The Cosmological Constant on the Recognition Lattice: Vacuum Energy, Record Creation and the Expansion of the Universe

Jonathan Washburn

Recognition Physics Institute, Austin, Texas

jon@recognitionphysics.org

doi: 10.5281/zenodo.23839822 (version 1.2) all versions: 10.5281/zenodo.23806713

version 1.1: 10.5281/zenodo.23814465; Pith preprint 10.67347/pith.preprint.2026.000025

version 1.0: 10.5281/zenodo.23806714; Pith preprint 10.67347/pith.preprint.2026.000021

Abstract

The expansion of the universe measures the energy of empty space. The Planck 2018 data give a vacuum energy density of 1.1×10^{-123} in Planck units; zero-point energies cut off at the Planck scale give about 10^{-2}, more than 10^{120} times too much. We ask what the recognition ledger's lattice path sum says. Its action, the Lorentzian Regge action, has no volume term and is homogeneous of degree one; its spacing is near the Planck length. With a volume term the flat lattice is a vacuum only when the constant is zero, so in the native amplitude, which the phase premise makes the ledger's, the bare cosmological constant is zero, not free. On a fixed finite lattice the one-loop vacuum phase tends, as the regulator is removed, to a count of signs over the nonzero modes, $\pi/4$ each and of order one per cell; by the action's homogeneity that count is the same at every size, for the graviton and massless fields. That the graviton's one-loop vacuum therefore generates no cosmological constant is a reading of the path sum's phase, measure and effective action, conditional in a theory whose massless graviton and unitary evolution are still open. Massive fields respond to the size, at an estimated order $(m/m_P)^2$, and nothing in the ledger yet cancels them. A fixed vacuum phase per cell drops out if every history has the same number of cells, and is a volume term if not. The homogeneous sector reduces exactly to four equations in record time, whose long-wave quadratic form is Friedmann's minisuperspace action. With null postings and a fixed tick a fixed lattice cannot expand, so under these readings the universe grows by adding records: a constant rate per record, about 5×10^{-61} per tick for the measured constant, is de Sitter expansion, and whole-number counts meet Friedmann's equation only in the limit of many records. How the ledger adds records and what the path sum generates beyond one loop are open; no value of the vacuum energy is derived.

1 The weight of empty space

Take one mode of a quantum field, a wave of angular frequency ω. Its lowest state carries the energy $\hbar\omega/2$. A field in a box has one such mode for each wave that fits, and adding their zero-point energies up to a largest wave number K gives, for a massless field in units $\hbar = c = 1$,

$$\rho_{\text{vac}} = \frac{1}{2}\int^{K} \frac{\mathrm{d}^3k}{(2\pi)^3}\,|k| = \frac{K^4}{16\pi^2}.$$

If this energy gravitates, it acts as a cosmological constant: a density that does not dilute as space expands [1, 2]. With K at the Planck scale, $K = 1/\ell_P$, it is 6.3×10^{-3} in units of the Planck density $\rho_P = c^5/(\hbar G^2)$. The expansion of the universe measures the actual value. The acceleration found in supernova distances [3, 4], and the Planck 2018 fit to the microwave background with $\Omega_\Lambda = 0.6847 \pm 0.0073$ and $H_0 = 67.36 \pm 0.54\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [5], give

$$\frac{\rho_\Lambda}{\rho_P} = \frac{3\,\Omega_\Lambda\,(H_0 t_P)^2}{8\pi} = 1.1 \times 10^{-123},$$

an energy scale $\rho_\Lambda^{1/4}$ of about 2.2 meV. The estimate is more than 10^{120} times too large. A cutoff in momentum is not Lorentz invariant, and a treatment that keeps the invariance replaces K^4 by the fourth power of each field's mass; with the masses of the Standard Model the estimate is then too large by more than fifty orders of magnitude instead of a hundred and twenty [6]. The continuum number also depends on the renormalization scheme: the vacuum energy needs a counterterm whose finite part is free, and with dimensional regularization the terms that remain depend on the renormalization scale [6]. No cancellation in continuum field theory is known to remove the difference without a parameter adjusted to many decimal places [2, 6]. On a lattice the cutoff is physical, and the estimate with K at the lattice scale is the one that matters.

This paper asks what the recognition ledger's lattice path sum says about the vacuum energy. In Recognition Science events are records kept on a ledger: four counters advance one step at a time, each step a *posting*, and the lattice of records they generate carries an order before it carries a geometry [7]. A real-time path sum over the geometries this lattice can carry, with the Lorentzian Regge action [8], is the candidate quantum theory of gravity of [9, 10]. Its spacing b is near the Planck length, $b \approx 1.92\,\ell_P$ under three identifications [11], so the lattice supplies a physical cutoff at just the scale where the estimate fails: with $K = \pi/b$ the same sum gives $4.5 \times 10^{-2}\,\rho_P$. If the ledger's vacuum gravitated the way the estimate assumes, the lattice would make the problem worse, not better.

What a lattice changes. The estimate makes two assumptions that a lattice path sum does not make for free. It assumes that the vacuum's energy enters the gravitational equations through the volume, as a term ΛV in the action. The Regge action has no such term: it is a sum over triangles of area times deficit angle, and it vanishes on every flat configuration whatever its volume. And the estimate assumes that expansion keeps the cutoff fixed and adds modes. On a lattice there are two ways to expand, and they differ exactly here: the edges can stretch while the number of records stays fixed, or records can be added at fixed spacing. The question therefore splits into three parts.

Table 1. The steps from the ledger to the vacuum energy and their status. "Proved" means proved as stated, with the premises listed in the section named; "identification" marks a physical premise that is not a consequence of the definitions; "open" marks a precise statement that is neither proved nor refuted.

step	status	where
the volume term	proved: the action has none; the trace identity $S = 2\Lambda V$ at stationary points, so flat space fixes $\Lambda = 0$; the weight is one along the flat scale ray; so the bare coefficient is zero in the native amplitude, which the phase premise makes the ledger's	Section 3
the vacuum	proved at one loop on a fixed finite lattice: as the regulator is removed, the vacuum phase tends to a count of signs over the nonzero modes, invariant under rescaling for the graviton and massless fields; at $\kappa = 1$ no first-order push from the shape if its density in the ordered limit is differentiable (inference); its reading as the gravitating vacuum stress is conditional; a vacuum phase per cell cancels if the number of cells is the same in every history and is a volume term if it varies. Massive fields respond to the size, at order $(m/m_P)^2$ (inference). Open: the value of the generated coefficient at the ledger's phase scale, and a cancellation for massive fields	Section 4
the expansion	proved: exact homogeneous reduction, Friedmann's minisuperspace Hessian at long wavelengths, a fixed lattice cannot expand; adding records at a constant rate is de Sitter expansion. Numerical: the nonlinear limit. Open: the nonlinear limit as a theorem, and how records are added	Section 5
the number	open: ρ_Λ in Planck units from the three steps; Planck 2018 as comparison	Section 6

1. *The volume term.* Does the ledger fix or forbid a term ΛV in the action, and with what coefficient (Section 3)?
2. *The vacuum of the path sum.* How does the one-loop vacuum of the lattice path sum respond to a change of scale, and how does it count per cell (Section 4)?
3. *The expanding lattice.* What are the homogeneous solutions of the lattice equations, what is record time cosmologically, and does the lattice expand by stretching or by adding records (Section 5)?

The number itself, ρ_Λ in Planck units, needs all three (Section 6).

What this paper establishes. The first part is settled in the native amplitude, the second at one loop for the graviton and massless fields up to a conditional reading, and the third has reached its decisive question. The native amplitude, which the phase premise makes the ledger's, carries no volume term, and the flat lattice is a vacuum only without one (Section 3). For the graviton and massless fields the limiting phase of the one-loop vacuum on a fixed finite lattice, a count of signs over its nonzero modes, does not change when the lattice is

rescaled, and at $\kappa = 1$ the shape exerts no first-order push on it, if its density in the ordered limit is differentiable at the flat metric (an inference; nothing is claimed at second order); that this vacuum then does not gravitate is a reading of the path sum's phase, measure and effective action, and it is conditional while the path sum has no massless graviton and no unitary evolution in record time [10]; massive fields do respond to the size, with a term estimated at order $(m/m_P)^2$ in Planck units; and a fixed vacuum per cell gravitates only if the number of cells differs between histories (Section 4). A fixed lattice cannot expand under the programme's readings of time and causality, so the universe grows by adding records, and how it adds them decides whether the vacuum gravitates; a constant rate of adding records would itself be a cosmological constant (Section 5). Table 1 lists the steps.

Relation to other work. Zel'dovich identified the vacuum energy of quantum fields with the cosmological constant [1], and Weinberg showed that no adjustment mechanism of a broad class removes it without fine tuning [2]; Martin reviews the problem in detail [6]. In unimodular gravity the cosmological constant is an integration constant rather than a coupling [12]. In causal set theory, where spacetime is a discrete order, a fluctuating cosmological term of the observed size arises from the number of elements [13]. Homogeneous cosmologies in Regge calculus go back to Collins and Williams [14] and Brewin [15]. The ledger route differs from all of these in its objects: an action without a volume term on a fixed lattice of records with a Planck-scale spacing, a vacuum defined by a real-time path sum, and a time, the record count, that is part of the lattice.

2 The lattice, its action and its scale

Status: what the vacuum question rests on

Proved in [9]. The Regge action on the Kuhn lattice has no volume term and is homogeneous of degree one in the squared edge lengths; for $\kappa > 1/2$ its admissible configurations form a cone; the path sum is finite at a stated cutoff on the edge lengths and diverges without it along every ray of the cone; the lattice Newton clause gives $G = 1/(8\pi s)$.

Proved in [10]. At $\kappa = 1$ the Born weight of the ledger's cost of curvature cannot be normalized over the whole cone of admissible configurations with null postings, with the uniform measure on squared lengths.

Identifications. $\kappa = 1$; the phase premise; one record length per rank, for $b \approx 1.92\,\ell_P$ [11].

Open in [10]. An action whose graviton is massless at one loop and whose evolution in record time is unitary at the lattice scale; the nonlinear long-wave limit. Every reading of a one-loop vacuum below as a gravitating stress rests on this theory.

We recall what the lattice path sum is; the proofs are in [9, 10].

Records and the lattice. Vertices are points of $\mathbb{Z}^4$ or of the side-N torus, with coordinates x_a that count the postings on four counters, and the *record rank* of a vertex is $\sum_a x_a$. Four counters advanced one step at a time from $(0,0,0,0)$ to $(1,1,1,1)$ have 24 arrival orders, the 24 simplices of Kuhn's triangulation of the unit cube [16, 17]. Every edge joins x to $x + d$ for one of fifteen classes $d \in \{0,1\}^4 \setminus \{0\}$, and an edge with $|d| = 1$ is a posting. A configuration

is a squared length ℓ_e on every edge. The flat lattice has the metric $g_\kappa = I - \kappa \mathbf{1}\mathbf{1}^\top$ in record coordinates, $\det g_\kappa = 1 - 4\kappa$, with $g_\kappa(d,d) = |d|(1 - \kappa|d|)$; at $\kappa = 1$ every posting is null and every longer edge is timelike, and that value is an identification [9, Sec. 2.2].

The action has no volume term. The action is Regge's,

$$S(\ell) = \sum_h A_h(\ell)\, \delta_h(\ell),$$

a sum over triangles h of the area A_h times the deficit angle δ_h, the amount by which the dihedral angles of the simplices around h fail to close. It is homogeneous of degree one, $S(c\ell) = cS(\ell)$ for $c \geq 0$ [9, Sec. 3.1], and it vanishes on every flat configuration, whatever its volume. There is no term ΛV: the action does not know how much spacetime there is, only how it is curved. For $\kappa > 1/2$ the admissible configurations, those whose simplices are timelike in all their faces, form a cone, and at $\kappa = 1$ the configurations with every posting null form the *null stratum*, also a cone [9, Sec. 3.1]. The overall scale of a configuration is therefore a coordinate on configuration space, the *scale ray*, and the action is linear along it.

The path sum is defined at a cutoff. The weight of a configuration is e^{iasS}, with a the phase scale and s the action scale; $\hbar = 1/a$ is a definition and the lattice Newton clause gives $G = 1/(8\pi s)$ [9, Sec. 3.5], the coupling of the tree-level exchange; the coupling of the long-wave limit of the quantum path sum is ZG, with a factor Z that is not yet computed [11, Sec. 2]. With every squared length bounded by a cutoff the path sum is finite [9, Thm. 3.1]. Without it the sum diverges, and the reason bears directly on this paper: along every ray $c \mapsto c\ell$ the phase is linear in c, so the integral over the scale has no limit [9, Sec. 3.2]. The ledger's cost of curvature does not supply the missing scale either. It is homogeneous of degree one and vanishes on flat configurations, so at $\kappa = 1$ its Born weight cannot be normalized over the whole null stratum with the uniform measure on squared lengths, the overall scale and the flat moduli included [10]. The lattice path sum is an effective theory at a fixed spacing. How large that spacing is in physical units is fixed under three identifications, $\kappa = 1$, one record length per rank and the phase premise: $b = (2/\sqrt{3})\,\lambda_{\mathrm{rec}} \approx 1.92\,\ell_P$ [11].

What is known about the quantum theory. At $\kappa = 1$ the flat operator has massless doublers at four corners of the Brillouin zone [10, Thm. 2.1], which a measure derived from the ledger's cost of curvature gaps [10, Thms. 5.4 and 5.5]. As it stands the path sum has no massless graviton in the one-loop model of [10] (a unit-weight gauge term, no ghost line, the Feynman regulator) [10, Thm. 3.3], and its record tick preserves no positive norm on its physical solutions [10, Thm. 8.1]; with the derived weight the record-time roots of its Gaussian equation are stable in the continuum limit at fixed weight strength, but not at the lattice scale, and that is not yet a norm-preserving evolution [10, Sec. 8.1]. Whether a repaired action gives a massless graviton and a unitary evolution at the lattice scale is open. The vacuum of Section 4 is the Gaussian vacuum of that path sum, with its regulator kept.

3 The volume term

Status: the volume term

Proved in [9]. The action has no volume term and is homogeneous of degree one in the squared lengths; the flat lattice solves the Regge equations.

Proved. A Kuhn simplex of the flat lattice has 4-volume $\sqrt{4\kappa-1}\,b^4/24$, and the volume is homogeneous of degree two. At every stationary point of $S-\Lambda V$, $S=2\Lambda V$; hence the flat lattice is stationary for $S-\Lambda V$ exactly when $\Lambda=0$ (Proposition 3.1). The path-sum weight is identically one along the flat scale ray, and a recognition cost charged on the cell volumes (a model) would hold the size without tilting it (Proposition 3.2).

Identification. That the ledger's amplitude is the weight e^{iasS}, fixed by the phase premise; it is used to read the results, not to prove them.

A cosmological constant on the lattice would be a term ΛV in the action, with $V=\sum_\sigma V_\sigma$ the total 4-volume. In the weight e^{iasS} the phase scale and the action scale enter only through their product, so in this section and the next we write a for as and measure Λ in units of s: the weight is e^{iaS}, and a cosmological term enters as $S-\Lambda V$. At long wavelengths $\sum_h A_h\delta_h$ approximates $\frac{1}{2}\int\sqrt{-g}\,R\,\mathrm{d}^4x$ [8], so the classical action $s(S-\Lambda V)$ is $(16\pi G)^{-1}\int(R-2\Lambda)\sqrt{-g}\,\mathrm{d}^4x$ with $G=1/(8\pi s)$: the Λ of $S-\Lambda V$ is the cosmological constant of $G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi G T_{\mu\nu}$, with no factor of two between the two conventions. The volume of a Kuhn simplex follows from its squared edge lengths. Take the four edges from one vertex and form their Gram matrix, with entries $\frac{1}{2}(\ell_{0i}+\ell_{0j}-\ell_{ij})$; for a Lorentzian simplex its determinant is negative, and $V_\sigma=\sqrt{-\det\mathrm{Gram}}/24$. On the flat lattice the four edges of a Kuhn simplex are a unimodular basis of $\mathbb{Z}^4$, so the Gram determinant is $\det g_\kappa=1-4\kappa$ in lattice units, and

$$V_\sigma=\frac{\sqrt{4\kappa-1}}{24}\,b^4,\qquad V_{\mathrm{cell}}=\sqrt{4\kappa-1}\,b^4,\qquad V=N^4\sqrt{4\kappa-1}\,b^4$$

on the side-N torus, $\sqrt{3}\,b^4$ per cell at $\kappa=1$. The volume is homogeneous of degree two in the squared lengths, $V(c\ell)=c^2V(\ell)$, while the Regge action is of degree one.

Flat space and a volume term do not mix. The difference in degree has a sharp consequence. Along the scale ray through any configuration, $S-\Lambda V$ becomes $cS-\Lambda c^2V$, and its derivative at $c=1$ is $S-2\Lambda V$. At a stationary point that derivative vanishes.

Proposition 3.1 (The lattice trace identity)**.** *At every configuration where $S-\Lambda V$ is stationary, $S=2\Lambda V$. In particular, for $N\geq 3$ and $\kappa>1/2$ the flat lattice is a stationary point of $S-\Lambda V$ if and only if $\Lambda=0$.*

The flat lattice solves the Regge equations: by Schläfli's identity the variation of the angles drops out of the first variation, which is $\sum_h(\partial A_h/\partial\ell_e)\,\delta_h$, and on the flat lattice every deficit vanishes [9, Thm. 4.1]. So $S=0$ there while $V>0$, and the identity leaves $\Lambda=0$. This is the lattice form of the trace of Einstein's equations, which in vacuum reads $R=4\Lambda$: flat spacetime solves them only without a cosmological constant.

The ledger's weight has none. The path sum weighs a configuration by e^{iasS}, with the phase scale fixed by the ledger's phase premise [11]. On the flat lattice and every rescaling of it the action vanishes.

Proposition 3.2 (No volume term in the weight)**.** *For $N \geq 3$, $\kappa > 1/2$ and every phase scale, the weight e^{iaS} equals 1 at every point $c\ell^0$, $c > 0$, of the flat scale ray, while $S - \Lambda V = -\Lambda c^2 N^4 \sqrt{4\kappa - 1}\, b^4$ there, which is constant along the ray only for $\Lambda = 0$. If the recognition cost $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ were charged on the ratio of each simplex's volume to its flat value (a model), the total along the ray would be $24N^4 J(c^2)$: zero, with zero slope, at $c = 1$, and nonnegative.*

So in the native amplitude e^{iaS} the bare cosmological coefficient is zero, not free; a nonzero one would make the flat lattice non-stationary and would be the only term in the action that distinguishes flat lattices by their size. A cost of volumes behaves differently: it holds the size like a spring and never tilts it the way Λ does. That the ledger's amplitude is the native one is the phase premise, a separate open step [11]. If some other reading of the ledger allowed a volume term, its coefficient would be built from the ledger's constants, and in the ledger's units the only scale is the record length. Such a coefficient is a pure number times $\lambda_{\mathrm{rec}}^{-4}$, a Planck-scale density: zero, or of Planck size unless its pure number is itself of order 10^{-123}, which would need a derivation of its own (INFERENCE: dimensional).

4 The vacuum of the path sum

Status: the vacuum

Proved. As the regulator is removed, the phase of the one-loop vacuum of the path sum tends to a count of signs over its nonzero modes, $\pi/4$ per mode, once the zero modes of the stationary set are removed; at finite regulator no mode's phase is exactly $\pm\pi/4$, and both sides of the regulator are kept (Proposition 4.1). For the graviton that limiting phase does not change when the lattice is rescaled (Proposition 4.2); nor does a massless scalar's on the flat lattice, while a massive scalar's does (Proposition 4.3). At $\kappa = 1$ a function of the flat metric that has the lattice's symmetries, including scale invariance, and is differentiable there has zero derivative along all six flat moduli, size and shape (Proposition 4.4). A fixed phase per cell cancels from every normalized expectation if every history has the same number of cells, and is a volume term $\Lambda = -\varphi/(av)$ if the number varies (Proposition 4.5). Beyond one loop the generated cosmological coefficient, the size response of the vacuum phase read as a volume term, is the response of its phase to the phase scale (Proposition 4.6).

Inference. The one-loop phase is the vacuum energy times the time extent, as for a single oscillator. The vacuum of a field of mass m gravitates at the order $(m/m_P)^2$ in Planck units, which nothing in the ledger yet cancels. A phase of order one per cell would be a volume term of Planck size. Applying Proposition 4.4 to the vacuum energy of the graviton and of massless fields needs its density in the ordered limit to be differentiable at the flat metric; massive fields break the scale invariance it uses.

Open. Target 4.7: the value of the generated coefficient at the ledger's phase scale. A cancellation of the vacuum energy of massive fields.

Identifications and models. $\kappa = 1$; the Feynman regulator; for matter, the lattice scalar of [9] with its mass lumping (model).

In a real-time path sum the vacuum energy is a phase. Take a single oscillator over a time T. Its Gaussian path integral is a product over the modes of the time interval, and each mode contributes $e^{i\pi/4}$ or $e^{-i\pi/4}$ according to the sign of its eigenvalue. As T grows, one eigenvalue turns negative every time ωT passes a multiple of π, and each change of sign shifts the phase by $\pi/2$; after a long time the phase has moved by $\omega T/2$, which is e^{-iE_0T} with $E_0 = \hbar\omega/2$. So the zero-point energy of a real-time path sum is a count: $\pi/4$ times the signature of the quadratic action, the number of positive minus the number of negative modes. Each Gaussian is defined with the regulator kept: an eigenvalue λ is replaced by $\lambda + i\varepsilon$, for which the integral converges when $\varepsilon > 0$, and every square root is the principal one, so that $(2\pi i/\lambda)^{1/2}$ has the argument $\pi/4$ for $\lambda > 0$ and $-\pi/4$ for $\lambda < 0$; Proposition 4.1 also records the other side of the regulator. For a finite set of real modes,

$$\int_{\mathbb{R}^n} e^{\frac{i}{2}x^{\top}Hx}\,\mathrm{d}x = (2\pi)^{n/2}\,|\det H|^{-1/2}\,e^{i\pi\,\mathrm{sig}(H)/4}.$$

The modulus is a positive weight; for an oscillator the phase is its vacuum energy times the time. That is why the vacuum energy is read from the phase below and the modulus is set aside: the modulus is the normalization of the Gaussian, carries no phase, and enters a sum over geometries as part of its measure, where it is treated with the measure [10, Sec. 5], not as an energy.

Proposition 4.1 (The one-loop vacuum is a count)**.** *Let the zero modes of the quadratic action, the gauge directions, the radial direction along the flat scale ray and any others, be the directions of the stationary set, integrated separately as in the stationary-phase law of [9], so that every remaining mode has an eigenvalue $\lambda \neq 0$. Each Gaussian factor $(2\pi i/\lambda)^{1/2}$ of such a mode equals $\sqrt{2\pi/|\lambda|}\,e^{i(\pi/4)\,\mathrm{sign}\,\lambda}$, so the one-loop constant of [9] is a positive modulus times $\exp\bigl(i\frac{\pi}{4}\sum_i \mathrm{sign}\,\lambda_i\bigr)$, the sum running over the nonzero modes. With the regulator kept, the phase of a mode is $\frac{1}{2}\arctan(x/\varepsilon)$ with x a positive multiple of λ: it is $\pm\pi/4$ at no finite ε, and it tends to $\frac{\pi}{4}\,\mathrm{sign}\,\lambda$ as $\varepsilon \to 0^+$ and to $-\frac{\pi}{4}\,\mathrm{sign}\,\lambda$ as $\varepsilon \to 0^-$.*

Read the same way, as the vacuum energy times the time extent (INFERENCE), this phase is of order one per cell, a density b^{-4}: the Planck-sized vacuum energy of Section 1. Whether it gravitates is a separate question.

The size cannot be felt at one loop. On a fixed lattice the quadratic action of the graviton is the Hessian of the Regge action, and the action is homogeneous of degree one in the squared lengths.

Proposition 4.2 (No response to the size)**.** *At every Lorentzian nondegenerate configuration y, one whose simplices are timelike in all their faces, and every $c > 0$, the Hessian of the Regge action satisfies $D^2S(cy) = c^{-1}D^2S(y)$. So the zero modes are the same at every size, and dividing every nonzero eigenvalue by c keeps the limiting phase of the one-loop vacuum, its count of signs, and multiplies its modulus by a power of $\sqrt{c}$; at finite ε the regulated phase of each mode changes, and only its limit is invariant.*

What Proposition 4.2 states is a scale invariance: on a fixed finite lattice the phase of the regulated Gaussian vacuum of the graviton, in the limit in which the regulator is removed,

the Fresnel count over its nonzero modes, does not change when every squared length is multiplied by the same factor, because the action is homogeneous. A cosmological constant is a response of the vacuum to the volume, a term in the effective action proportional to it. Reading the invariance as "at one loop the graviton generates no cosmological constant" identifies this phase with the vacuum's gravitating stress, and that identification is conditional: on the reading of the path-sum phase as an energy, on the measure, and on the effective action of a path sum that does not yet have a massless graviton or a unitary evolution in record time [10]. Under that reading, the graviton's one-loop vacuum generates no volume term on a fixed lattice. The vacuum energy $K^4/(16\pi^2)$ of Section 1 assumed the opposite: that the vacuum is a density, so that more volume means more vacuum energy.

Massive fields feel the size. The argument uses the homogeneity of the quadratic action, and a massive field breaks it. When the squared edge lengths are multiplied by c, the kinetic part of a lattice scalar's action scales by c and its mass part by c^2.

Proposition 4.3 (Massive fields feel the size)**.** *For a lattice scalar ϕ of mass m on a configuration with squared edge lengths y, and every $c > 0$, $S_m[cy, \phi] = c\, S_{\sqrt{c}\,m}[y, \phi]$: the vacuum at size c is the vacuum at size 1 with the mass multiplied by $\sqrt{c}$. On the flat lattice a massless scalar's one-loop phase is therefore the same at every size. A massive scalar's is not: at $\kappa = 1$ on the torus of side 16, for every $m \neq 0$ the one-loop phase at the size $c = 1/m^2$ is smaller than at $c = 1/(100\, m^2)$, because a timelike mode of the lattice changes the sign of its eigenvalue between the two.*

The proof, and the spectrum on the side-16 torus, are in Appendix A.

The part of the phase that does not depend on the mass, the Planck-sized count of order one per cell, still does not respond to the size. The part that depends on the mass does. Its magnitude is estimated at order m^2 per cell in lattice units, the squared mass times the density of lattice modes near the light cone: a cosmological term with $\Lambda \ell_P^2$ of order $(m/m_P)^2$, about 2×10^{-45} for the electron and 2×10^{-34} for the top quark (INFERENCE: the coefficient is not computed). This is the lattice form of the term $m^2 K^2$ of the continuum estimate, and on the lattice it is physical, because the cutoff is. Nothing in the ledger as formulated cancels it; a cancellation between bosons and fermions would, and none is derived. So, under the reading of the phase as the gravitating vacuum stress, the lattice removes the largest term of the vacuum problem, the Planck-sized one, from the gravitational sector and from massless fields, and leaves the terms that come from mass.

No first-order push from the shape. At $\kappa = 1$ the lattice symmetry says more, at first order. The flat configurations with null postings form a family with six parameters, the constant metrics with zero diagonal in posting coordinates, which change the size and the shape of the cell while keeping every posting null.

Proposition 4.4 (No first-order push on the null flat lattice)**.** *Let f be a function of the constant metric in posting coordinates that is invariant under the 24 relabellings of the axes and under scaling, and differentiable at $I - \mathbf{1}\mathbf{1}^\top$. Then its derivative vanishes along every strain with zero diagonal, that is, along all six moduli of the null flat family. Nothing is stated about its second derivatives. Both conditions matter: a function meeting them can respond to a strain that moves*

the postings, and away from $\kappa = 1$ and $\kappa = 1/4$ such a function can respond along a strain with zero diagonal.

For the graviton and massless fields, if the density of the vacuum in the ordered limit is differentiable at the flat metric, the only first-order vacuum stress this symmetry leaves at $\kappa = 1$ is proportional to the identity in posting coordinates: four null fluxes along the postings, which couple only to the posting lengths, and $\kappa = 1$ holds those at zero (INFERENCE, until that differentiability is shown). So at $\kappa = 1$ the vacuum exerts no first-order push on the size or the shape of the cell; its response to the shape at second order is not constrained here.

When a vacuum per cell gravitates. What a fixed lattice does not do, a growing one might. The vacuum phase is a sum over modes, close to a fixed amount φ per cell. Whether that amount acts on the geometry depends on one property of the path sum.

Proposition 4.5 (A clock cancels the vacuum)**.** *In a sum over a finite family of histories, if any two histories with the same record time have the same number of cells, so that the record count is a clock, a fixed phase per cell multiplies every history by the same factor and drops out of every normalized expectation. If the number of cells differs between histories, a phase φ per cell is a volume term with $\Lambda = -\varphi/(av)$, for a cell of 4-volume v; it vanishes only if $\varphi = 0$.*

So, read as the vacuum's coupling to gravity, a fixed vacuum phase per cell gravitates only through the variation of the number of records between the histories of the path sum. With a fixed number it is a constant phase and has no effect, which is how a constant zero-point energy behaves in quantum mechanics without gravity. With a varying number it is a cosmological constant, of Planck size if φ is of order one (INFERENCE), and zero only if φ vanishes. Which case holds is a question about how the ledger adds records (Section 5).

Beyond one loop. Because $S(c\ell) = cS(\ell)$, a change of the lattice's overall size is the same as a change of the phase scale, $a \to ac$, and this holds for the whole path sum, not only at one loop.

Proposition 4.6 (The generated coefficient)**.** *The path sum over the lattice rescaled by $c > 0$ equals the path sum at phase scale ac. For a finite measure on which the action is bounded, and wherever Z is off the closed negative real axis, the derivative of the phase of the path sum Z with respect to a is the real part of the expectation value of the action, $\partial_a \arg Z = \mathrm{Re}\langle S\rangle$. Define the generated cosmological coefficient as $\Lambda_{\mathrm{gen}} = -\mathrm{Re}\langle S\rangle/(2V)$ for $V \neq 0$; then the phase of the rescaled path sum changes at $c = 1$ at the rate $-2a\Lambda_{\mathrm{gen}}V$, as the phase of a volume term with that coefficient would. On the null stratum with a cutoff $L \geq 0$ on the squared edge lengths, $Z_{cL}(a) = c^d Z_L(ac)$, and the phase of Z_{cL} changes at $c = 1$ at the same rate: the generated coefficient is also the response of the vacuum phase to the cutoff.*

Reading the whole size response of the vacuum phase as a volume term, the cosmological constant the lattice generates is a definite property of its path sum: how much the phase of the vacuum moves when the phase scale, or equivalently the cutoff, moves.

Target 4.7 (The generated cosmological coefficient)**.** On the null stratum at $\kappa = 1$, with every squared edge length at most a fixed cutoff L and the ledger's phase scale a, determine

the number X such that the path sum on the side-N torus is nonzero for all large N and its generated coefficient, measured against the flat volume $N^4\sqrt{3}$, tends to X as $N \to \infty$.

The target is not trivially satisfiable: it asks for the path sum to be nonzero on all large tori, where the coefficient is defined, and at most one value of X meets it.

5 The expanding lattice

Status: the expansion

Proved. A homogeneous isotropic configuration is four functions of record time, with all 24 simplices at a rank congruent, and it solves the Regge equations, with a volume term and matter, exactly when four reduced equations hold at every rank (Proposition 5.1). At long wavelengths the quadratic part of the lattice action on homogeneous strains is the Hessian of the minisuperspace action of a Friedmann universe, whose lapse equation is Friedmann's constraint with $G = 1/(8\pi s)$ (Proposition 5.2). With null postings and a fixed tick per rank, a fixed lattice cannot expand (Proposition 5.4). If records are added instead, a constant creation rate per record is de Sitter expansion, and a power-law rate fixes the equation of state (Proposition 5.5).

Numerical. The nonlinear long-wave limit of the reduced action holds at the rate $1/N^2$ for the profiles computed.

Inference. The theorems hold on the torus, periodic in rank; an open line of record rank has the same local equations, since every statement is local in rank once N exceeds the width of a simplex. Under the readings below, an expanding universe grows by adding records.

Proved. With whole-number counts, agreement with Friedmann's equation at every tick and a vacuum alone forces the count to grow by a whole factor each tick, a Planck-sized cosmological constant (Proposition 5.6); in the limit of many records a small one is allowed, and the guards of Target 5.7 hold.

Open. Target 5.3: the nonlinear Friedmann limit. Target 5.7: a law of homogeneous record creation whose count is the same in every history and meets Friedmann's equation in the limit of many records.

Identifications. $\kappa = 1$; the posting order as the causal order (postings stay null); proper time counts records (a fixed tick per rank); the constant cell read as the Friedmann cell; dust as matter on the diagonal edges.

A cosmology on the lattice starts with configurations that look the same at every place and in every direction. Translations that preserve the record rank, the vectors v with $\sum_a v_a = 0$, move a vertex within its slice of fixed rank, and the 24 permutations of the counters preserve the triangulation and the rank. For an expanding universe the carrier is a spatial torus of rank-preserving translations times an open line of record rank; the theorems below are proved on the side-N torus, periodic in rank as well. A configuration with these symmetries gives an edge a squared length that depends only on the rank t of its first vertex and its number n of postings: four functions $f_n(t)$.

Proposition 5.1 (The homogeneous sector)**.** *For the configuration with squared lengths $f_n(t)$, all 24 Kuhn simplices at a vertex of rank t carry the same ten squared lengths, and on the side-N torus its Regge action is $24N^3$ times a sum over ranks of a density. For $N \geq 3$, if every simplex is timelike in all its faces, the configuration is stationary for the full lattice action if and only if four*

reduced equations, one for each $n = 1, \dots, 4$, hold at every rank. The same holds for the action $sS - \Lambda V + \sum_e g_{n_e}(\ell_e)$ with a volume term and matter on the edges, such as dust on the diagonal edges. The flat family $f_n = \alpha n(1 - \kappa n)$ solves the equations without them.

The reduction is exact: no symmetric-criticality argument is assumed, because the first variation is re-indexed edge by edge (Appendix B).

The cell of an expanding lattice. A configuration with $f_n = \alpha n + \beta n^2$ for constants α, β is flat, with metric $\alpha I + \beta \mathbf{1}\mathbf{1}^\top$ in record coordinates. On the slice directions, $\sum_a v_a = 0$, this metric is α times the Euclidean one, so the spatial scale factor is $a = \sqrt{\alpha}$. Along the time direction $\mathbf{1}$, one rank step is a quarter of $\mathbf{1}$, of squared length $(\alpha + 4\beta)/4$, so the lapse per rank step is n with $n^2 = -(\alpha + 4\beta)/4$. Comparing with $g_\kappa = I - \kappa \mathbf{1}\mathbf{1}^\top$,

$$\kappa = -\frac{\beta}{\alpha} = \frac{1}{4} + \frac{n^2}{a^2},$$

The metric is Lorentzian for every $\kappa > 1/4$, but its simplices are timelike in all their faces, the condition the theorems below use, exactly when $\kappa > 1/2$, that is, when the lapse per rank exceeds $a/2$.

Proposition 5.2 (Friedmann at long wavelengths)**.** *For $\kappa > 1/2$, the long-wave coefficient of the lattice action on a homogeneous strain $\mathrm{d}\alpha\, I + \mathrm{d}\beta\, \mathbf{1}\mathbf{1}^\top$ with wave vector $\omega(1,1,1,1)$ is $-6\omega^2 \mathrm{d}\alpha^2/\sqrt{4\kappa - 1}$, with no term in $\mathrm{d}\beta$, the direction of the lapse. This is the Hessian of $-6a\dot{a}^2/n$ per site of a slice, the textbook minisuperspace Lagrangian of a Friedmann universe. With matter of density ρ, the lapse equation of $s(-6a\dot{a}^2/n) - 2na^3\rho$ is*

$$\left(\frac{\dot{a}}{na}\right)^2 = \frac{8\pi G}{3}\rho, \qquad G = \frac{1}{8\pi s},$$

and resting dust on the diagonal edges has $\rho = 2m/a^3$, with $2a^3 n$ the 4-volume per vertex.

The derivation of the coefficient is in Appendix C. Newton's constant enters the cosmology as it enters Newton's law in [9], through the action scale, and no new constant appears. What Proposition 5.2 gives is the quadratic, long-wave level. The full statement is nonlinear.

Target 5.3 (The Friedmann limit)**.** For periodic profiles $f_n(t) = \alpha(s)n + \beta(s)n^2$, $s = (t + n/2)/N$, three times differentiable with $\alpha > 0$ and $\alpha + 2\beta < 0$, N times the reduced action per site tends to $\int_0^1 -\frac{3}{2}\,\alpha'^2/(\sqrt{\alpha}\,n)\,\mathrm{d}s$ as $N \to \infty$, the Friedmann action.

The target holds at zeroth order: constant profiles have zero action at every $N \geq 3$. Computed for profiles of amplitude 0.2 and 0.6 at $\kappa = 1$ and 0.3 at $\kappa = 5/4$, conformal and with an independent lapse, the limit holds at the rate $1/N^2$ without relaxing the two massive lattice modes (NUMERICAL, Appendix D). Regge calculus has treated closed Friedmann universes on fixed triangulations of the three-sphere [14, 15]; what is new here is the record-time reading and the exact reduction on the Kuhn lattice.

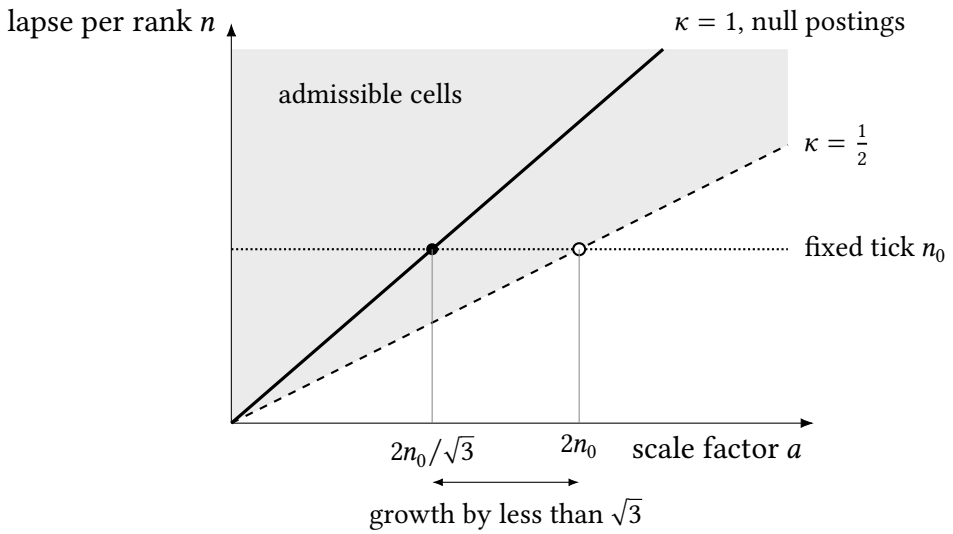


Figure 1. The expansion trilemma, for the cells $\alpha(I - \kappa \mathbf{1}\mathbf{1}^\top)$ of a homogeneous lattice, in the plane of the scale factor a and the lapse per rank n, where $\kappa = \frac{1}{4} + n^2/a^2$. Cells are admissible, every simplex timelike in all its faces, in the shaded region above the dashed line $\kappa = \frac{1}{2}$ (the cell metric itself is Lorentzian for every $\kappa > \frac{1}{4}$); postings are null on the solid line $\kappa = 1$. A fixed tick n_0 (dotted) meets the null line at one scale factor, $a = 2n_0/\sqrt{3}$ (filled point), and leaves the admissible region at $a = 2n_0$ (open point). So null postings and a fixed tick together allow no expansion; a fixed tick alone keeps $a < 2n_0$, so from the cell with null postings the scale factor grows by less than a factor $\sqrt{3}$; and null postings alone make the lapse grow with a: record time is conformal time.

A fixed lattice cannot expand. The relation $\kappa = \frac{1}{4} + n^2/a^2$ decides what record time can mean, and the answer constrains the whole cosmology.

Proposition 5.4 (The expansion trilemma)**.** *For the cell $\alpha(I - \kappa \mathbf{1}\mathbf{1}^\top)$: null postings ($\kappa = 1$) and a fixed lapse per rank n_0 fix the scale factor, $a^2 = 4n_0^2/3$. With a fixed lapse alone the cell stays admissible, every simplex timelike in all its faces, only while $a < 2n_0$, that is, while $\kappa > 1/2$, so from the cell with null postings the scale factor can grow by less than a factor $\sqrt{3}$, and the bound is sharp. With null postings alone, $n^2 = \frac{3}{4}a^2$: record time is conformal time, and the proper duration of a rank grows with the scale factor.*

Figure 1 shows the three conditions in one plane.

The programme reads proper time as a count of records, a fixed tick per rank, and reads the posting order as the causal order, postings null. Under both readings together a fixed lattice cannot expand at all. So if the universe expands, it does so by adding records (INFERENCE): new cells at the fixed spacing, not longer edges. This is also the reading behind the spacing of [11], one record length per rank. The framework's existing account of record growth describes records added at the frontier of a growing region; a homogeneous universe needs records created throughout the bulk.

Expansion by adding records. If the universe grows by adding records of fixed size, its expansion is a count. With R the number of records in a comoving region and $\Gamma = \dot{R}/R$ the rate at which records are created per record, the next statement is kinematic and needs no field equation beyond Friedmann's.

Proposition 5.5 (Records and the expansion)**.** *For records of fixed size, $a \propto R^{1/3}$ and $H = \Gamma/3$. A constant creation rate per record is de Sitter expansion, and Friedmann's equation then requires the constant density $\rho = \Gamma^2/(24\pi G)$. A power law $R \propto t^p$ is a perfect fluid with equation of state $w = 2/p - 1$: $p = 2$ is dust, $p = 3/2$ radiation, $p = 3$ a coasting universe, $p = 1$ a stiff fluid.*

In this reading a cosmological constant is kinematic: it is a constant rate of record creation per record, whatever the vacuum phase does. A constant rate Γ is the cosmological constant $\Lambda = \Gamma^2/3$. The measured value, $\Lambda\ell_P^2 = 2.85 \times 10^{-122}$, corresponds to $\Gamma t_P = 2.92 \times 10^{-61}$, or 4.9×10^{-61} per tick of $2\sqrt{\ln 2}$ Planck times; today's total expansion rate corresponds to $\Gamma = 3H_0$, 5.9×10^{-61} per tick (comparisons). The question why the cosmological constant is small becomes the question why a record gives rise to a new record with a probability of about 5×10^{-61} per tick: one new record per record in about 3×10^{60} Planck times (INFERENCE: the reading of de Sitter expansion as record creation). The framework's existing descriptions of record growth do not answer it: each of their record counts is a fixed function of record time, a clock in the sense of Proposition 4.5, and each grows at the frontier of a region rather than throughout its bulk; read as bulk counts, they would give $w = 1$ or $w = -1/3$, not the $w \approx -1$ that is measured.

For a background to be a cosmology the count has to match the matter it carries: a record count $R(t)$ and a density $\rho(t)$ give a Friedmann universe exactly when $(\dot{R}/3R)^2 = (8\pi G/3)\rho$ at every time. Dust, with $\rho = D/R$ for a fixed amount D, then needs $R = 6\pi G D t^2$ exactly, and a count that grows linearly in time, such as two records per tick, fails for dust: read as a bulk count it is a stiff fluid.

The ledger's count is a whole number, and that changes what the requirement can ask. Over one tick of duration τ_0 the count grows from $R(t)$ to $R(t+1)$, so for records of fixed size the expansion rate averaged over the tick is $H(t) = \ln\big(R(t+1)/R(t)\big)/(3\tau_0)$. Asking $H(t)^2 = (8\pi G/3)\rho$ exactly at every tick fails for a small cosmological constant, for a reason that is arithmetic, not physical.

Proposition 5.6 (Exact agreement forces a Planck-sized constant)**.** *Let a whole-number count of records in a region grow without decreasing, and let the region hold a vacuum of density $\rho_v > 0$ and nothing else. If $H(t)^2 = (8\pi G/3)\rho_v$ at every tick, the count grows by the same whole factor, at least 2, at every tick, so $\rho_v \geq (\ln 2)^2/(24\pi G\tau_0^2)$; with the tick of the identifications, $\tau_0^2 = 4 \ln 2\, G$ in Planck units, the cosmological constant $\Lambda = 8\pi G\rho_v$ satisfies $\Lambda G \geq \ln 2/12$.*

A constant rate needs the ratio $R(t+1)/R(t)$ to be the same rational number at every tick, and a whole-number count can keep a fixed rational ratio for ever only if the ratio is a whole number. The measured value, $\Lambda\ell_P^2 = 2.85 \times 10^{-122}$, is about 10^{120} times below the bound. So the requirement asks for Friedmann's equation in the limit of many records, where the description by a Friedmann universe applies anyway.

Target 5.7 (How the ledger adds records)**.** Derive the ledger's law of record creation, and show that it gives each history of the path sum, of which there is at least one, a count of the records in each comoving region at each record time, positive, never decreasing and the same in every region, such that (i) the count at each record time is the same in every history, and (ii) in every history and region

$$\frac{H(t)^2}{(8\pi G/3)\,\rho(t)} - 1 \longrightarrow 0 \qquad (t \to \infty), \qquad \rho(t) = \rho_v + \frac{D}{R(t)},$$

for the ledger's matter content: a constant vacuum density ρ_v and dust of fixed amount D per region.

The matter content has to be the ledger's own, supplied independently of the count. Fitted to the count, ρ_v would let any count whose growth ratio settles meet clause (ii), and the target would test nothing.

The limit admits a small cosmological constant. For every $x > 0$, adding $\lceil xR(t) \rceil$ records each tick meets clause (ii) with the vacuum density $(\ln(1+x))^2/(24\pi G\tau_0^2)$, which tends to zero with x, and the relative defect is at most a multiple of one over the number of records created per tick, a multiple that grows like $1/x^2$ as x shrinks; a count that grows as the square of the number of ticks meets it with dust alone, a matter era. The target is not trivially satisfiable: a static count fails it for every positive density, the count of two records per tick fails it with dust, and an ensemble whose histories grow by different factors fails clause (i), although each of its histories can meet clause (ii) for a vacuum of its own. What the limit does not test is the matter era of a universe with a vacuum: when $\rho_v > 0$ the dust term vanishes from the limit, so the target asks the late expansion to be right, not the history before it.

Clause (i) is where the two halves of this paper meet. By Proposition 4.5, a count that is the same in every history makes the vacuum phase per cell drop out of every normalized expectation, and, read as the vacuum's gravitating stress, the lattice vacuum then does not gravitate; if the count varies between histories, the vacuum phase is a volume term, of Planck size if the phase per cell is of order one, and zero only if it vanishes per cell. The target asks for the first case, in which the cosmological constant is the kinematic rate of Proposition 5.5.

6 The number

Status: the vacuum energy
Open. ρ_Λ in Planck units from Targets 4.7, 5.3 and 5.7, and the equation of state of whatever term results.
Comparison only. Planck 2018: $\rho_\Lambda/\rho_P = 1.1 \times 10^{-123}$, $\Lambda\ell_P^2 = 2.85 \times 10^{-122}$, $w = -1.03 \pm 0.03$.

No value of the vacuum energy is derived in this paper. This section records what a derived value would meet. The numbers below are measurements; none of them enters any statement above.

The measured value. The Planck 2018 analysis of the microwave background, with lensing, gives $\Omega_\Lambda = 0.6847 \pm 0.0073$ and $H_0 = 67.36 \pm 0.54\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [5]. In Planck units,

$$\frac{\rho_\Lambda}{\rho_P} = \frac{3\,\Omega_\Lambda (H_0 t_P)^2}{8\pi} = 1.13 \times 10^{-123}, \qquad \Lambda\ell_P^2 = 3\,\Omega_\Lambda (H_0 t_P)^2 = 2.85 \times 10^{-122},$$

with $H_0 t_P = 1.18 \times 10^{-61}$. Both forms appear in the literature as "10^{-122}". Combined with supernovae and baryon acoustic oscillations, the Planck analysis finds the equation of state $w = -1.03 \pm 0.03$ for a constant w [5]. The baryon acoustic oscillation measurements of DESI, combined with the microwave background and supernovae, prefer an equation of state that

changes with time, at a significance between 2.5 and 3.9 standard deviations depending on the supernova sample [18]. A derivation that produces a term with $w = -1$ exactly makes a different prediction from one that produces a slowly varying term, and the data are beginning to distinguish them. Nothing in this paper predicts an equation of state other than $w = -1$: the DESI result is context for a derivation still to come, and no statement here rests on it.

In the ledger's own units the measured value is a single number, the vacuum energy density in units of $\hbar c/\lambda_{\mathrm{rec}}^4$. With ρ_Λ the mass density, so that $\rho_\Lambda c^2$ is the energy density, $X = \rho_\Lambda c^2 \lambda_{\mathrm{rec}}^4/(\hbar c)$ lies between 8.4 and 9.0×10^{-123} (comparison, at tree level; with the long-wave factor Z of [11] the measured values fix XZ^2 instead). The same number fixes Newton's constant: with H_0 and Ω_Λ measured, a derived X gives $G = \pi X c^5/\big(6(\ln 2)^2 \hbar H_0^2 \Omega_\Lambda\big)$ to the 2% precision of $H_0^2\Omega_\Lambda$, under the identifications of [11]. A derivation of the vacuum energy on the ledger would therefore also be a prediction of G.

What a derivation must reach. A derived vacuum energy has to come out near 10^{-123} in Planck units with nothing adjusted. By Section 3 the bare term is zero in the native amplitude, under the phase premise, and by Section 4 the limiting phase of the graviton's one-loop vacuum on a fixed lattice does not respond to its size; a fixed vacuum phase per cell acts only through the variation of the number of cells between histories (Proposition 4.5). If it does act, Proposition 4.5 gives $\Lambda = -\varphi/(av)$, with v the 4-volume of a cell. Take $b = \beta\lambda_{\mathrm{rec}}$, a cell of 4-volume $v = \sqrt{3}\, b^4$ ($\kappa = 1$), the phase scale $a = \ln 2/(2\pi\lambda_{\mathrm{rec}}^2)$ of the phase premise and $\ell_P^2 = \lambda_{\mathrm{rec}}^2/(4\ln 2)$, the Planck length of the tree-level coupling (identifications). Then

$$\Lambda\ell_P^2 = -\frac{\varphi}{av}\,\ell_P^2 = -\varphi \cdot \frac{2\pi\lambda_{\mathrm{rec}}^2}{\ln 2} \cdot \frac{1}{\sqrt{3}\,\beta^4\lambda_{\mathrm{rec}}^4} \cdot \frac{\lambda_{\mathrm{rec}}^2}{4\ln 2} = -\frac{\pi\varphi}{2(\ln 2)^2\sqrt{3}\,\beta^4},$$

so matching the measured value needs $|\varphi| < 3 \times 10^{-122}$ for β between 1 and $2/\sqrt{3}$; with the long-wave coupling ZG the right side is multiplied by Z. Massive fields are a separate obstacle: their vacuum responds to the size (Proposition 4.3), at an order estimated as $(m/m_P)^2$, some 10^{77} times the measured value of $\Lambda\ell_P^2$ already for the electron, and a derivation has to show what cancels it. A small nonzero value therefore needs a large number, and the ledger has one: the number of records. The age of the universe is about 8×10^{60} Planck times, $H_0 t_P$ is close to the inverse of that number, and the measured value is $\rho_\Lambda/\rho_P \approx 0.08\,(H_0 t_P)^2$. If records were created at random, so that their number in a region of 4-volume V fluctuated between histories by about $\sqrt{V/b^4}$, the per-cell phase of Proposition 4.5 would act as a cosmological term about $\sqrt{b^4/V}$ times its Planck-scale value per cell, which for the 4-volume of the observable universe is of the observed order; the fluctuating cosmological term of causal-set theory has this form [13]. This is a statement about sizes, not a derivation (INFERENCE). If instead the measured acceleration is kinematic, a constant rate of record creation (Proposition 5.5), the number to derive is that rate: $\Gamma t_P = 2.92 \times 10^{-61}$, about 5×10^{-61} new records per record per tick. Either way the answer to Target 5.7 has to be derived, with its rate and its statistics, and then compared.

7 What remains

The cosmological constant problem has a definite form on the ledger's lattice. For the graviton it has a conditional answer at one loop: no bare term in the native amplitude, which the phase premise makes the ledger's, and on a fixed finite lattice a one-loop vacuum phase whose limit as the regulator is removed, a count of signs over the nonzero modes, does not change when the lattice is rescaled. That this phase is the vacuum's gravitating stress is a reading of the path sum's phase, measure and effective action, which waits on a massless graviton and a unitary evolution in record time [10]. What remains is ordered by consequence.

1. *How the ledger adds records* (Target 5.7). Under the programme's readings of time and causality a fixed lattice cannot expand (Proposition 5.4), so the universe grows by adding records. Whether their number is the same in every history decides whether a fixed vacuum phase per cell gravitates (Proposition 4.5), and the rate at which they are added is itself an expansion law: a constant rate is a cosmological constant (Proposition 5.5), about 5×10^{-61} per record per tick for the measured one. With whole-number counts the law can meet Friedmann's equation only in the limit of many records if the cosmological constant is small (Proposition 5.6).
2. *The vacuum of massive fields.* A field of mass m breaks the homogeneity that protects the graviton's vacuum: its vacuum responds to the size (Proposition 4.3), at an order estimated as $(m/m_P)^2$ in Planck units. A cancellation, or a reason it does not act, has to be derived.
3. *Beyond one loop* (Target 4.7). The value of the generated coefficient $-\mathrm{Re}\langle S\rangle/(2V)$ of the full path sum at the ledger's phase scale, at a fixed cutoff as the torus grows (Proposition 4.6).
4. *The nonlinear Friedmann limit* (Target 5.3). Computed at the rate $1/N^2$; the theorem is open.
5. *The shape response.* That the vacuum density in the ordered limit is differentiable at the flat metric, so that Proposition 4.4 applies to it, and its response to the shape at second order.
6. *The premises.* The phase premise, which makes the native amplitude the ledger's [11]; $\kappa = 1$; and the two readings behind Proposition 5.4, that proper time counts records and that the posting order is the causal order.

The continuum estimate fails because it treats the vacuum as a density that grows with the volume. The ledger's lattice gives the vacuum a different form: an action that cannot see volume, a vacuum whose limiting phase is a count of signs, and a universe that grows by adding records. On that form, whether the graviton's vacuum gravitates turns on one question about records, whether their number is the same in every history; the rate at which records are added is itself a cosmological constant; and the vacuum of massive fields still has to be cancelled.

Table 2. The premises of each statement in the paper.

statement	premises
no volume term; degree-one homogeneity; cone of configurations	$\kappa > 1/2$ [9, Sec. 3.1]
finite path sum at a cutoff; divergence along the scale ray without it	[9, Thm. 3.1, Sec. 3.2]
no normalizable Born weight of the cost of curvature over the whole null stratum	$\kappa = 1$; the uniform measure on squared lengths [10]
$G = 1/(8\pi s)$	every $\kappa > 1/2$ [9, Sec. 3.5]
$b \approx 1.92\,\ell_P$	$\kappa = 1$; the phase premise; one record length per rank (identifications) [11]
Propositions 3.1 and 3.2	$N \geq 3$, $\kappa > 1/2$; the native amplitude for the reading (identification); the cost of cell volumes (model)
Propositions 4.1 and 4.2	the nonzero modes, after the zero modes of the stationary set are removed; the regulator, both sides kept (model), and its limit $\varepsilon \to 0$; a fixed finite lattice
no volume term generated by the graviton's one-loop vacuum	Proposition 4.2; the reading of the vacuum phase as the gravitating vacuum stress, conditional on the phase, the measure and the effective action of [10]
the Λ of $S - \Lambda V$ is the continuum Λ	Regge's long-wave normalization $\sum_h A_h \delta_h \approx \frac{1}{2}\int \sqrt{-g}R$ [8]; $G = 1/(8\pi s)$
Proposition 4.3	the lattice scalar of [9] with its mass lumping (model); $\kappa = 1$ and the side-16 torus for the change of phase; the order $(m/m_P)^2$ is an inference
Proposition 4.4	$\kappa = 1$; relabelling and scale invariance; differentiability at the flat metric, which for the vacuum density is an inference
Proposition 4.5	a finite family of histories; a fixed phase per cell
Proposition 4.6	a finite measure with the action bounded on it; Z off the closed negative real axis; $V \neq 0$; a cutoff $L \geq 0$ on the squared lengths for the cutoff form
Propositions 5.1 and 5.2	$N \geq 3$, cells timelike in all their faces ($\kappa > 1/2$), differentiable matter terms; the Friedmann reading of the constant cell and of dust (identifications); an open line of record rank (inference)
Proposition 5.4	proper time counts records; postings null (identifications)
Proposition 5.5	records of fixed size; Friedmann's equation
Proposition 5.6	whole-number, never-decreasing counts; a vacuum alone; records of fixed size; the tick of the identifications for $\Lambda G \geq \ln 2/12$
the generated coefficient, the Friedmann limit, record creation	open (Targets 4.7, 5.3 and 5.7)

A The massive scalar on the side-16 torus

This appendix proves Proposition 4.3. The scalar of [9] has on each simplex the action

$$-\frac{\sqrt{|\det G|}}{48}\,(\mathrm{d}\phi)^{\top}G^{-1}\mathrm{d}\phi-\frac{\sqrt{|\det G|}}{240}\,m^2\sum_{v}\phi_v^2,$$

with G the Gram matrix of the simplex's edge vectors from its first vertex, computed from the squared lengths, $\mathrm{d}\phi$ the differences of ϕ along those edges, and the mass term shared equally among the five vertices (MODEL: the mass lumping); the sign is that of $-\frac{1}{2}\int\sqrt{|g|}\,(g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi+m^2\phi^2)$ in the signature $(-,+,+,+)$.

The scaling law. Multiplying every squared length by $c>0$ multiplies G by c, so $\sqrt{|\det G|}$ by c^2 and G^{-1} by c^{-1}. The kinetic term scales by c and the mass term by c^2, so $S_m[cy,\phi]=cS_{\sqrt{c}\,m}[y,\phi]$. A positive factor changes the sign of no eigenvalue, so the one-loop phase at size c is the phase at size 1 with the squared mass $\mu^2=cm^2$. At $m=0$ it is the same at every size.

The flat lattice. On the flat lattice at $\kappa=1$ the action is diagonal in the Bloch waves of the side-N torus, and on the wave with wave vector k its eigenvalue is a negative multiple of $\mathrm{kin}(k)+\mu^2$, where

$$\mathrm{kin}(k)=\sum_a(2-2\cos k_a)-\tfrac{1}{3}\Big(2-2\cos\sum_a k_a\Big)$$

is the massless symbol of [9] divided by $\sqrt{3}$. As μ^2 grows every eigenvalue decreases, so a sign can change only from positive to negative and the one-loop phase $\frac{\pi}{4}\sum\operatorname{sign}\lambda$ can only fall. On the torus of side 16 the wave $n=(1,1,1,1)$, $k_a=\pi/8$, has $\mathrm{kin}=4\big(2-2\cos\frac{\pi}{8}\big)-\frac{2}{3}=\frac{22}{3}-4\sqrt{2+\sqrt{2}}\approx-0.0577$: it is timelike, and its eigenvalue is positive at $\mu^2=1/100$ and negative at $\mu^2=1$. So for every $m\neq0$ the phase at the size $c=1/m^2$ is smaller than at $c=1/(100\,m^2)$, which is Proposition 4.3. Of the 65536 waves of the side-16 torus exactly two are timelike, $n=(1,1,1,1)$ and $(15,15,15,15)$; the phase is $-65532\,\pi/4$ at $\mu^2=1/100$ and $-65536\,\pi/4$ at $\mu^2=1$, smaller by π, and the massless phase is $-65531\,\pi/4$, with one zero mode. The program scalar_spectrum16.py lists the whole spectrum [19].

B The homogeneous sector

This appendix proves Proposition 5.1. Work on the side-N torus with $N\geq3$. The rank of a vertex x is $t(x)=\sum_a x_a$, read mod N, and a homogeneous isotropic configuration gives the edge from y to $y+d$, $d\in\{0,1\}^4\setminus\{0\}$, the squared length $f_{|d|}(t(y))$.

All simplices of one rank are congruent. The Kuhn simplex at x with order σ has the chain $p_0=x$, $p_k=x+e_{\sigma(1)}+\cdots+e_{\sigma(k)}$, so $t(p_k)=t(x)+k$. Its edge from p_i to p_j $(i<j)$ has displacement $e_{\sigma(i+1)}+\cdots+e_{\sigma(j)}$, with $j-i$ postings, and starts at rank $t(x)+i$. Its squared length is therefore

$$L_t(i,j)=f_{j-i}(t+i),\qquad t=t(x),$$

the same for all $24N^3$ simplices whose first vertex has rank t, whatever the order σ.

The hinge census. A triangle of the triangulation has vertices y, $y+e_A$, $y+e_A+e_B$ for disjoint nonempty sets of axes A, B, with $|A|=a$ and $|B|=b$; there are 50 such triangles at each vertex. A simplex contains it when its chain passes through the three vertices in this order: the chain starts at $y-e_C$ for a set C of the remaining $m=4-a-b$ axes, adds the axes of C, then those of A, then those of B, then the rest, each group in any order. The number of simplices around the triangle is

$$\sum_{c=0}^{m}\binom{m}{c}\,c!\,a!\,b!\,(m-c)! = (m+1)!\,a!\,b!.$$

In a simplex the local triangle (i,j,k) has $a=j-i$ and $b=k-j$, so the number of simplices around it is

$$n_{ijk} = (5-k+i)!\,(j-i)!\,(k-j)!,$$

which for the ten local triangles $ijk = 012, 013, 014, 023, 024, 034, 123, 124, 134, 234$ is 6, 4, 6, 4, 4, 6, 6, 4, 4, 6. It depends only on (a,b), so it is the same at every position the triangle takes in the simplices around it. For $N \geq 3$ these simplices are distinct on the torus and a triangle is determined by its three vertices, so the census is the same as on $\mathbb{Z}^4$.

The reduced action. Write the deficit of a hinge h as $\delta_h = \sum_{\sigma\ni h}\big(2\pi/n_h - \theta_{h,\sigma}\big)$, sharing 2π equally among the n_h simplices around it. Since the area A_h is the same in each of them,

$$S = \sum_h A_h\delta_h = \sum_{\text{simplices }\sigma}\ \sum_{c} A_c(\sigma)\Big(\frac{2\pi}{n_c} - \theta_c(\sigma)\Big),$$

the inner sum over the ten local triangles c of σ. For a homogeneous configuration the inner sum depends only on the rank t of the simplex; call it the density

$$D(t) = \sum_c A_c(L_t)\Big(\frac{2\pi}{n_c} - \theta_c(L_t)\Big).$$

There are $24N^3$ simplices at each rank, so $S = 24N^3\sum_t D(t)$.

The first variation, edge by edge. The Schläfli identity, $\sum_h A_h\,\mathrm{d}\theta_h = 0$ summed over each simplex, removes the variations of the angles: for every edge e,

$$\frac{\partial S}{\partial \ell_e} = \sum_{h\ni e}\delta_h\,\frac{\partial A_h}{\partial \ell_e} = \sum_{\sigma\ni e} F_{i(\sigma,e)}(\sigma), \qquad F_i(\sigma) = \sum_{c\ni i}\Big(\frac{2\pi}{n_c} - \theta_c(\sigma)\Big)\frac{\partial A_c}{\partial L_i}(\sigma),$$

where $i(\sigma,e)$ is the local edge of σ that is e; the second form shares each deficit as above. Now count the simplices that contain a given edge e from y to $y+d$, with $|d|=n$ postings and $t(y)=t$, as a given local edge (i_1,i_2) with $i_2-i_1=n$. The order σ must put the n axes of d in positions $i_1+1,\dots,i_2$, in any of $n!$ ways, and the other $4-n$ axes in the remaining positions, in any of $(4-n)!$ ways; the first vertex is then $x = y - e_{\sigma(1)} - \cdots - e_{\sigma(i_1)}$, of rank $t-i_1$. So each local edge of width n occurs $n!\,(4-n)!$ times, always in a simplex of rank $t-i_1$, and

$$\frac{\partial S}{\partial \ell_e} = n!\,(4-n)!\;E_n(t), \qquad E_n(t) = \sum_{(i_1,i_2)\colon i_2-i_1=n} F_{(i_1,i_2)}\big(L_{t-i_1}\big).$$

The derivative along every edge of the lattice depends only on its rank and its number of postings. The configuration is therefore stationary for the full lattice action, along all $15N^4$ edge lengths and not only along homogeneous variations, exactly when $E_n(t) = 0$ for $n = 1, \dots, 4$ at every rank: the four reduced equations. No principle of symmetric criticality is used; the variation along each edge has been computed.

Volume and matter. The same count applies to $V = \sum_\sigma \mathrm{vol}(\sigma) = 24N^3 \sum_t \mathrm{vol}(L_t)$, with $\mathrm{vol} = \sqrt{-\det \mathrm{Gram}}/24$: $\partial V/\partial \ell_e = n!\,(4-n)!\,W_n(t)$ with $W_n(t) = \sum_{i_2 - i_1 = n} \partial \mathrm{vol}/\partial L_{(i_1,i_2)}(L_{t-i_1})$. An edge action $\sum_e g_{|d_e|}(\ell_e)$ contributes $g_n'(f_n(t))$. For $sS - \Lambda V + \sum_e g_{|d_e|}(\ell_e)$ the reduced equations are

$$s\,n!\,(4-n)!\,E_n(t) - \Lambda\, n!\,(4-n)!\,W_n(t) + g_n'\big(f_n(t)\big) = 0, \qquad n = 1, \dots, 4,$$

at every rank; resting dust of mass m on the diagonal edges is $g_4(\ell) = -m\sqrt{-\ell}$, with $g_4'(\ell) = m/(2\sqrt{-\ell})$.

The flat family. For $f_n = \alpha n(1 - \kappa n)$ every simplex is a piece of the one flat metric $\alpha(I - \kappa \mathbf{1}\mathbf{1}^\top)$, the angles around each hinge fill its normal plane and every deficit vanishes. By the first form of $\partial S/\partial \ell_e$ every derivative vanishes, so $E_n(t) = 0$: the flat family solves the equations without a volume term or matter.

Computations. cosmo_torus_check.py builds the side-3 and side-4 tori from scratch and checks each step on a random homogeneous configuration near the flat one at $\kappa = 1$: the congruence (exactly), the census, $S = 24N^3 \sum_t D(t)$ (to 2×10^{-12}), and $\partial S/\partial \ell_e = n!(4-n)!E_n(t)$ on every edge (to 3×10^{-15}, against derivatives of order 0.2), the last also for the volume and for dust; cosmo_reduce.py and cosmo_eqs.py give D and the equations at 30 digits [19].

C The long-wave coefficient of a homogeneous strain

This appendix proves the first part of Proposition 5.2. Label each edge by its midpoint. The quadratic part of the lattice action around the flat configuration $g = I - \kappa \mathbf{1}\mathbf{1}^\top$ is then a Hermitian symbol $H(u)$, a 15×15 matrix for each wave covector u, and the quadratic form it defines on real edge variations is even in u. A metric strain h, a symmetric 4×4 matrix, acts on an edge of class d as $\delta \ell_d = d^\top h d$. The long-wave identity of [9] is that, for $\kappa > 1/2$, the term of second order in u of the quadratic form on metric strains is

$$-\tfrac{1}{4}\sqrt{|\det g|}\ \mathrm{FP}_g(u)(h,h),$$

where, with $S = g^{-1}$,

$$\mathrm{FP}_g(u)(h,h) = (u^\top S u)\,\mathrm{tr}(ShSh) - 2\,u^\top ShShSu + 2\,\mathrm{tr}(Sh)\,u^\top ShSu - (u^\top Su)\,\mathrm{tr}(Sh)^2$$

is the linearized Einstein form, and the term of order zero vanishes on metric strains.

The homogeneous strain. Take $h = \mathrm{d}\alpha\, I + \mathrm{d}\beta\, J$ with $J = \mathbf{1}\mathbf{1}^\top$ and $u = \omega \mathbf{1}$. On an edge of n postings it gives $\delta\ell = \mathrm{d}\alpha\, n + \mathrm{d}\beta\, n^2$, a homogeneous isotropic variation of the configuration of

Proposition 5.1. Everything lies in the algebra spanned by I and J, where $(pI+qJ)(p'I+q'J) = pp'I + (pq' + qp' + 4qq')J$, $\mathrm{tr}(pI + qJ) = 4p + 4q$ and $u^\top(pI + qJ)u = \omega^2(4p + 16q)$. The inverse metric is $S = I + cJ$ with $c = \kappa/(1-4\kappa)$; put $r = 1 + 4c = 1/(1-4\kappa)$. Then $Sh = aI + bJ$ with $a = \mathrm{d}\alpha$ and $b = c\,\mathrm{d}\alpha + r\,\mathrm{d}\beta$, and

$$u^\top Su = 4\omega^2 r, \quad \mathrm{tr}(ShSh) = 4a^2 + 8ab + 16b^2, \quad \mathrm{tr}(Sh) = 4(a+b),$$
$$u^\top ShSu = 4\omega^2 r(a+4b), \qquad u^\top ShShSu = 4\omega^2 r(a+4b)^2.$$

Substituting,

$$\mathrm{FP} = 4\omega^2 r\big[(4a^2+8ab+16b^2) - 2(a+4b)^2 + 8(a+b)(a+4b) - 16(a+b)^2\big] = -24\,\omega^2 r a^2 = \frac{24\,\omega^2\,\mathrm{d}\alpha^2}{4\kappa-1}.$$

Every term with b cancels, so $\mathrm{d}\beta$ does not appear. This had to happen: at $u = \omega\mathbf{1}$ the strain $J = uu^\top/\omega^2$ is a linearized change of coordinates, $u\xi^\top + \xi u^\top$ with $\xi = u/(2\omega^2)$, on which the linearized Einstein form and all its cross terms vanish. With $|\det g| = 4\kappa - 1$ the long-wave coefficient is

$$-\tfrac{1}{4}\sqrt{4\kappa-1}\cdot\frac{24\,\omega^2\,\mathrm{d}\alpha^2}{4\kappa-1} = -\frac{6\,\omega^2\,\mathrm{d}\alpha^2}{\sqrt{4\kappa-1}},$$

as Proposition 5.2 states.

The Friedmann Hessian. The metric $\alpha I + \beta J$ has spatial scale factor $a = \sqrt{\alpha}$ on the slices $\sum_a \nu_a = 0$ and lapse n per rank step with $n^2 = -(\alpha + 4\beta)/4$, the squared length of $\frac{1}{4}\mathbf{1}$. For $\mathrm{d}s^2 = -n^2\mathrm{d}t^2 + a^2\mathrm{d}y^2$ the Einstein–Hilbert density is, up to a total derivative, $\sqrt{-g}\,R = -6a\dot{a}^2/n$ per unit coordinate volume; in the normalization in which $\sum_h A_h\delta_h$ is $\frac{1}{2}\int\sqrt{-g}\,R$, and with the coordinate volume 2 of a site of the slice lattice $\{\nu \in \mathbb{Z}^4 : \sum_a \nu_a = 0\}$, the Lagrangian per site and per rank is $L = -6a\dot{a}^2/n$. In the variable $\alpha = a^2$ it reads $L = -\frac{3}{2}\,\dot{\alpha}^2/(\sqrt{\alpha}\,n)$. At the flat cell, $\alpha = 1$ and $n = \sqrt{4\kappa-1}/2$, the second variation along $\delta\alpha = \mathrm{d}\alpha\, e^{i\omega t}$ is

$$\frac{\partial^2 L}{\partial\dot{\alpha}^2}\,\omega^2\,\mathrm{d}\alpha^2 = -\frac{3\,\omega^2\,\mathrm{d}\alpha^2}{n} = -\frac{6\,\omega^2\,\mathrm{d}\alpha^2}{\sqrt{4\kappa-1}},$$

the lattice coefficient. The lapse has no kinetic term, and its mixed second derivatives with α carry a factor $\dot{\alpha}$, which vanishes on the static cell; so the lapse direction, $\mathrm{d}\beta$, has no term on either side.

Friedmann's constraint. With matter of density ρ the action per site and rank is $s(-6a\dot{a}^2/n) - 2na^3\rho$, since $2na^3$ is the 4-volume per vertex. Its derivative in n vanishes when $6sa\dot{a}^2/n^2 = 2a^3\rho$, that is $(\dot{a}/(na))^2 = \rho/(3s) = (8\pi G/3)\rho$ with $G = 1/(8\pi s)$. Resting dust of mass m on the diagonal edges, one per vertex and of proper length $4n$, contributes $-4mn$ per site and rank, which is $-2na^3\rho$ with $\rho = 2m/a^3$.

Computations. fp_homog.py repeats the algebra above in exact arithmetic. cosmo_symbol.py differentiates the four reduced equations at the flat cell for $\kappa = 1, 5/4, 2$; cosmo_midpoint.py reads the resulting symbol with midpoint labels and finds the (α,α) entry divided by ω^2 tending to $-6/\sqrt{4\kappa-1}$ (-3.4638 at $\omega = 0.005$ for $\kappa = 1$, against -3.4641) while the entries with $\mathrm{d}\beta$ vanish at this order, with or without relaxing the two massive directions; cosmo_schur.py finds the same coefficient with first-vertex labels after relaxing them [19].

D The Friedmann limit, computed

Target 5.3 is open; this appendix records the computation behind its numerical status. The program evaluates the lattice value L_N, N times the reduced action per site, for the profile $f_n(t) = \alpha(s)n + \beta(s)n^2$, $s = (t + n/2)/N$, with $\alpha = 1 + A\sin(2\pi s)$, on tori of side $N = 40$, 80 and 160, and compares it with the Friedmann action $\int_0^1 -\frac{3}{2}\,\alpha'^2/(\sqrt{\alpha}\,n)\,\mathrm{d}s$, $n^2 = -(\alpha + 4\beta)/4$. Each profile is run with a conformal lapse, $\beta = -\kappa\alpha$, and with an independent one, $\beta = -\kappa\alpha - A\cos(2\pi s)/4$; the two massive lattice modes are not relaxed. In all six runs, amplitude $A = 0.2$ and 0.6 at $\kappa = 1$ and $A = 0.3$ at $\kappa = 5/4$, the difference r_N from the Friedmann action has r_N/r_{2N} between 4.00 and 4.04: the rate is $1/N^2$. At $N = 160$, $N^2 r_N$ is 90.71 and 110.10 at $A = 0.2$, 1001.9 and 1162.2 at $A = 0.6$, and 412.3 and 457.6 at $\kappa = 5/4$, and the extrapolation $(4L_{160} - L_{80})/3$ lies within 10^{-6}, 1.4×10^{-4} and 1.4×10^{-5} of the Friedmann action in the three cases (NUMERICAL). The coefficients $N^2 r_N$ are not physical curvature-squared coefficients: the massive modes are not relaxed and no map from lattice to continuum fields is fixed. The programs cosmo_longwave.py and longwave_residuals.py produce these numbers [19].

References

[1] Ya. B. Zel'dovich, The cosmological constant and the theory of elementary particles, *Sov. Phys. Usp.* **11**, 381–393 (1968).

[2] S. Weinberg, The cosmological constant problem, *Rev. Mod. Phys.* **61**, 1–23 (1989).

[3] A. G. Riess *et al.*, Observational evidence from supernovae for an accelerating universe and a cosmological constant, *Astron. J.* **116**, 1009–1038 (1998), doi:10.1086/300499.

[4] S. Perlmutter *et al.*, Measurements of Ω and Λ from 42 high-redshift supernovae, *Astrophys. J.* **517**, 565–586 (1999), doi:10.1086/307221.

[5] N. Aghanim *et al.* (Planck Collaboration), Planck 2018 results. VI. Cosmological parameters, *Astron. Astrophys.* **641**, A6 (2020), doi:10.1051/0004-6361/201833910.

[6] J. Martin, Everything you always wanted to know about the cosmological constant problem (but were afraid to ask), *C. R. Physique* **13**, 566–665 (2012).

[7] J. Washburn, The record cell of the four-dimensional lattice: time, Lorentzian signature and the Regge action near flatness, in *The Formation of the Universe from Recognition: The Integrated Manuscript and Its Focused Papers*, Zenodo (2026), doi:10.5281/zenodo.23738724.

[8] T. Regge, General relativity without coordinates, *Nuovo Cimento* **19**, 558–571 (1961).

[9] J. Washburn, Linearized Einstein gravity from Lorentzian Regge calculus on the Kuhn lattice, and the divergence of its one-loop graviton limit, Zenodo (2026), doi:10.5281/zenodo.23764491; Pith preprint doi:10.67347/pith.preprint.2026.000017.

[10] J. Washburn, Quantum gravity from the recognition ledger: weighted one-loop graviton responses with finite real parts, Einstein gravity at long wavelengths, and the massless-graviton problem, Zenodo (2026), doi:10.5281/zenodo.23806703; Pith preprint doi:10.67347/pith.preprint.2026.000018.

[11] J. Washburn, Newton's constant and the recognition ledger: Einstein's equations, the graviton, and the one number that fixes G, Zenodo (2026), doi:10.5281/zenodo.23806705; Pith preprint doi:10.67347/pith.preprint.2026.000019.

[12] M. Henneaux and C. Teitelboim, The cosmological constant and general covariance, *Phys. Lett. B* **222**, 195–199 (1989).

[13] M. Ahmed, S. Dodelson, P. B. Greene and R. Sorkin, Everpresent Λ, *Phys. Rev. D* **69**, 103523 (2004), doi:10.1103/PhysRevD.69.103523.

[14] P. A. Collins and R. M. Williams, Dynamics of the Friedmann universe using Regge calculus, *Phys. Rev. D* **7**, 965–971 (1973).

[15] L. Brewin, Friedmann cosmologies via the Regge calculus, *Class. Quantum Grav.* **4**, 899–928 (1987).

[16] H. Freudenthal, Simplizialzerlegungen von beschränkter Flachheit, *Ann. of Math.* **43**, 580–582 (1942).

[17] H. W. Kuhn, Some combinatorial lemmas in topology, *IBM J. Res. Dev.* **4**, 518–524 (1960).

[18] A. G. Adame *et al.* (DESI Collaboration), DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations, *JCAP* **02** (2025) 021, doi:10.1088/1475-7516/2025/02/021.

[19] J. Washburn, Computations for *The cosmological constant on the recognition lattice*, version 1.2: programs, outputs and run instructions, file CosmologicalConstantRecognitionLattice_computations_v1.2.zip, SHA-256 d668f41ba0f34a659b43bc0568644805a2a05bd90e72621fb2116614abc58672, in the Zenodo record doi:10.5281/zenodo.23839822 (2026).