answers, as the planar example below shows. All homology groups use integer coefficients.

Definition 5.3 (Nontrivial loop-complement homology)**.** Dimension D detects nontrivial linking if there is a topological embedding $f : S^1 \hookrightarrow S^D$ for which

$$H_1\big(S^D \setminus \operatorname{im} f; \mathbb{Z}\big) \neq 0. \tag{27}$$

Theorem 5.4 (Dimension Selected by Loop Linking)**.** *Among nonnegative integer dimensions,* (27) *holds for some embedded circle exactly when $D = 3$.*

Proof We use the classical Alexander-duality proof. For $D \geq 2$, a compact locally contractible subset $K \subset S^D$ gives

$$\widetilde{H}_1(S^D \setminus K; \mathbb{Z}) \cong \widetilde{H}^{D-2}(K; \mathbb{Z}). \tag{28}$$

Here $K = \operatorname{im} f$ is homeomorphic to S^1, so its reduced cohomology is $\mathbb{Z}$ in degree one and zero in degree zero or any degree at least two. Since reduced and unreduced homology agree in degree one, (28) gives a nonzero group precisely for $D - 2 = 1$. The duality theorem applies to topological embeddings, including those without a smoothness assumption (Hatcher 2002, Section 3.3).

There is no embedding of a circle in S^0. An embedding of S^1 into S^1 has open image by invariance of domain and closed image by compactness; connectedness makes its image all of S^1. Its complement is empty and has zero first homology.

For existence in dimension three, regard S^3 as $\{(z, w) \in \mathbb{C}^2 : |z|^2 + |w|^2 = 1\}$ and take the circle $w = 0$. Its complement is homeomorphic to the open unit disk times S^1 by $(z, w) \mapsto (z, w/|w|)$, and retracts to the second factor. Its first homology is therefore $\mathbb{Z}$. □

A concrete representative is the pair of circles $K = \{(e^{i\theta}, 0) : 0 \leq \theta < 2\pi\}$ and $L = \{(0, e^{i\theta}) : 0 \leq \theta < 2\pi\}$ in $S^3 \subset \mathbb{C}^2$. They form the standard Hopf link. Under the retraction of $S^3 \setminus K$ onto the circle factor described in the proof, L maps with degree one. Thus $[L]$ generates $H_1(S^3 \setminus K; \mathbb{Z}) \cong \mathbb{Z}$. With consistent orientations, the linking number is $+1$ (and changes sign if an orientation is reversed). Displacing or deforming L within the complement cannot reduce this nonzero reading to zero. The retained information is a topological relation between traces; the theorem does not assign them a metric length or a duration.

The transport requirements are simultaneously realizable in this complement. Put

$$c(z, w) = \frac{w}{|w|}, \qquad T_{(z,w)} = R_{c(z,w)}.$$

These operations are continuous, commute with the source update U, and preserve the norm of every signal. For any uniform source cycle α, the closed probe $\gamma(t) = (0, \alpha(t))$ satisfies $T_{\gamma(t)} f = R_{\alpha(t)} f$. Thus the complement supports both the linking class and the complete comparison transport that reads it. Conversely, Proposition 5.2 and Theorem 5.4 exclude such full-domain transport with source-cycle enactment in every other sphere dimension.

For a smooth oriented pair in S^3, the integer linking number can be read by counting signed crossings of L through a spanning surface for K, transverse to L. The Hopf pair gives one signed crossing. A physical example of this type of readout comes from magnetostatics. In vacuum, consider an isolated steady current in an ideal thin closed wire. Ampère's law identifies the magnetic circulation around a probe circuit with the signed current through a surface spanning the circuit (Feynman et al. 1964, Ch. 13). For one positive threading the magnetic circulation is $\mu_0 I$, where I is the current. Reversing the circuit changes the sign. This example explains how an apparatus could read a relation between paths. Magnetostatics already describes fields in three-dimensional space. Identifying the current path with a completed recognition would require a separate physical argument.

5.3 Euclidean space and the point at infinity

Take $M = \mathbb{R}^D$ with $D \geq 2$. In a plane, let K be the unit circle and let L be a larger concentric circle. The exterior of K retracts onto L, so $[L] \neq 0$ in the plane complement. In S^2, adding the point at infinity caps the outside of L with a disk disjoint from K. The same probe now bounds. Thus a nonzero reading alone also permits a planar realization.

A physical condition distinguishes the two situations: any two positions outside the completed trace can be joined by a path that avoids it. In $\mathbb{R}^D$, this condition together with a nonzero complement reading selects exactly $D = 3$. To see this, compute the complement in the sphere with *both* the trace and infinity removed:

$$H_1(\mathbb{R}^D \setminus K; \mathbb{Z}) \cong \widetilde{H}^{D-2}(K \sqcup \{\infty\}; \mathbb{Z}).$$

For $D = 2$ the group is $\mathbb{Z}$, but a Jordan curve separates the plane. For $D = 3$ the group is $\mathbb{Z}$ and the complement is path connected. A standard circle and its meridian realize both requirements. For $D \geq 4$ the group is zero. The circle cannot embed in $\mathbb{R}^0$ or $\mathbb{R}^1$. The path-access condition thus eliminates precisely the planar alternative left by the homology reading.

There is also a result without a spherical background. Among closed, connected, orientable smooth manifolds of dimension at least two with vanishing ambient first integral homology, a tame circle can have a complement with nonzero first homology exactly in dimension three (Pardo-Guerra and Washburn 2026a). The ambient hypothesis prevents the probe from registering a pre-existing hole in its surroundings. The same work constructs a space from refined records: with its specified linear refinement and metric, the completion is a solid D-cube. Passing to its interior and then compactifying that interior produces S^D. Those two operations choose how the boundary is treated. Even this explicit construction therefore leaves a physical choice to identify.

In both results D belongs to the space through which the probe can move. To infer the dimension of an observed physical region, the two traces must describe material positions, and their permitted motions must represent the ways a probe can move in that region. A probe confined to a guide can report a feature of the guide even when a path around it is available in the surrounding space.

6 Which Requirements Exclude the Alternatives

The alternatives identify what the operational requirements contribute. Holding the represented asking histories fixed while changing a comparison, joining, or retention rule isolates a structure that the corresponding selection theorem excludes.

Write Π for the three operational claims Dist, Weak, Iter in Section 2. A consequence of Π must hold in every interpretation of that signature satisfying those claims. The following proposition distinguishes the record signature from the complete physical specification.

Proposition 6.1 (Limits of the Asking Signature)**.** *Suppose two worlds run every asking history in exactly the same way but assign different additional readings. Every*

predicate depending only on that operational signature has the same truth value in both worlds. Consequently, if a proposed cost, hierarchy, or retention condition holds in one world and fails in the other, it is not definable by a predicate of that signature alone. If the second world asks and satisfies Π*, it is also a countermodel to deriving that condition from* Π*.*

Proof The two signatures are equal, so substituting one for the other leaves every predicate of the signature unchanged. A condition with different truth values in the two worlds cannot be such a predicate. A derivation from Π would, by soundness, hold in the second world, contradicting the failure of the condition there. □

The following examples keep the permitted posting, retention and equality-comparison histories fixed while changing a numerical or spatial operation.

An alternative phase reference. Keep the same posting, retention, and equality-comparison histories, and even the same complex-linear residual apparatus. Replace the quarter-phase reference $R_2 = 2kab$ by the full-swing reference $R_{\mathrm{full}} = 4kab$. At $a/b = 2$ the full-swing meter reads $J(2)/2 = 1/8$. The quarter-phase meter reads $J(2) = 1/4$. The alternative violates the quarter-phase normalization but leaves the underlying record interpreter unchanged. The change concerns the reference unit: both readings have the same reciprocal shape, and the declared quarter-phase normalization selects $F = J$.

A doubling hierarchy. Let the levels be $s_n = 2^n$ and the numerical update be multiplication by two. This is additive and preserves positive comparisons. Its first new level is the sum of two level-one parts, so the join is balanced and costs zero. It fails (22). It also fails the seed-sum condition of Theorem 4.2: $1 + 2 = 3$ never occurs on the doubling ladder. The records representing the two parts may still be distinct.

A four-dimensional spatial realization. Keep the same permitted record and comparison histories, but realize a completed trace as a standard embedded circle $K \subset S^4$. Alexander duality gives $H_1(S^4 \setminus K; \mathbb{Z}) = 0$. The record interpreter is unchanged, while the nonzero complementary linking requirement (27) fails. This realization preserves the asking behavior while losing the specified spatial retention. More concretely, no continuous family satisfying (24) on that whole complement can enact (26) on a closed probe. Proposition 5.2 would produce a nonzero homology class, contradicting the calculation. Thus the exclusion can be stated in terms of source transport before a linking class is assumed.

The countermodels show why the complete specification matters: the record signature leaves quantitative and spatial readings open, whereas the stated operational requirements fix the selected structure. Their philosophical role is to locate the source of necessity. A structural conclusion becomes unavoidable within a class of realizations only when an operational premise rules out the competing case; the record histories alone do not make that selection. These examples establish dependence on the indicated requirements, not mutual independence of every added premise.

Alternative	Additional requirement that fails	Conclusion no longer forced
Full-swing reference	Quarter-phase reference at $\kappa = 2$ (Section 3)	Canonical normalization $F = J$ (alternative: $J/2$)
Doubling hierarchy	Seed-sum closure, or the reversible binary census (Section 4)	Golden scaling φ
Circle embedded in S^4	Global source-cycle transport, hence nonzero complement homology (Section 5)	Selection of dimension three

Table 1 Countermodels with the same record-comparison signature. Each changes a named operational requirement and exhibits the corresponding excluded alternative.

7 A Parameter-Free Foundation for Physical Theory

Every realization of the conserved comparison, comparison-preserving hierarchy, and spatial retention operations specified above has the same selected cost, ratio, and dimension:

$$(J(x),\ \varphi,\ D) = \left(\frac{x + x^{-1}}{2} - 1,\ \frac{1+\sqrt{5}}{2},\ 3 \right).$$

Detector gain, positive reporting units, and the initial hierarchy level can vary without changing these relations. The arithmetic construction is likewise invariant under the explicit translation between its finite-record realizations. This is the sense in which the paper identifies the forced core. Its uniqueness concerns these structural invariants across the stated class of operations.

7.1 What Parameter-Free Means

The selected dimensionless relations contain no adjustable shape or scale-ratio parameter. In the comparator, an arbitrary continuous report becomes $(2/\kappa)J$; the normalized cost is always J. The reference convention $\kappa = 2$ makes the raw report equal to that normalized cost. In the hierarchy, the positive initial level s_0 sets an overall unit, while every step has the forced ratio φ. In the spatial realization, the retained homology class selects the integer $D = 3$.

Neither κ nor s_0 is fitted to obtain these selected values. Their presence records the difference between a structural law and its numerical expression in a chosen unit. Absolute physical calibration, a metric, and a dynamical law require their own derivations. A fixed framework can also admit many states and histories. Parameter-free structure concerns the governing relations; a prediction for a particular system additionally uses its state and boundary data.

7.2 Physical Identification and Empirical Reach

A physical comparison can be identified by independently setting the operands, measuring same-phase and quarter-phase energies on matched preparations, and forming the ratio in Theorem 3.2. Theorem 3.3 provides an independent characterization through composition and local calibration. The hierarchy identification specifies its additive reading, the signed comparisons preserved by the update, and the two input occurrences producing the first new level. Theorem 4.2 uses closure of the seed sum on that orbit. Proposition 4.3 instead uses reversible evolution of a two-register integer census. Both derive the unequal join and its positive cost.

For dimension, Proposition 5.2 supplies the missing intermediate mechanism: the actual source coefficient determines a global phase, and uniform repetition forces its nonzero winding. Its application requires that the transport operators describe physical operations throughout the complement, and that a closed material probe enacts the entire cycle. Sampling the source at eight instants does not establish that condition. The ambient domain must also be identified. For a sphere the duality argument applies directly. For $M = \mathbb{R}^D$ the circle must leave passage between all complement positions open. Internal signal registers and a guide's restricted paths do not by themselves determine the dimension of observed space. No measurements are reported here.

The remaining universal claim has a precise form: every physically admissible realization of comparison, formation, and retention must admit the operational representation used by these theorems. The present results prove uniqueness once that representation is supplied, derive several of its requirements from simpler operations, and construct a spatial realization. Establishing the representation for all physically possible carriers and dynamics is the further step needed for an exclusive theory of everything.

7.3 Relation to Other Foundational Arguments

The starting point has a connection to Hintikka's performative treatment of the cogito (Hintikka 1962): an act of examination itself supplies an occurrence with distinguishable representations. The finite-record arithmetic is related to Dedekind's use of mappings and iteration in the construction of number (Dedekind 1888; Washburn and Zlatanović 2026a). These connections concern the operations through which the objects are constituted.

Operational reconstructions likewise show how requirements on physical operations constrain a theory. Hardy's reconstruction uses continuity of reversible transformations between pure states to distinguish the quantum case (Hardy 2001). Chiribella, D'Ariano, and Perinotti give purification a central role in deriving finite-dimensional quantum theory (Chiribella et al. 2011). The forcing results here address comparison cost, hierarchy, and spatial retention through their specified operational requirements. The dimensionality arguments of Whitrow (1955) and Tegmark (1997) use different physical constraints; the criterion here is retained homology in a circle complement.

The proofs use classical mathematical tools: initiality, rational separation of additive readings, and Alexander duality. The paper's foundational contribution lies in

how these tools are used. It distinguishes the minimal record operations, the additional operational commitments that select a quantitative or spatial structure, and the invariant conclusions shared by all realizations meeting those commitments. The countermodels keep the lower-level record histories fixed while varying the added structure, so they identify where an assumption acquires explanatory force. In this sense, deriving a physical structural constraint is not merely displaying a familiar formula: it is identifying the class of permitted operations and the alternative that class excludes.

This is a question of structural determination, not logical completeness. Gödel's theorems concern provability in effectively axiomatized formal systems (Gödel 1931); the issue here is which physical relations follow from the identified operations.

8 Conclusion

A conserved comparison with an internal phase reference has the unique normalized cost J. Composition and unit local curvature select the same function independently. Preservation of signed comparisons forces a common hierarchy scale; binary generation with seed-sum closure, or a reversible binary census, fixes that scale to φ and derives the positive cost of the first unequal join. Continuous clock-compatible transport throughout the complement, preserving the source reference norm and enacting a uniformly repeated cycle along a closed probe, produces a nonzero spatial path reading. Its homology class fixes dimension three in a sphere, and in Euclidean space when the trace leaves its complement path connected.

Each exclusion has an operational cause. Conservation removes the energy exponent and relative register weights. Internal referencing removes detector gain. Composition fixes the normalized cost. Preserved comparisons and identifiable joins fix the hierarchy. Global transport and source-cycle enactment force retained linking. These results give a common, parameter-free core across their realizations. Connecting the core to the full physical claim requires showing that actual comparison, formation, and spatial retention implement these operations throughout their physical domains. Once that identification is established, the corresponding structural values admit no further choice.

Statements and Declarations

Funding. No funding was received for this work.
Competing interests. The author declares no competing interests.
Data availability. No datasets were generated or analyzed in this theoretical study. No supplementary materials accompany the manuscript.

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