

Making Provision Possible

Applied Recognition Economics

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Abstract

A meal can arrive while its recipient goes hungry. A repair can succeed while exhausting its worker. Economics must follow provision to the people who need it and bear its costs. We develop the economic description of Recognition Science's physical ethics, retaining resources, affected persons and historical obligations on the same recognition structure. In a two-person economy, we follow production through assisted intake and funded internal responses, then characterize every care schedule that maintains both people. Searching the available actions also reveals choices that preserve present maintenance while removing every successful continuation. The helper's separate capacity matters to the physical operation as well: for conserving Love on two bonds, we derive the unique least-cost endpoint permitted by their individual renewal budgets. Together, these results identify distinctions that an adequate economic description must preserve. The examples state their physical calibrations and authority conditions; responsibility for a created loss further requires control, knowledge and competing requirements.

1 From a meal to an economy

Alice has ingredients. Ben needs a meal. Between those facts lie preparation, storage, transport and eating. Each takes time. Some take work. Alice must remain able to continue, and the food must reach Ben in a form he can use. A delivery receipt settles none of this by itself.

Economics studies how conscious subjects, individually and together, organize the means of living and acting within shared physical reality and its moral law. Production changes what is available. Ownership and agreements affect who may use it. Exchange changes its distribution. Their consequences reach the subjects whose lives and actions depend on those means.



At every stage: enough material and capacity, the authority to act,
and the requirements of each affected subject.

Figure 1: Provision requires the entire route. Storage can lose food; delivery can leave a recipient unable to eat. Each stage has its own physical and ethical conditions.

We take the Recognition Science (RS) account of one universal conscious reality as a starting premise. Its economic description has two foundations. First, describing the whole must preserve the needs, effects and obligations within it; describing its members must preserve their shared physical constraints and count each resource once. Second, physical moral law governs their relations. Actions change the states to which that law applies. They do not choose the law.

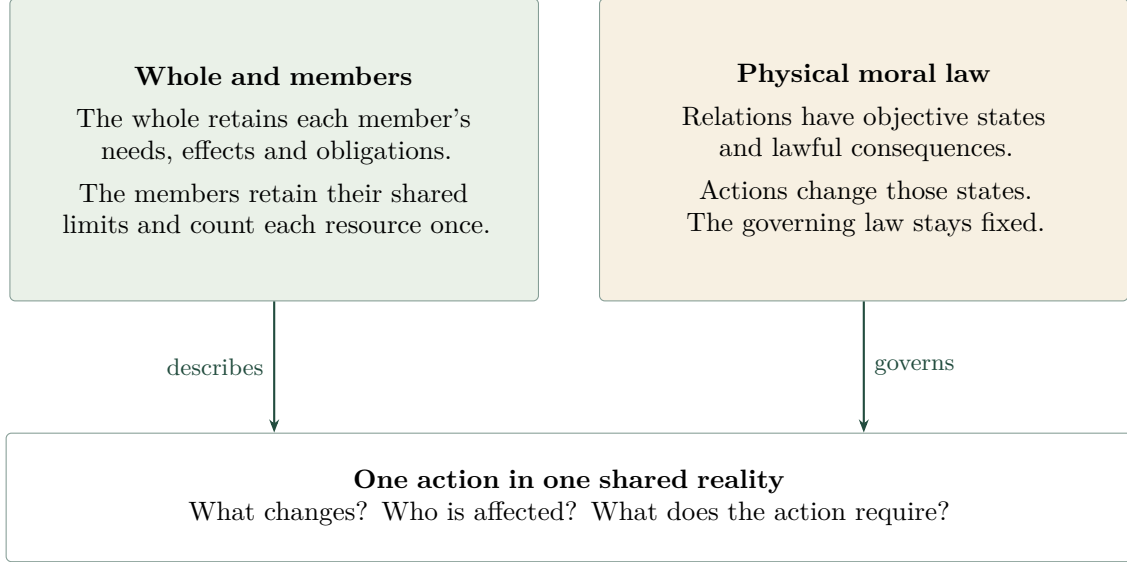


Figure 2: Both foundations apply to the same action. A description of the whole keeps its members' needs and effects; a description of the members keeps their common physical limits. Physical moral law governs the relations retained in either description.

Alice's work remains hers. A gain to Ben cannot make her injury disappear. Physical ethics gives the account its starting distinctions: the state of a relation, who acts on it, what actually happens, and what continued action requires [4]. Economics follows those facts through production and provision. We check each person's reserve throughout the plan, retain the events that establish obligations, and read the represented relational effects under the fixed recognition cost law. Their connection is developed in Section 2. A model that leaves out a person's experience or responsibility has an incomplete account of that person.

Being affected and being able to act are different roles. A subject who cannot choose an action can still have needs and bear its effects. The finite set of subjects used below need not consist entirely of actors. A household or ecosystem can also be described as a group, and groups can overlap. Identifying any such system as a subject requires an account of its own experience and persistence, together with its members' relations. The preservation results below apply once the subjects have been identified. They do not establish a boundary of personhood or a hierarchy of consciousness.

The first example follows two people through production, storage, transfer and consumption. It shows how the same final total can conceal a loss of one person's ability to continue. Accepted agreements and control determine which plans can execute. Assisted care extends the account to a recipient who cannot act; it follows the provision and both people's internal responses without combining their costs. Repeated care determines every successful feeding timing in the example. It also identifies choices that leave both people maintained now while destroying their remaining means of completion. These physical distinctions constrain the account of responsibility. A maintenance model then yields an exact interval of permissible assistance under its stated physical interpretation. The final delivery calculation asks what these distinctions reveal in an operational workload.

Money can organize claims and exchanges. Splitting an account creates neither another meal nor another subject. A balanced account can accompany enough food or no food. The underlying distinction between physical possibilities, persons and historical obligations is developed in [3].

Sen distinguishes starvation from aggregate food supply [10]. Related work studies social com-

mitments [11], interacting actions and necessary information [7, 8], and courier satisfaction [9]. Here performance, authored permissions and each person’s requirements constrain one course of action. Material balance, finite minimization and indistinguishability establish what its economic description must preserve.

2 What economics inherits from physical ethics

Alice prepares a meal for Ben under accepted terms. Four facts matter: the meal exists; Alice did the work; Ben received it; and their undertaking gives that performance a particular consequence. A description that keeps only the meal loses authorship and obligation. A description that keeps only the agreement loses whether anybody was fed. Physical ethics requires an account of the same event from all of these sides.

Morality and economics describe the same physical relations and transformations. The moral account concerns what an action does to subjects and their relations, and what follows for the acting subjects. The economic account concerns how the means of living and acting can be produced, preserved, used and shared under those requirements. Its distinctive work is constructive: finding an available course of action, including the time, resources, permissions and responses that make it possible. Objective moral law fixes what a judgment must respect. Economic analysis determines what can be done with the available means, and how doing it changes the same subjects and relations. The distinction is between questions asked of one reality.

The common object is a network of recognition relations. We call its representation a ledger and each represented relation a bond. A subject’s moral state reads that shared network at the subject’s position and time, with its available capacity retained separately. Relational cost is derived from the subject’s incident relations. When an action changes the network, each affected subject’s change is read from the same before-and-after states. Physical and ethical descriptions must therefore agree on which transformation occurred. Section 5 gives the exact correspondence used in the finite model.

2.1 Relations, authors and events

The record construction in [4] compares two parties’ records of one relation. On positive records, write their figures as u and v . Their disagreement has ratio $r = u/v$ and recognition cost

$$J(r) = \frac{1}{2}(r + r^{-1}) - 1 = \frac{(r - 1)^2}{2r}.$$

Equal figures have zero disagreement cost; interchanging the ends leaves the cost unchanged. The foundation defines these figures as the two parts of one compared recognition pair. This fixes the cost once the physical magnitudes are identified; the definition alone does not identify arbitrary real records with those magnitudes. Each application must supply that physical map. The service and maintenance examples below state their maps explicitly.

The records also have authors. In the two-record construction, a party’s consent to a write is its authorship of its own side. A party can author a change that increases its cost, or benefit from a change it did not author. Agreement is therefore not recoverable from the sign of a burden change. In an economic application, authorship must be connected to actual authorization, including the terms and action being authorized. A work grant below retains that connection; the number measuring an effect cannot supply it.

What happened is a further fact. In the foundation's exchange example, two runs have identical claims and identical histories of writing them. A unit was delivered in one run and was never delivered in the other. Under the same accepted terms, the amounts due differ. Neither current balances nor their complete writing history can determine the obligation. The event record must retain the performance, its parties and its terms. For a performed unit sale at price q , with valid repayments totaling a and authorized releases totaling w , the remaining amount is $q - a - w$. A debtor's denial is no release by the creditor.

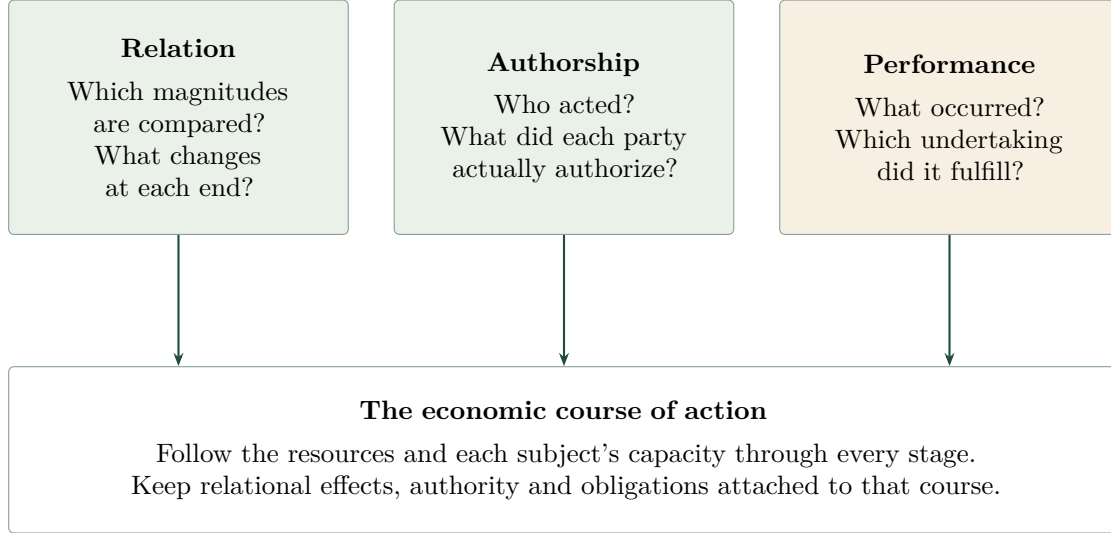


Figure 3: The ethical foundation determines what an economic description must retain. These are different readings of one course of events. No balance or total replaces the actors, the performance or the subjects affected.

This gives economic modeling a precise requirement. A reduced state must retain, or allow recovery of, every fact that changes a physical possibility, an authorization or an obligation. Sections 3 and 4 build such a state for the finite examples. Section 5 states the conditions under which a decision survives a change of description.

2.2 Why agreement cannot stand in for truth

Suppose both parties record one unit as due although nothing was performed and nothing is due. With a unit baseline their figures are $(2, 2)$. They agree, and $J(2/2) = 0$. Now one party corrects its own figure to 1. The records become $(1, 2)$, and the cost rises to $J(1/2) = 1/4$. One record has become accurate while disagreement has increased. This is the false-consensus example proved in [4].

This result matters directly to an economic planner. Minimizing disagreement alone can preserve a shared false claim. Nor can an aggregate cost say who performed the work or who can release a debt. The service planner below therefore has a stated domain and decision specification. Its rule forbidding positive cost changes has this limited decision role; the full moral law also concerns the truth and authority of what is done. Historical truth and authority remain separate requirements of the economic description.



Underlying event history unchanged: no performance, no amount due.

Figure 4: Agreement, accuracy and cost are distinct. A rule that permits only reductions in disagreement would reject this first correction. A complete ethical judgment must retain the event evidence as well as the cost.

2.3 Continuing agency has physical requirements

An obligation must be considered with the means of acting on it. The foundation gives a precise resource argument. Let an agent begin with reserve $R_0 > 0$, renew $b > 0$ per interval, and pay upkeep $c_k \geq 0$ during interval k . Its remaining reserve is

$$R_n = R_0 + \sum_{k < n} (b - c_k).$$

If $c_k \geq b + d$ throughout, with $d > 0$, then $R_n \leq R_0 - nd$. Strictly positive reserve cannot last through $n = \lceil R_0/d \rceil$. In the foundation's held-ratio model, upkeep is $J(e^\sigma) + J(e^{-\sigma}) = 2(\cosh \sigma - 1)$. A continuing agent cannot sustain upkeep above its renewal indefinitely. Continuing supply can support a persistent cost; finite initial reserve alone cannot.

The normative step in [4] uses two stated premises. The continuation premise says that a deliberator is committed to continuing as an agent. The instrumental principle uses the strong transmission form: commitment to an end carries over to every necessary condition of that end. Together these premises commit the agent to avoiding the sustained deficit. A weaker instrumental principle would require a different argument for that commitment; the physical exhaustion bound remains. The resource inequality is a physical consequence; the premises supply the practical conclusion. The foundation proves that the continuation premise follows neither from the instrumental principle alone, nor from a commitment to keeping options open, nor from a commitment to a necessary condition of continuing.

Coupling makes the argument interpersonal. If a neighbor's condition imposes unavoidable upkeep on the agent, that condition can determine whether the agent continues. The resulting commitment belongs to the agent whose continuation is at stake. It does not automatically assign the neighbor a particular action or give anyone authority over the neighbor's resources.

Economics must now determine the available actions. Can food arrive before the reserve runs out? Does helping Ben leave Alice able to continue? Who controls intake, and what if they do not perform it? These are questions about the actual means of meeting requirements. Section 6 retains the worker's reserve alongside the repair benefit. The maintenance model later derives the interval of assistance compatible with both parties' stated conditions. A universal duty of rescue requires the further account of need, available alternatives, authority and all affected subjects; no transfer amount or cost score supplies those facts by itself.

These requirements persist across descriptions of personhood. Describing a conscious whole cannot erase a member's exhaustion, authorship or claim. Describing the members cannot duplicate the

meal or the capacity of their common source. The represented subjects include those who cannot operate the planner; their needs and effects remain in the state. Identifying consciousness at a given scale supplies the subjects to whom this account applies; aggregation must then preserve the effects and obligations within that scale.

2.4 Love with separate limits on what each person can bear

Love acts on the same recognition ledger. For the conserving two-bond operation considered here, the multipliers move toward their geometric mean while their product is retained. Each bond carries the charge $J(x)$ defined above; the pair carries the sum of those charges. With no other constraint, the total recognition cost of the pair is least at equality. When the two bonds have different owners, each owner's capacity to sustain its own charge also matters, and equality can ask more of one person than that person can bear.

Consider an initial pair $(1, r^2)$ with $r \geq 1$ and any conserving endpoint $(x, r^2/x)$ with $x > 0$. The two participants have separate nonnegative renewal budgets B_0 and B_1 . Each budget covers its owner's own incident cost; they are not a common reserve. Set

$$U(B) = 1 + B + \sqrt{B(B+2)}, \quad u = U(B_0), \quad v = U(B_1). \quad (1)$$

$U(B)$ is the largest multiplier whose charge a budget B covers.

Theorem 1 (Least recognition cost within both renewal budgets). *A positive conserving endpoint satisfying $J(x) \leq B_0$ and $J(r^2/x) \leq B_1$ exists exactly when $r^2 \leq uv$. When it exists, its unique least-total-cost endpoint is*

$$x_* = \max\{r^2/v, \min\{r, u\}\}, \quad (x_*, r^2/x_*). \quad (2)$$

The total compared is $T(x) = J(x) + J(r^2/x)$ on this same two-bond ledger.

Proof. For $z > 0$ and $B \geq 0$, the inequality $J(z) \leq B$ is $z^2 - 2(1+B)z + 1 \leq 0$. The roots of this quadratic are $U(B)$ and $1/U(B)$, since their sum is $2(1+B)$ and their product is 1, so $J(z) \leq B$ exactly when $1/U(B) \leq z \leq U(B)$. Coverage of the endpoint therefore means $1/u \leq x \leq u$ and $1/v \leq r^2/x \leq v$, that is,

$$\max\{1/u, r^2/v\} \leq x \leq \min\{u, r^2v\}.$$

Because $u, v \geq 1$ and $r \geq 1$, the three comparisons $1/u \leq u$, $1/u \leq r^2v$ and $r^2/v \leq r^2v$ hold automatically, so this interval is nonempty exactly when $r^2/v \leq u$, which is $r^2 \leq uv$. Under that condition the displayed x_* lies in the interval: $\min\{r, u\} \geq 1 \geq 1/u$ and $r^2/v \leq u$ put x_* in $[1, u]$, and $x_* \geq r^2/v$ together with $x_* \leq \max\{r^2/v, r\} \leq r^2$ put r^2/x_* in $[1, v]$. So both charges are covered.

For the total, write $T(x) = \frac{1+r^2}{2}(x/r^2 + 1/x) - 2$. For any positive x, y ,

$$T(x) - T(y) = \frac{(r^2 + 1)(x - y)(xy - r^2)}{2r^2xy}.$$

If $y < x \leq r$ then $xy < r^2$ and $T(x) < T(y)$; if $r \leq y < x$ then $xy > r^2$ and $T(x) > T(y)$. So T strictly decreases up to r and strictly increases after r . On a closed interval the least value of such a function is taken at the point of the interval nearest r , and at no other point. That point is $\max\{L, \min\{r, M\}\}$ for the interval $[L, M]$ found above. Since $r \leq r^2v$, we have

$\min\{r, M\} = \min\{r, u\}$; since $\min\{r, u\} \geq 1/u$, the lower clamp is r^2/v . This is (2). In words: if r is covered, it is the unique minimum; if the first owner's ceiling u is below r , the largest covered x is u ; if the second owner's ceiling forces $x > r$, the smallest covered x is r^2/v . Strict monotonicity on each side of r gives strict improvement over every other covered endpoint. $\square$

For example, take $r = 2$, $u = 3/2$ and $v = 3$, so the budgets are $B_0 = J(3/2) = 1/12$ and $B_1 = J(3) = 2/3$. Equalizing at $(2, 2)$ would charge each owner $J(2) = 1/4$, which exceeds the first owner's budget. The covered minimum is $x_* = \max\{4/3, \min\{2, 3/2\}\} = 3/2$, the endpoint $(3/2, 8/3)$, with charges $1/12$ and $25/48$, each within its owner's budget. Reversing the two budgets gives $x_* = 8/3$ and the endpoint $(8/3, 3/2)$: the same total, with the other person carrying the larger charge. The separate capacities determine the least cost both participants can sustain.

The constraints have a physical meaning through the constant charged renewal process of Section 2.3. Hold each endpoint multiplier fixed, with no fresh exposure, a positive renewal budget and a nonnegative opening reserve for each owner. Both owners continue indefinitely exactly when their initial reserves are positive and their budgets cover their own charges. A covered charge leaves a positive reserve positive. A charge exceeding its budget creates a fixed deficit per interval, which eventually exhausts any finite reserve. Thus the covered endpoints in Theorem 1 are precisely the sustainable endpoints for positive initial reserves and budgets in this process.

The recurring-care application in Section 9 has a different renewal law: fresh exposure, food intake, work and separately funded repairs occur each interval. Its timing and authority conditions must be checked for those actions. The result here establishes the conserving endpoint and each owner's charge on the shared physical ledger. It does not supply the actuator, work permission or complete judgment authorizing a particular transfer.

3 Production and continuation

A usable meal and the ability to prepare it are separate quantities. Material accounting tracks the food. A person's reserve tracks the capacity left to act. Consuming food can replenish reserve; maintenance and work draw it down.

Time is divided into intervals. Inputs and intake occur at the opening of an interval; production outputs appear at its end. Maintenance and labor draw reserve during the interval. Requirements due at the opening checkpoint are checked before new intake. This timing matters. Food made at the end cannot support work already performed during that interval.

Let P and R be finite sets of persons and resource slots. At a checkpoint the physical state is

$$s = (t, x, c, w, e, d, k).$$

The entries keep seven facts separate:

t	Current time	x_r	Stock in resource slot r
c_p	Person p 's cumulative intake	w_p	Person p 's cumulative labor
e_p	Person p 's remaining reserve	d	Discarded material
k_j	Completed jobs of type j		

Stock, cumulative intake, labor, discard and job counts are nonnegative integers; reserve is an integer and must remain positive for continuation. The unit of quantity is chosen for the application. A slot can identify a particular physical unit when performance must be credited separately.

An operation a specifies inputs I_r , outputs O_r , intake u_p , labor ℓ_p , waste v and job completions. Intake and waste use the common material unit; each toy meal below contains one such unit. Resource weights ω_r and intake conversions γ_p are positive integers. Conversion maps intake to reserve and is distinct from material weight. Maintenance m_p , labor expenditure per unit λ_p , and interval capacities are nonnegative. Permitted recipes constitute the technology. A batch adds its recipes' quantities before the input check, so two concurrent jobs cannot spend the same opening stock.

Write $b_r = x_r - I_r + O_r$. At an expiry checkpoint, all b_r in that expiring slot is lost; otherwise its loss z_r is zero. An enabled operation has fresh opening stock, $I_r \leq x_r$, labor within capacity, an allowed recipe and the material balance

$$\sum_r \omega_r I_r = \sum_r \omega_r O_r + \sum_p u_p + v. \quad (3)$$

The update is

$$x'_r = x_r - I_r + O_r - z_r, \quad c'_p = c_p + u_p, \quad (4)$$

$$e'_p = e_p + \gamma_p u_p - m_p - \lambda_p \ell_p, \quad w'_p = w_p + \ell_p, \quad (5)$$

$$d' = d + \sum_r \omega_r z_r + v, \quad t' = t + 1.$$

Read the reserve equation as opening capacity plus usable intake, less maintenance and work. Keeping a material balance cannot replace this person-by-person check. Completed-job counts increase by the operation's completions. Reserve is an application variable for available capacity; it is not a universal physiological scale. Material accounting covers the declared system boundary. A recipe using fuel or water needs those inputs or an explicitly modeled inflow.

Proposition 2 (Material conservation). *For any enabled operation, the total*

$$M(s) = \sum_r \omega_r x_r + \sum_p c_p + d$$

satisfies $M(s') = M(s)$.

Proof. The input bounds make the subtraction in (4) ordinary subtraction. Multiplying by ω_r , summing and adding the intake and discard updates gives

$$\begin{aligned} M(s') - M(s) &= - \sum_r \omega_r I_r + \sum_r \omega_r O_r - \sum_r \omega_r z_r + \sum_p u_p + \sum_r \omega_r z_r + v \\ &= 0 \end{aligned}$$

by (3). □

Food can become waste while accounted material stays constant. Conservation constrains what can happen; it does not measure usefulness.

3.1 The whole interval counts

Suppose reserve is 1, maintenance costs 1, each unit of intake restores one reserve unit, and no intake occurs in the first interval. Reserve reaches zero. Intake of 2 in the second interval would leave a positive final reserve of 1, but the earlier failure remains. An endpoint score misses it.

A continuation is a complete sequence that keeps every person's reserve positive and meets every due requirement at every checkpoint. Each action must also be possible and permitted when it is taken. For a state predicate $\text{Req}(s)$ recording requirements due now, and a permission predicate $A(s, a)$, define this recursively:

$$\begin{aligned} C(s, []) &\iff (\forall p, e_p > 0) \wedge \text{Req}(s), \\ C(s, a :: \alpha) &\iff (\forall p, e_p > 0) \wedge \text{Req}(s) \wedge \text{Enabled}(s, a) \wedge A(s, a) \wedge C(T(s, a), \alpha). \end{aligned} \quad (6)$$

T is the update above. A horizon of n requires a list of exactly n operations, even when the current obligations have already been met. Positive reserve at its final checkpoint means capacity remains there. It guarantees nothing beyond that horizon; extending the window requires checking the additional states.

Proposition 3 (Capacity needed for an interval). *For $\gamma_p > 0$, positive ending reserve is equivalent to*

$$u_p > \frac{m_p + \lambda_p \ell_p - e_p}{\gamma_p}. \quad (7)$$

If intake occurs first and the subsequent drain is constant and nonnegative, positive ending reserve ensures positive reserve throughout the interval. Every prefix of a continuation in (6) ends with positive reserve for every person.

Proof. Rearranging (5) and dividing by positive γ_p gives (7). At fraction $q \in [0, 1]$ of the interval, reserve is

$$e_p + \gamma_p u_p - q(m_p + \lambda_p \ell_p) \geq e_p + \gamma_p u_p - m_p - \lambda_p \ell_p > 0.$$

For the prefix claim, apply the first conjunct of (6) after each recursive step, including its terminal case. $\square$

Need exists before any promise to satisfy it. Equation (7) determines an input necessary for continued capacity under the stated law. It does not identify another person's duty to provide that input.

3.2 The same total can leave someone without capacity

Alice and Ben start with reserves (6, 5). There are two units of ingredients, three prepared meals at Alice's location and one at Ben's. Every unit has material weight one. Each interval costs each person two reserve units; a meal restores one. Alice can prepare two more meals in the first interval by using the ingredients and one unit of work. Here work costs one reserve unit per unit, transfer requires no labor, and storage is lossless. After preparation, the meal stocks are (5, 1) and reserves are (3, 3).

There are two possible continuations. In the second interval, transfer one meal to Ben or leave the food where it is. Either action leaves reserves (1, 1). In the third interval, let Alice and Ben consume (2, 2) after the transfer, or (3, 1) without it. Figure 5 shows the result.

Every operation meets its stock, recipe and work-capacity conditions. In the first course the reserves are (6, 5), (3, 3), (1, 1) and (1, 1). In the second, only the last pair changes, to (2, 0). Both conserve all six units of material. Their final time, each resource stock, total intake, total reserve, discarded material, each person's work and completed-job counts agree. Permission and due requirements are unrestricted in this example, so the distinction is precisely individual continuation.

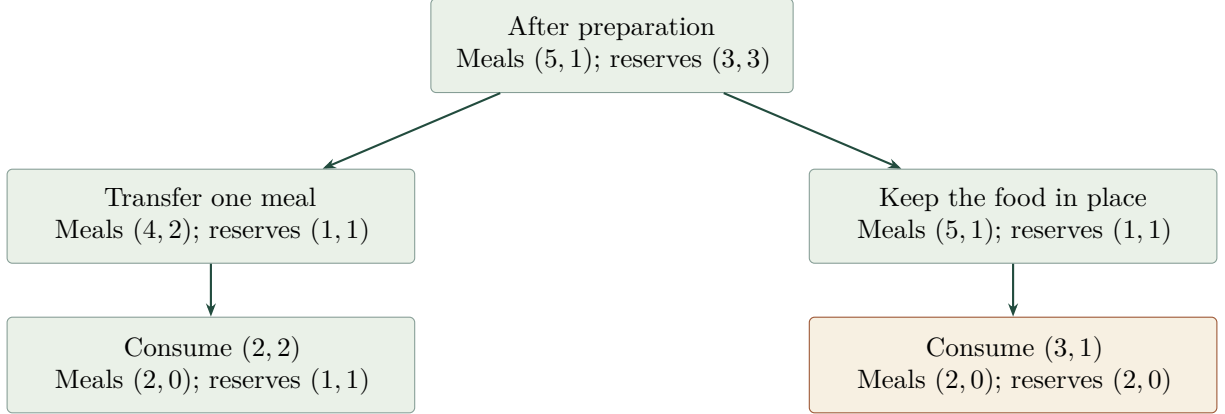


Figure 5: Both courses end with two meals in the same location, four consumed meals, total reserve two and the same work. Only the left course preserves positive reserve for each person at every checkpoint. The zero on the right records Ben’s loss of capacity in this model.

It follows that no rule using only that final report can decide whether every person’s reserve is positive. Such a rule would receive the same input for these two outcomes and would have to return different answers. Retaining each person’s reserve removes this particular ambiguity. Adding historical obligations requires retaining the information on which those obligations depend as well.

3.3 When a simpler description is sufficient

Some details can be removed without changing the decision. In the unrestricted physical model, cumulative work, intake and completion counts can be omitted when all future rules depend only on time, current stock and each person’s reserve. An agreement limiting cumulative work would change this conclusion.

Proposition 4 (Preservation of a course of action). *Let $q : S \rightarrow V$ describe concrete states by reduced states. Both systems use the same actions. Write their transitions as T and $\bar{T}$, their checkpoint conditions as G and $\bar{G}$, and their permitted-operation conditions as B and $\bar{B}$. Suppose, for every state s and action a ,*

$$q(T(s, a)) = \bar{T}(q(s), a), \quad G(s) \iff \bar{G}(q(s)), \quad B(s, a) \iff \bar{B}(q(s), a).$$

A fixed finite action list satisfies every checkpoint and operation condition from s if and only if it does so from $q(s)$. Existence of such a list of any fixed length is also preserved.

Proof. An empty list checks only G , so the claim follows from its displayed equivalence. For $a :: \alpha$, the initial checkpoint and operation conditions correspond. The transition equality identifies the next reduced state, and induction applies to α . Taking existence over lists of the prescribed length proves the last assertion. $\square$

For (6), G includes each person’s positive reserve and the current due requirements; B includes enabledness and permission. Thus a report that removes a deciding fact cannot satisfy the displayed equivalences. A description can retain that fact directly or retain enough information to recover it. The proposition gives sufficient conditions, without asserting a unique smallest description.

4 What an agreement changes

Two kitchens can have the same food on hand and the same account balances while owing different deliveries. One has already fulfilled Ben’s order. The other has fulfilled a different order. Current stock cannot recover that difference. This is the economic consequence of the performance result in [4]: claims, even with their complete writing history, cannot determine what a party owes when they omit what was done.

Keep the accepted history H fixed at the planning cutoff. Each undertaking names its parties and terms. A proposal becomes active only when the specified counterparty accepts those exact proposed terms through the stated protocol. A pending offer is different from an accepted obligation. Identifying the real authors of recorded events is part of an application’s evidence. The history retains the exercise of an authority, rather than inferring agreement from a favorable effect. A factual correction, an authorized release and a completed delivery have different consequences even if a summary labels all three a reduction in a balance.

Enrich a physical state to

$$y = (s, \rho, g, B),$$

where $\rho : U \rightarrow Q \cup \{\perp\}$ allocates each canonical physical unit $u \in U$ to at most one delivery undertaking $q \in Q$, g_q records used work allowance, and B retains past breaches. A full action specifies its jobs, its requested performance credits and its selected work grants. The jobs determine the physical operation of Section 3.

A valid delivery credit requires an unused unit, an outstanding accepted quantity, and an actual located crossing connected to that action’s stock movement. The source and destination slots must belong to the same canonical unit. The action withdraws its one source unit and leaves its one destination unit. Separate checks exclude duplication of that unit across slots or outputs. Credits are processed in sequence, updating ρ after each one. Repeating a credit request cannot turn one consumable performance into two.

An active work grant names a worker, a time window, breaks and a cumulative cap. Any positive labor must cite the worker’s grant. The full occupied interval must fit its window, avoid its breaks and leave cumulative use below the cap. In this model each person supplies at most one unit of active work per interval. Permission to use up to one interval does not require using it.

Delivery requirements are checked at every due checkpoint. Service undertakings have a different condition: the asset must remain ready and accessible to the named person throughout each covered interval. Withdrawing a tool at the opening makes it unavailable during that interval, even when it returns at the end.

Let $\text{Step}(H, y, a)$ be the conjunction of enabled physical execution, resource authority, the service condition, valid work grants, canonical-unit checks and valid sequential credits. The enriched transition $F_H(y, a)$ applies the physical update, processes the credits and adds work use. It leaves B unchanged. Accepted terms remain fixed. Recovery can reduce a present shortfall while retaining an earlier missed deadline.

Proposition 5 (Retaining history preserves the physical route). *Suppose a sequence of n full actions satisfies Step at every interval, positive reserves at all $n + 1$ checkpoints, and every due delivery and physical requirement. Its sequence of physical operations is a continuation of length n with the same physical states, resource authority and service conditions.*

Proof. For $n = 0$, the initial checkpoint satisfies the required conditions. For $n + 1$, take the first

full action's physical operation. Its enabledness, authority and service conditions are conjuncts of Step. The physical component of F_H is exactly T . Apply induction to the remaining actions and append the resulting continuation. $\square$

The richer route retains its delivery allocation and used work allowances. Projecting to physical stock forgets them. The implication therefore supplies no reconstruction of obligations from inventories alone.

5 Choosing among plans that can actually be carried out

An ethical effect can have several physical realizations. Repairing a shared entrance with agreed labor and repairing it by assigning someone else's labor can produce the same ready entrance. Selecting the common benefit cannot confer the missing agreement.

5.1 Reading effects on each person

The ethical description keeps a finite set $\mathcal{B}$ of directed bonds. A bond joins two persons or connects a person to themselves. Each carries a positive recognition ratio x_b . Ratio one has zero cost. Departures above and below one are measured by the same fixed law: $J(x) = J(1/x)$. In log coordinates, $J(e^r) = \cosh r - 1$, which increases with $|r|$.

The application must establish which physical relation a bond represents and what its ratio measures. In the entrance example, the ratio reads the condition of a shared service. In the maintenance example, it reads departure of a supplied compartment from its reference condition. These interpretations determine the states to which the law applies.

Let L be the common ledger of these bonds and ratios. Let $\mathcal{B}_p$ be the bonds incident to person p , counting a self-loop once. The recurring quantities are the fixed cost law, the cost on that person's relations, and the ledger's total absolute strain:

$$J(x) = \frac{x + x^{-1}}{2} - 1, \quad D_p(L) = \sum_{b \in \mathcal{B}_p} J(x_b), \quad \Sigma(L) = \sum_{b \in \mathcal{B}} |\log x_b|. \quad (8)$$

An effect changes log ratios by η_b , giving $x'_b = x_b \exp \eta_b$. Harm to p is $h_p = D_p(L') - D_p(L)$. The cost law is fixed; the ledger and effect specify the states to which it applies. Identifying hunger, a broken entrance or expended labor with those quantities requires a physical interpretation of the actual relations. The service ledger below measures one represented effect. The full moral account also retains each person's other affected relations, experience, capacity and history.

The same transformation in both descriptions. Let $y \rightarrow y'$ be a physically executed transition, including its relevant history. On a fixed set of bond endpoints, let $\chi_b(y)$ be the physically interpreted log ratio, so $x_b(y) = e^{\chi_b(y)}$. The corresponding ethical effect must retain the actual actor and set

$$\eta_b = \chi_b(y') - \chi_b(y), \quad x_b(y)e^{\eta_b} = e^{\chi_b(y')} = x_b(y').$$

The equality follows by adding exponents. Thus applying the effect to the first state's ledger gives exactly the second state's ledger, and h_p is the difference of incident costs on those same states. Fixed endpoints and positive ratios are essential to this representation. Its algebra checks consistency

once the physical interpretation is supplied; it grants no authority to cause the transition. The entrance and maintenance examples make their interpretations explicit.

Matching the foundation's units. The two-record construction in [4] divides a relationship's cost equally between its two distinct ends. It assumes that the reading depends on the before-and-after ratios, is local and invariant under renaming and reversal, and charges the network's change exactly once. With distinct endpoints on every bond and every endpoint included, its personal reading is

$$\hat{D}_p = \frac{1}{2}D_p, \quad \hat{h}_p = \frac{1}{2}h_p, \quad \sum_p \hat{D}_p = \sum_b J(x_b).$$

Our D_p retains the full incident cost used by the bond-ledger decision rule [5]. The sign of harm and the ordering of maximum harm are unchanged by this common positive factor. The exponential tie-break is not generally invariant under it: $\exp(D_p) = \exp(2\hat{D}_p)$. Substituting $\exp(\hat{D}_p)$ would specify a different comparison. Self-loops have one incident person and are outside the foundation's two-distinct-party chart; the convention in (8) counts them once. Section 5.3 keeps the resulting incidence accounting explicit.

5.2 Admission, then comparison

Three different questions enter a decision. Can this plan run through its whole time window with the required permissions? Does its effect satisfy the ethical admission conditions? If several plans satisfy both, which does the selection rule select? Keeping these questions separate prevents a desirable endpoint from supplying missing authority to reach it.

Let P_0 be a nonempty finite list of stated persons, without duplicates. Among them, the protected set contains the actor and those who can suffer a burden the actor can change from the actor's own incident bonds. At zero harm cap, an admissible effect is by the stated actor, satisfies $\Sigma(L') \leq \Sigma(L)$, and has $h_p \leq 0$ for every protected person in P_0 . A nonincrease in a person's burden is an effect condition. It is distinct from that person's authored agreement to work.

Coverage is part of this construction. Include the actor and every endpoint of a changed bond in P_0 , and suppose the action changes only bonds incident to the actor. Every person on a changed bond is then protected; every person whose incident bonds are unchanged has exactly zero harm. Admissibility therefore gives $h_p \leq 0$ for every agent in the ledger, including any unlisted agent. Taking all endpoints of all active bonds supplies this coverage without knowing the action in advance. This is completeness relative to the represented ledger. Establishing that it includes every real affected person and causal channel is a further requirement of the physical realization. Acting on one's incident bonds supplies no missing permission to use another person's labor or resources.

For admissible effects the selection rule minimizes, lexicographically,

$$S(L') = \left(\max_{p \in P_0} h_p, - \sum_{p \in P_0} [1 - \exp(D_p(L'))], \Sigma(L') \right). \quad (9)$$

The second coordinate uses local value $-D_p$ and the adopted transform $v \mapsto 1 - \exp(-v)$. It equals $\sum_{p \in P_0} \exp(D_p(L')) - |P_0|$, so, for the fixed person list, minimizing it is equivalent to minimizing the sum of exponentiated incident costs. First compare the worst burden change. If it ties, compare the sum of exponentiated personal costs. Only a further tie reaches total absolute strain. Thus a later coordinate never compensates for a worse earlier one. We adopt the zero-cap filter and lexicographic

order developed in [5], taking the maximum over the full stated roster P_0 . The earlier paper takes that maximum over protected persons. The two conventions agree when everyone listed is protected. Under the coverage condition above, an unprotected listed person has zero harm; including that person can turn a negative maximum into zero and move a comparison to the second coordinate. The roster and this convention are therefore part of the decision specification. Neither the filter nor the order follows from the cost function alone. We use that procedure on the stated service and maintenance interpretations. The broader physical ethics in [4] also requires authorship and evidence of performance; its false-agreement example shows why a zero-cap test on disagreement alone cannot decide every moral question. The selection theorem below establishes that the specified procedure preserves eligibility.

Fix the actual history H , a finite list of candidate plans $\mathcal{A}$, an eligibility predicate E_H , and an effect map $f : \mathcal{A} \rightarrow \{\text{effects}\}$. Eligibility retains the plan's own full-window execution and permission conditions. Define

$$\mathcal{A}_H^* = \{\alpha \in \mathcal{A} : E_H(\alpha) \text{ and } f(\alpha) \text{ is admissible}\}.$$

A chosen plan is an element of $\mathcal{A}_H^*$ whose effect score is no greater, in lexicographic order, than every other element's score.

Theorem 6 (Plan-specific selection). *If $\mathcal{A}_H^*$ is nonempty, a chosen plan exists and retains its own eligibility. If $f(\alpha) = f(\beta)$ but $E_H(\beta)$ is false, selection of α does not make β a chosen plan.*

Proof. Lexicographic order on three real coordinates is total and transitive. A nonempty finite list has a least score: retain its first element, compare each subsequent element and retain whichever has the smaller score. Induction on list length proves that the retained element is no greater than every visited element. Apply this to $\mathcal{A}_H^*$. Membership supplies the selected plan's eligibility. An ineligible β is absent from $\mathcal{A}_H^*$ regardless of its effect. $\square$

The calculation chooses a plan available under the actual history. It cannot select a different past in which somebody agreed. A restricted candidate list also yields a restricted comparison; it establishes no optimum over plans that were omitted.

5.3 What the whole must retain

A bond can affect both its endpoints while remaining one bond. The cost assigned to each person in (8) therefore answers a different question from a cost counted once per bond. This distinction becomes necessary when describing a group.

For a set of members $S \subseteq P_0$, let $\mathcal{B}(S)$ be the union of their incident bonds. Let $\mathcal{B}_{\text{int}}(S)$ contain the bonds whose two endpoints are distinct members of S . A self-loop is incident to its one endpoint once. Define

$$U_S(L) = \sum_{b \in \mathcal{B}(S)} J(x_b), \quad I_S(L) = \sum_{b \in \mathcal{B}_{\text{int}}(S)} J(x_b).$$

Proposition 7 (Member costs and shared bonds). *For every finite group S ,*

$$\sum_{p \in S} D_p(L) = U_S(L) + I_S(L). \tag{10}$$

Merging all endpoints in S into a single group endpoint, while retaining every bond and its ratio, gives that endpoint incident cost $U_S(L)$.

Proof. Exchange the finite sums over persons and bonds. A bond outside $\mathcal{B}(S)$ contributes zero. A bond touching one distinct member contributes once. A bond touching two distinct members contributes twice, once through U_S and once through I_S . A self-loop contributes once. After merging, precisely the bonds in $\mathcal{B}(S)$ touch the group endpoint, each once. $\square$

Subtracting (10) before and after an action gives the corresponding identity for the sum of member harms. These are readings of the same bonds. They create no additional food, energy or other resource. In a resource inventory, the same stored unit represented in two overlapping groups remains one unit. The groups are views of the inventory, with their overlap retained.

Now take an actor 0 and two other persons 1, 2. Three bonds have ratios

$$(x_0, x_1, x_2) = (2, 2, 1) \longrightarrow (1, 5/2, 1).$$

Compare two fixed arrangements of their endpoints:

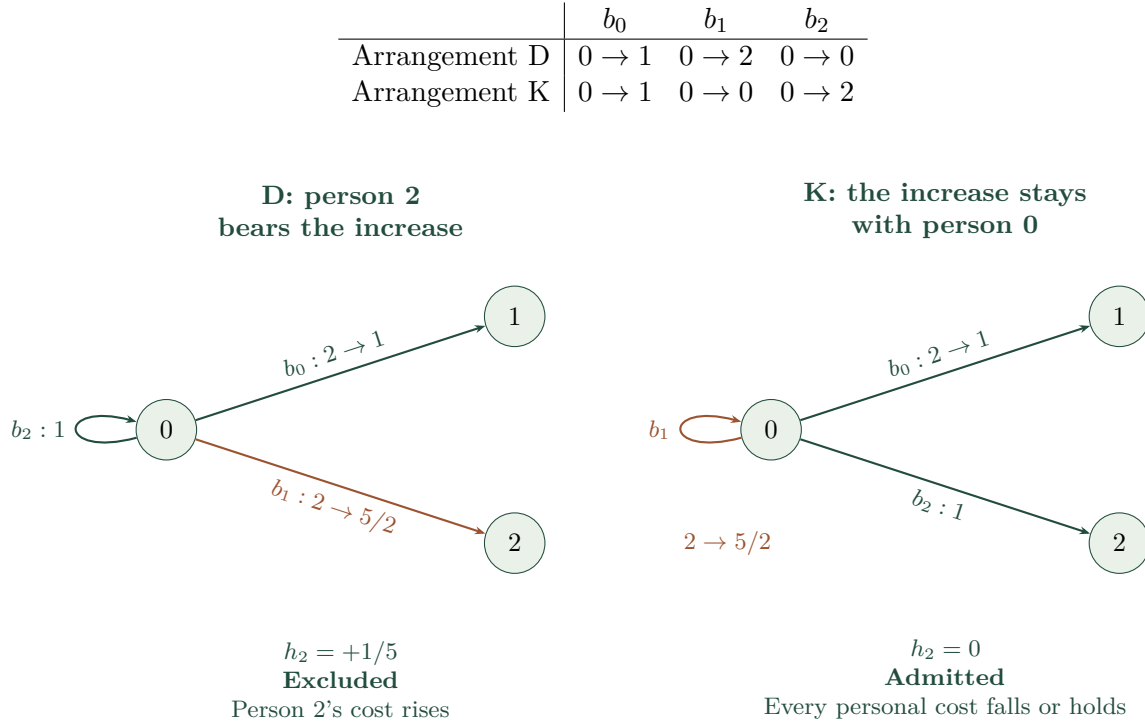


Figure 6: Identical bond changes, different personal effects. In both arrangements the cost counted once per bond falls from $1/2$ to $9/20$. Person 0's net cost falls in both, despite the increase on b_1 . Merging the three persons erases the fact that excludes D: in D the increasing bond also touches person 2. Endpoints stay fixed during each action. The ratio-one bond keeps person 2 represented in K at zero cost.

No endpoint changes during either action. In arrangement D person 2 is the destination of b_1 , whose cost rises. In arrangement K person 2 is the destination of b_2 , which stays at ratio one. That bond retains person 2 at zero cost and gives $h_2 = 0$. Since

$$J(2) = \frac{1}{4}, \quad J(1) = 0, \quad J(5/2) = \frac{9}{20},$$

the cost counted once per bond falls from $1/2$ to $9/20$ in both arrangements. Total absolute strain

falls from $\log 4$ to $\log(5/2)$. Their effects on the three persons are

	h_0	h_1	h_2
Arrangement D	$-1/20$	$-1/4$	$1/5$
Arrangement K	$-1/20$	$-1/4$	0

All three persons are protected by the stated zero-cap rule. Arrangement D fails that rule because person 2 is harmed; K meets it. Merging all three persons makes every bond a self-loop at the group endpoint. The complete merged ledgers, including bond identities and ratios, then agree before and after the action in the two arrangements.

Proposition 8 (A merged ledger can lose the judgment). *No predicate of only the merged before-and-after ledger pair recovers member-level admissibility for both arrangements D and K.*

Proof. The input ledger pairs are equal, so any such predicate has the same truth value on both. The calculation above gives different member-level admissibility judgments, a contradiction. $\square$

The lost fact is who bears the second bond's increased cost. Keeping every bond and every ratio does not recover it once the endpoints have been merged. A group description that also retains its original member relations avoids this loss. These examples use the stated bond-cost reading and decision rule; deciding an actual assistance case also requires the physical effects and the plan's eligibility.

5.4 Preserving selection and standing

For the rule in (9), a sufficient ethical description retains the same person roster, the same actor-match condition and protected set, the harm cap, every listed person's before-and-after cost, and before-and-after absolute strain. These data determine every admissibility test and every score. A plan description must preserve its eligibility under the actual history as well.

Proposition 9 (Selection through a reduced description). *Let q map concrete plans to reduced plans. Suppose the reduced candidate list is exactly $q(\mathcal{A})$, and for every $\alpha \in \mathcal{A}$ the reduced description preserves its eligibility and the ethical data just listed. Then α is chosen from $\mathcal{A}$ if and only if $q(\alpha)$ is chosen from $q(\mathcal{A})$. Every chosen reduced plan has a chosen concrete representative.*

Proof. Preserving the listed data preserves each person's harm, each admissibility condition and the three score coordinates. Eligibility is also equivalent for every plan and its image. Each reduced alternative has a preimage in $\mathcal{A}$, so comparison against all eligible admissible alternatives is equivalent in the two lists. Membership in the image supplies a representative; the equivalence just proved makes that representative chosen. $\square$

To meet these hypotheses, the map can identify two plans only when the specified deciding facts agree. An agreed assignment and an unagreed assignment can have the same physical endpoint and effect while differing in eligibility. Mapping them to one record cannot satisfy the proposition's eligibility condition. Their actual histories must remain distinguishable wherever the agreement changes the judgment.

Account splitting gives a different case. Let d decode an account representation into a physical and personal state. A refinement r that splits accounts while preserving $d(r(z)) = d(z)$ preserves every

decision read only from that decoded state. This follows by substitution into the decision function. Neither the resource inventory nor the roster of subjects changes.

Duplicating a person in the roster can change the rule instead. Start with two bonds $0 \rightarrow 1$ and $0 \rightarrow 2$, both at ratio 2. Two admissible repairs change the ratios to $(1, 2)$ or $(2, 1)$. Both have worst harm zero and final strain $\log 2$. Put $u = 1 - \exp(1/4) < 0$. With roster $[0, 1, 2]$, both have summed local value $2u$ and tie. If duplicate person entries were allowed, roster $[0, 1, 1, 2]$ would give the first value $2u$ and the second $3u$, so the first alone would be chosen. This calculation explains why an account label cannot create another entry in the list of persons. Both the formal complete roster and the applied reader use distinct person identities.

5.5 The path to an acceptable endpoint

An endpoint effect does not retain intermediate burdens. Consider one bond joining Alice and Ben, so each person's burden is the full J of its ratio. Two successive log changes, $\log 2$ and $-\log 4$, give

$$2 \longrightarrow 4 \longrightarrow 1, \quad J : \quad \frac{1}{4} \longrightarrow \frac{9}{8} \longrightarrow 0.$$

Direct repair followed by no change gives $2 \rightarrow 1 \rightarrow 1$. The final ledgers agree, including the bond and its parties. Their common endpoint effect is a log change of $-\log 2$, which lowers both burdens and total absolute strain and satisfies the zero-cap condition. The first route nevertheless raises each person's burden by $9/8 - 1/4 = 7/8$ in its first step. That step fails the zero-cap condition; the subsequent restoration satisfies it relative to the raised state.

This calculation uses the stated ledger operations. A physical application must identify which intermediate ledgers its actions actually realize. Once they are identified, a plan's eligibility can retain the conditions at those intermediate states as well as its final score.

Proposition 10 (Successive zero-cap checks). *Let $L_0, \dots, L_n$ be ledgers obtained by successive effects of a fixed actor, each applied to the current ledger. Use the same list P_0 at every step. If every effect is admissible with zero harm cap relative to its current ledger, and $p \in P_0$ remains protected at every step, then*

$$D_p(L_b) \leq D_p(L_a) \quad (0 \leq a \leq b \leq n).$$

Proof. At step $t < n$, admissibility and protection give $D_p(L_{t+1}) - D_p(L_t) \leq 0$. Chaining these inequalities from a to $b - 1$ gives the result; the case $a = b$ is equality. $\square$

The baseline is refreshed at each step. Checking only against L_0 would allow a later increase that remained below the initial burden. The proposition supplies a sufficient condition for nonincrease throughout the checked sequence. Choosing the physical interval and deriving which temporal conditions govern a particular action remain part of its ethical interpretation.

6 Repair without a service promise

Consider a broken shared entrance. Alice has actually proposed one interval of repair work, and the counterparty has accepted those exact work terms. There is no undertaking to deliver a unit or provide a service. Asset authority and access for both Alice and Ben are separately supplied. Two plans are available: repair then wait, or wait for both intervals.

Each person starts with reserve 4. Maintenance costs 1 per interval; the repair costs Alice one additional reserve unit. The repair transforms one broken asset into one ready asset, occupies a tool and returns that tool at the interval's end. It creates no second entrance. The reserve paths are

	initial	after one interval	after two intervals
repair: Alice, Ben	(4, 4)	(2, 3)	(1, 2)
wait: Alice, Ben	(4, 4)	(3, 3)	(2, 2).

(11)

All checkpoints are positive. Both people can use the ready entrance during the second interval. The repair interval itself supplies no continuous ready service.

The declared service interpretation places one bond between Alice and Ben, with ratio 2 while the asset is broken and 1 while it is ready. Thus

$$D_A = D_B = J(2) = \frac{2 + 1/2}{2} - 1 = \frac{1}{4} \quad \longrightarrow \quad J(1) = 0.$$

The repair's change in log ratio $-\log 2$ gives $2 \exp(-\log 2) = 1$, matching the physical terminal state's service interpretation. Waiting has zero change in log ratio and leaves the burden unchanged. Both effects are admissible. Their maximum harms are respectively $-1/4$ and 0, so (9) strictly selects repair before either tie-break is used.

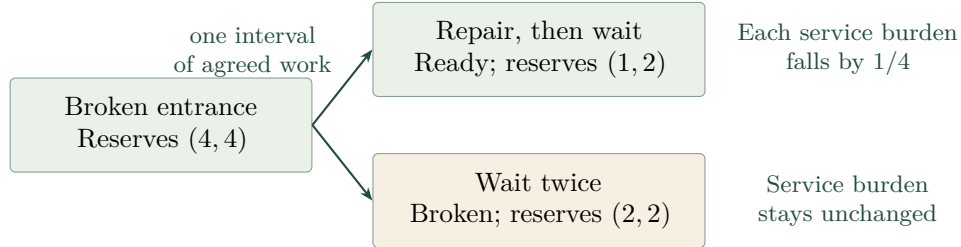


Figure 7: Two eligible plans in the same history. The repair's benefit and the worker's extra reserve expenditure remain separate facts.

Now change only Alice's initial reserve to 3. Repair would end with reserves (0, 2); waiting ends with (1, 2). The favorable service score has not changed. Eligibility has. Theorem 6 selects waiting from the surviving candidates. Here the worker is protected from exhaustion by the positive-reserve requirement; the service score itself does not measure her reserve expenditure.

This is a positive choice without a prior service promise. The burden interpretation measures the stated service condition, not total human welfare; Alice's expenditure is retained in the physical path. Deriving the scope of a duty to rescue requires the actual need, available means, authorship and governing physical moral law. Neither the service coding nor its selected candidate menu supplies that whole derivation. The continuation argument of Section 2.3 explains why the worker's future capacity belongs in the same account. It does not, by itself, assign Alice a duty to make every physically available repair.

The interpretation also fixes what the score can distinguish. Two eligible repairs with the same service ledger receive the same score even if one leaves Alice more reserve. The physical model records that difference. Making it part of an ethical comparison requires an interpretation of the additional effect on Alice.

A longer required window can distinguish the plans without altering that score. Start with reserves (5, 5) and an accepted grant covering up to two work intervals, shared by the two candidates in this

comparison. Compare repair followed by waiting with tool inspection followed by repair. Inspection withdraws and returns the existing tool and uses one interval of Alice’s labor. Both plans are eligible through two intervals. Their reserves end at $(2, 3)$ and $(1, 3)$ respectively, and both end with the entrance ready. The first plan also provides readiness earlier; equal endpoint ledgers do not mean equal service histories.

Require one more interval of survival, keeping the same technology. The first plan can wait, ending at $(1, 2)$. The second cannot continue under *any* enabled operation: every operation supplies zero intake, while Alice’s reserve of 1 must pay maintenance of 1 and nonnegative labor cost. Both plans are chosen under the two-interval requirement; only the first is chosen under the three-interval requirement. The extra interval is an explicit requirement, not a derived duty to maximize reserve. Waiting preserves the ready stock, so the same service interpretation applies at both endpoints.

More generally, adding a componentwise nonnegative reserve offset preserves any fixed continuing operation list, provided authorization and requirements are preserved by that offset. Availability, technology and freshness do not change. Equation (5) retains the offset after each operation, and positivity follows by induction. This is inclusion of physical possibilities, not a preference theorem. Arbitrary reserve-sensitive permissions or requirements need not satisfy its premises.

6.1 An unavoidable shortage

Suppose each of two people has reserve 1, maintenance 2, and conversion 2 per whole meal. One meal is available. Equation (5) gives $e'_p = 2u_p - 1$. Positive reserve requires integer $u_p \geq 1$ for each person, hence $u_A + u_B \geq 2$. Conservation permits at most one meal. No allocation meets both needs.

If a delivery was promised to Ben, keeping the meal and delivering it have different historical consequences, even though both leave someone without continuation. The physical contradiction identifies the conflict. It supplies no priority rule between those persons. Settling that rule requires the governing interpersonal ethics, including the role of an existing promise under unavoidable scarcity. An empty feasible set is evidence of a physical conflict; neither the selection theorem nor the continuation argument converts it into a verdict about whose claim takes priority.

7 Whose action makes the plan work?

A cooperative schedule may say that Alice delivers and Ben eats. Alice can perform the first action. The second belongs to Ben. A guarantee must respect that difference.

Use a turn-based protocol with a specified actor $\tau(t)$ for each interval. Every job has an actor. A person’s full action can contain only that person’s jobs, performance credits they control and their own work grants. In particular, a job with positive labor or autonomous intake for p must belong to p . This leaves room for effects on other people through delivery or shared assets. Assisted intake requires its own physical model.

A policy tells a person which action to choose from the information available to them. A guarantee must also account for the actions other people can choose. Each person observes $o_p(y)$ and has a private observation history h_p . Write $h = (h_p)_{p \in P}$ for the collection of those histories. A policy $\pi(h_p, o_p(y))$ produces a full action. Legal actions satisfy ownership, the protocol’s permission

condition and Step. Let $L_p(h, y)$ be their set. Put

$$\text{Good}(y) \iff (\forall p, e_p > 0) \wedge \text{Req}(H, y), \quad h_p^+ = o_p(y) :: h_p, \quad h^+ = (h_p^+)_{p \in P}.$$

Req includes the due delivery and physical requirements of Section 4. Legality itself does not require that every person's future reserve stays positive. Otherwise a guarantee could be obtained by deleting the very responses it must assess.

For controller c with fixed policy π , put $a_\pi = \pi(h_c, o_c(y))$ and define

$$\text{Ctrl}_0(\pi, h, y) \iff \text{Good}(y). \quad (12a)$$

On the controller's turn, $\tau(t) = c$,

$$\text{Ctrl}_{n+1}(\pi, h, y) \iff \text{Good}(y) \wedge a_\pi \in L_c(h, y) \wedge \text{Ctrl}_n(\pi, h^+, F_H(y, a_\pi)). \quad (12b)$$

On another person's turn, with $p = \tau(t) \neq c$,

$$\begin{aligned} \text{Ctrl}_{n+1}(\pi, h, y) \iff & \text{Good}(y) \wedge L_p(h, y) \neq \emptyset \\ & \wedge [\forall a \in L_p(h, y), \text{Ctrl}_n(\pi, h^+, F_H(y, a))]. \end{aligned} \quad (12c)$$

The universal quantifier includes each declared legal response by the other actor. Requiring a nonempty response set prevents success by vacuity.

Proposition 11 (A controlled plan has an executable branch). *If $\text{Ctrl}_n(\pi, h, y)$ holds, there is an enriched legal trace of length n satisfying Good at every checkpoint. Every collection of other-person policies whose encountered choices are legal also produces such a trace when the controller uses π .*

Proof. Induct on n . The zero case is $\text{Good}(y)$. At a controller turn, its chosen action is legal and the recursive condition holds at its successor. At another person's turn, nonemptiness supplies an action and the universal clause supplies the recursive condition. This proves existence. For a given collection of other-person policies, use its actual legal action instead; the same universal clause applies. The physical trace follows by the projection in Section 4. $\square$

The condition in (12) is sufficient for all observation-consistent response policies. It permits independent legal choices at each branch. Failure under this stronger response condition need not imply failure under a narrower fixed-policy restriction.

7.1 Delivery and nourishment

In a two-person provision instance, initial reserves are $(5, 2)$. Maintenance costs one reserve unit per person per interval; delivery costs Alice one further unit, and consuming the meal supplies Ben two units. Alice's authorized delivery leaves reserves $(3, 1)$ and a meal at Ben's location. The accepted delivery is now fulfilled. During Ben's next interval, consuming the meal yields reserves $(2, 2)$; waiting yields $(2, 0)$. Both are legal physical responses in the broad protocol.

Alice's delivery makes continuation possible. It cannot secure nourishment against Ben's permitted waiting. Once the meal is delivered, Ben can secure continuation through his own consumption policy. Both subsequent choices preserve the completed delivery credit. Lack of control over uptake therefore neither defeats the fact of delivery nor erases an accepted duty to deliver.

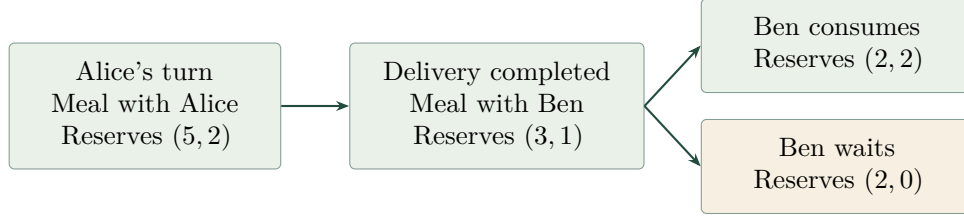


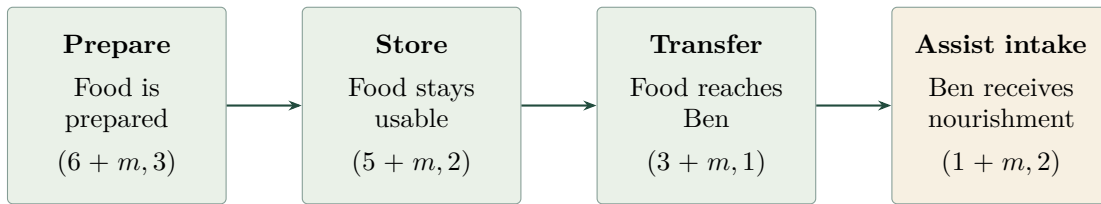
Figure 8: A completed delivery opens an option for its recipient. It does not transfer authorship of the recipient's next action to the deliverer.

8 Care when the recipient cannot act

Ben cannot eat without assistance. Requiring him to take a turn would exclude the person who needs the help, so Alice prepares the meal, keeps it usable, brings it to him and assists with intake. Ben performs no work and makes no decision during these intervals. His body can still respond to the nourishment. Her provision draws on her reserve; his internal response draws on his. Both people remain in the account.

Consider the following instance of (5). Each person spends one reserve unit per interval on maintenance. Each unit of Alice's work costs her one further unit. Consuming the meal adds two units to Ben's reserve. His labor capacity is zero. There is one unit of material, initially in the ingredient slot; preparation and transfer move it through the food slots, and intake consumes it. There is no spoilage or external inflow. The allowed technology schedules preparation, storage, transfer and assisted intake in four consecutive intervals. Storage may also be repeated while the food is held. The application requires Ben to have consumed one meal by the end of the fourth interval.

Alice begins with $8 + m$ reserve units and Ben with 4, where $m \in \mathbb{N}$ is additional capacity available to Alice. A work grant permits her three units of labor during these four intervals. A separate, previously established authorization permits assisted intake. Ben need not issue another instruction during the plan. This is a case with prior authorization; emergency authority is a different question.



Pairs give Alice's and Ben's reserves after each interval.
Alice performs all three work units. Ben performs none.

Figure 9: Care is a complete physical history. Under care, both people's reserves remain positive through intake even when $m = 0$. Whether they can continue through the next maintenance response is a further calculation.

The reserves in Figure 9 follow by repeated substitution into (5). Preparation and transfer each subtract two units from Alice and one from Ben. Storage subtracts one from each. Assisted intake subtracts two from Alice and adds a net one to Ben. The meal is counted once, and its consumption leaves no food stock. If Alice instead stores the delivered meal during the fourth interval, the

reserves become $(2 + m, 0)$. That operation remains physically available in the declared technology, but Ben has received no intake and the application's intake requirement is unmet.

8.1 A physical response belongs to the person who undergoes it

To connect this provision history to the cost calculus, specify what the capacity pays for. Let each person have one internal recognition relation, with distinct identities for the two relations. At a maintenance exposure its ratio is 2, so its cost is $J(2) = 1/4$. A funded internal response restores that ratio to 1, removes the cost and spends $1/4$ capacity. An unfunded response leaves the ratio and the unspent capacity unchanged. This response law and the recurring exposure are supplied physical interpretations for the example. They are not inferred from the presence of food.

This repair rule pays $J(x)$ once for the internal relation counted in (8). The held-ratio upkeep in Section 2.3 instead charges both ends, $J(x) + J(1/x) = 2J(x)$, at every window. At ratio two those rules give $1/4$ for a funded repair and $1/2$ for the two-ended upkeep. They use the same cost function with different stated payment mechanisms.

Measure capacity as reserve divided by four. On a funded response, the expenditure is the one reserve unit charged for maintenance in (5), expressed in the cost unit and counted once. The two rules differ when funding is absent. The signed reserve rule records an unpaid maintenance bill as a deficit. The response law spends nothing and leaves the burden unpaid: Ben at zero reserve stays at zero, whereas another interval of the signed rule would take him to -1 . We use the reserve rule during the four provision intervals and the funded-response law at the next maintenance exposure. This explicit change of model makes no claim that the unfunded transitions coincide. The response comparison starts at that common exposure, with opening capacities

$$(B_A, B_B) = \begin{cases} ((m+1)/4, 1/2), & \text{after care,} \\ ((m+2)/4, 0), & \text{after omission.} \end{cases}$$

Alice can fund her response in either case. Ben can fund his exactly when he received care. The resulting capacities and changes in relational cost are therefore

$$\begin{aligned} (E_A, E_B)_{\text{care}} &= (m/4, 1/4), & (\Delta C_A, \Delta C_B)_{\text{care}} &= (-1/4, -1/4), \\ (E_A, E_B)_{\text{omit}} &= ((m+1)/4, 0), & (\Delta C_A, \Delta C_B)_{\text{omit}} &= (-1/4, 0). \end{aligned} \tag{13}$$

At zero capacity Ben stays in the description, together with his unpaid burden and unmet intake. An observation of his condition remains meaningful even though it no longer defines a positive-capacity agent state.

The two responses can be represented sequentially on one shared state. Write their states as L_0, L_1, L_2 , with Alice's response first. Her transformation changes only her internal relation; Ben's changes only his. For either person p ,

$$[C_p(L_1) - C_p(L_0)] + [C_p(L_2) - C_p(L_1)] = C_p(L_2) - C_p(L_0).$$

The intermediate costs cancel. Each change is measured where it actually occurs, and neither person's repair is charged to the other. Here each funded response removes and pays $1/4$; an unfunded response changes neither the burden nor the capacity. Cost therefore falls by exactly the amount of capacity spent, giving the further identity

$$E_p - \Delta C_p = B_p.$$

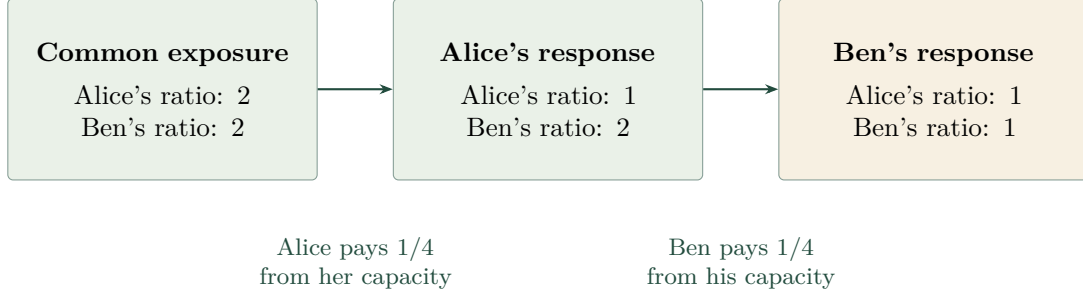


Figure 10: The response after care, when both repairs are funded. Provision makes Ben's response possible; it does not make that internal response Alice's action. The arrows represent a supplied automatic response law, not two deliberative choices.

Thus the same physical response has a relational effect and a capacity expenditure. Removing the burden does not restore the capacity used to remove it. The response steps satisfy the cost-based admission conditions of Section 5 with both people represented. This judgment starts at L_0 ; attributing the environmental exposure and judging the preceding provision history require their own causal account.

8.2 When care preserves both people's capacity

With $m = 0$, care removes both represented burdens but leaves Alice at zero capacity after the next response. With $m = 1$, it leaves both people at $1/4$. Omission leaves Ben at zero for every m . A comparison using only the terminal burdens prefers care in both cases: its maximum over persons of ΔC_p is $-1/4$, compared with 0 for omission. It cannot distinguish funded continuation from Alice's exhaustion without keeping her capacity as well.

Both candidate operations fit Alice's existing work grant. Omission is permitted in this menu; assisted intake also requires the separate care authorization.

Proposition 12 (Authorized care with continued capacity). *In the two-plan menu consisting of assisted intake and omission, keep the technology, response law and work grant above fixed. Let a mean that assisted intake is authorized, and let $m \in \mathbb{N}$ be Alice's additional initial reserve. A permitted plan leaving both people with strictly positive capacity after the response exists exactly when a holds and $m > 0$. Whenever it exists, assisted intake is the unique such plan.*

Proof. Omission is permitted but leaves $E_B = 0$ by (13), so it never meets the capacity condition. Assisted intake is permitted exactly when a holds. Its capacities are $m/4$ and $1/4$, which are both strictly positive exactly when $m > 0$. These exhaust the menu. $\square$

The result identifies a complete available route to the stated continuation target, including a recipient who cannot act for himself. It also locates the two distinct ways this menu can lack such a route: missing authority or insufficient helper reserve. Giving Alice more reserve changes the latter condition. It does not establish the former. Choosing omission when it is the only permitted option still leaves Ben's need unmet.

The common description retains Ben's need under his identity and Alice's expenditure under hers, as well as the events that establish authority. It includes Ben as an affected subject even though he cannot deliberate during the plan. These are the distinctions the economic account carries forward

from its moral foundation. The continuation target here is an explicit application requirement. Establishing when assistance is morally required calls for the foundation’s practical principles to be applied to the full history, including the alternatives and the effects of withholding care. A preferred endpoint alone cannot perform that derivation.

8.3 Whose effects count over time?

Alice changes Ben’s later condition by making a response possible. To follow that effect, compare care and omission from the same opening state, under the same physical response law. The difference then comes from her provision choice. Ben’s response remains his. A description limited to their separate internal relations at one instant would miss this connection.

This extends the moral foundation’s capacity question: can the actor change what another subject bears? When the alternatives range over all the original actions on the actor’s own relations, including doing nothing, the comparison gives exactly the original immediate-action answer. An action that changes burden differs from doing nothing; if two actions give different burdens, at least one differs from the original state. Over time, the comparison also follows the physical responses those actions make possible. The actor remains represented even when the actor’s own burden is unchanged. A difference at an earlier checkpoint is retained when final conditions happen to coincide.

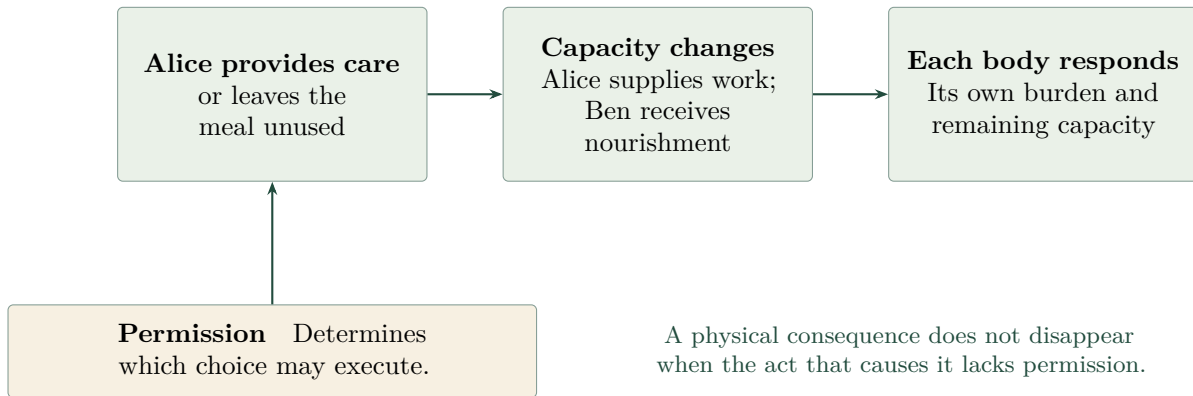


Figure 11: One controlled choice can affect another person’s later response. The comparison keeps the opening state and response law fixed. Permission governs execution; it does not determine whether the physical effect exists.

The duration matters. Continue the supplied response law without another meal or other replenishment. Each exposure doubles the ratio left by the previous round. A funded response restores it to one and pays its cost; an unfunded response leaves the exposed ratio in place. A round consists of one exposure followed by the response it can fund. The index $n = 0$ denotes the state before the first exposure, with ratio one and cost zero. The earlier L_0 is the exposed intermediate state inside round one; its ratios are two. Thus round one ends at the response already calculated in (13). Starting with capacity $k/4$, where $k \in \mathbb{N}$, the state after n rounds is

$$x_n = 2^{\max\{n-k, 0\}}, \quad E_n = \frac{\max\{k - n, 0\}}{4}. \quad (14)$$

The first k responses each spend $1/4$. After that, capacity is zero and the unrepaired ratio doubles at every exposure. Resetting it to two each time would discard the accumulating burden.

The cost law is $J(x) = (x + x^{-1})/2 - 1$. All ratios in this continuation are at least one, on which J is strictly increasing.

Proposition 13 (The later cost of helping). *Keep the care model above, with m additional units of Alice’s initial reserve and no later replenishment. At every positive response horizon n , Ben’s incident cost is lower under care than omission. Alice’s incident cost is greater under care exactly when $n > m+1$. Care therefore has no greater incident cost for either person exactly when $n \leq m+1$.*

Proof. The opening capacity units are $(m+1, 2)$ under care and $(m+2, 0)$ under omission. Equation (14) makes Ben’s exponents $\max\{n-2, 0\}$ and n , respectively; the former is smaller exactly when $n > 0$. Alice’s exponents are $\max\{n-m-1, 0\}$ and $\max\{n-m-2, 0\}$; the former is greater exactly when $n > m+1$. The function $J(x)$ is strictly increasing for $x \geq 1$, so these exponent comparisons give the stated cost comparisons. At $n = 0$, before any exposure, all four costs vanish. $\square$

For $m = 1$, after three rounds the costs are $(1/4, 1/4)$ under care and $(0, 49/16)$ under omission. Care has the lower total, but Alice bears a cost she would otherwise have avoided. The earlier improvement for Ben remains real. So does the later cost to Alice. Her reserve was already lower before that cost appeared; retaining capacity records prevents the delay from hiding her expenditure.

If assisted intake lacks permission, omission is the only legal option in this menu. That removes a choice, not the physical effect care would have had on Ben. His unmet need stays in the account. Neither the calculated benefit nor the absence of another permitted plan creates authority to act.

These results establish whose modeled condition the choice changes and when. The response law and its interpretation still need physical validation for the intended application. A duty to provide care requires the governing practical principles to address these histories, including the helper’s later condition and available renewed provision. The causal comparison supplies facts for that judgment; it does not replace it.

9 Keeping care possible

Care is not a single delivery. Each interval brings new exposure, and each repair draws on the capacity available then. Provision after an unrepaired exposure must meet a larger repair cost than provision at a balanced opening. A continuing care plan therefore tracks two quantities for each person: the burden ratio and the capacity available to repair it. The response law applies at every interval, with that interval’s actual intake and work supplying the net provision.

For each person, let $x_t \geq 1$ be the burden ratio and $E_t \geq 0$ the available response capacity at the beginning of interval t . In the units used above, net provision is

$$q_t = \frac{v i_t - \ell w_t}{4},$$

where i_t is actual intake, v its reserve value per unit, w_t the person’s actual work, and ℓ its reserve cost per unit. Maintenance is paid by the response below. Subtracting the earlier fixed maintenance bill as well would count it twice.

Exposure is an environmental change, followed by the separately described response. It changes the ratio to $2x_t$. Write $c_t = J(2x_t)$ for the cost at that exposed state, with $J(x) = (x + x^{-1})/2 - 1$.

The available capacity there is $E_t + q_t$. The same funded response then gives

$$(x_{t+1}, E_{t+1}) = \begin{cases} (1, E_t + q_t - c_t), & c_t \leq E_t + q_t, \\ (2x_t, E_t + q_t), & c_t > E_t + q_t. \end{cases} \quad (15)$$

Physical eligibility requires $E_t + q_t \geq 0$, enough material for the stated intake, and the permitted work and care actions. A failed repair remains a possible outcome. Its burden is carried into the next interval. The doubling, conversion and response rules specify this example's physical interpretation.

The same physical transformation. Each represented participant has one internal bond in a shared ledger, with its multiplier and that participant's capacity retained separately. Exposure doubles these multipliers. Actual intake and work supply the net provision q_t above. An affordable repair applies log-strain $-\log(2x_t)$ to that participant's bond and debits its charge from that participant's capacity. If the repair is unaffordable, that repair step changes neither quantity. Bond endpoints stay fixed.

Proposition 14 (Shared execution of the care history). *For a finite set of distinct represented participants with positive opening bond multipliers, perform each interval's exposure and provision, then attempt each participant's repair once in any order. Reading the shared ledger at each participant gives exactly (15) through every finite history and every prefix. Each participant pays only for its own executed repair; failed repairs pay nothing.*

Proof. On a positive exposed multiplier, the affordable action gives $2x_t \exp(-\log(2x_t)) = 1$ and subtracts $J(2x_t)$ from that participant's available capacity. The unaffordable branch retains the exposed multiplier and capacity. These are the two branches of (15). A repair changes neither another participant's distinct bond nor its capacity. Passing once through all participants therefore gives the same individual response regardless of order. Induction on intervals proves the history equality, including every prefix. The expense in each branch is exactly its executed charge or zero. $\square$

The response equation thus reads the physical ethical action on the shared ledger. The construction retains this application's internal-bond carrier, exposure law, provision conversion and maintenance-payment convention. Its distinct participant identifiers represent the supplied subjects; they do not establish subjecthood. Material availability, actual authority and the governing duty remain requirements of the complete action history.

9.1 How much provision is needed?

The protection target combines nonincreasing cost at completed checkpoints with the positive reserve required for continuation in Section 3. Starting at $x = 1$ with capacity $1/4$, repair spends the entire capacity and leaves the ratio balanced. Starting with $1/8$, the same exposure cannot be repaired: capacity remains $1/8$ and the ratio becomes two. A balanced condition and a positive reserve answer different questions.

Proposition 15 (Funding the next protected checkpoint). *For (15) with $x_t \geq 1$, the completed interval does not increase incident cost exactly when*

$$c_t \leq E_t + q_t.$$

Nonincreasing cost together with strictly positive remaining capacity holds exactly when this inequality is strict.

Proof. If repair is funded, the new cost is $J(1) = 0 \leq J(x_t)$ and the remaining capacity is $E_t + q_t - c_t$. If it is unfunded, the new ratio is $2x_t > x_t$. The function J is strictly increasing on $[1, \infty)$, so cost rises. These are the two branches of (15). $\square$

Starting from $x_0 = 1$, every protected checkpoint is balanced, so every repair costs $1/4$. Repeated substitution gives the exact conditions

$$E_0 + \sum_{t < k} q_t - \frac{k}{4} > 0 \quad (1 \leq k \leq N). \quad (16)$$

The full set of prefix inequalities is necessary and sufficient for nonincreasing completed-interval cost and positive capacity throughout all N intervals. This is a joint condition: after any failed repair, the next opening ratio exceeds one and its repair no longer costs $1/4$. The forward direction uses Proposition 15 at each step. Conversely, the inequalities fund the first repair with positive capacity, then each successive repair by induction. A large final delivery cannot compensate for a missed earlier condition.

The inequalities apply to every represented person, including the helper and a bystander. They specify a physical protection target at the completed checkpoints. Exposure itself still raises the burden before repair. Splitting the interval into exposure and repair preserves its outcome and expenditure, but retains that intermediate effect.

9.2 An undertaking to maintain care

Consider four intervals with three identified meals already prepared from one recipe, and three separate questions: which meal can be used, how much work Alice has accepted, and whether assisted intake is authorized. Each feed consumes one meal, gives Ben $1/2$ capacity and costs Alice $1/4$ capacity in work. Every funded maintenance response costs another $1/4$ to its own person. Ben performs no work. Alice holds one work grant covering at most two work intervals, with at most one job in any interval, and assisted intake has separate authorization. The application permits no credit requests: a meal must be in stock when it is used, and work must be permitted and within capacity when it is done.

We say a person is *maintained* at a checkpoint when the completed ratio equals one and the capacity is strictly positive. This requires both the completed repair and the capacity to continue.

Both ratios initially equal one. Let capacities start at $(E_A, E_B) = (2, 1/4)$. Feeding in the first and third intervals, and waiting in the others, gives Figure 12. Each checkpoint is balanced with positive capacity. Two meals have been consumed, one remains, and Alice has used her two accepted work intervals. The remaining meal does not extend that grant.

Suppose Alice has also accepted an undertaking to keep Ben maintained at checkpoints 1, 2, 3 and 4. The terms specify a condition for Ben, rather than a feeding schedule. Here uptake is assisted

and the internal response is automatic under the stated law, so Alice's provision can secure that condition. When Ben controls uptake, the distinction in Section 7 applies. Equation (16) determines what provision can meet the condition, and in this application it takes an exact form that we can state for both people at once.

Theorem 16 (Maintenance throughout a care history). *Start the process (15) with both ratios equal to one and capacities H for Alice and P for Ben. For a finite sequence of intervals, let $j_t \in \mathbb{N}$ be the number of feeds in interval t and $n(k) = \sum_{t \leq k} j_t$ the number of feeds in the first k intervals. Both people are maintained at every positive checkpoint if and only if, for every k up to the length of the sequence,*

$$H - \frac{k + n(k)}{4} > 0, \quad P - \frac{k}{4} + \frac{n(k)}{2} > 0. \quad (17)$$

Physical validity and authorization of the action sequence are separate requirements.

Proof. At a maintained opening the exposure takes the ratio to 2 and the repair cost is $c = J(2) = 1/4$. An interval with j feeds supplies net provision $q = -j/4$ to Alice and $q = j/2$ to Ben. By (15), Alice is repaired with strictly positive capacity exactly when $E_A - j/4 - 1/4 > 0$, and Ben exactly when $E_B + j/2 - 1/4 > 0$; when these hold, the successor capacities are those two expressions and both ratios return to one.

Induct on k . Suppose both people are maintained at checkpoints $1, \dots, k-1$ with capacities $H - (k-1 + n(k-1))/4$ and $P - (k-1)/4 + n(k-1)/2$, which is the opening state when $k=1$. Then both are maintained at checkpoint k exactly when the two inequalities (17) hold at k , and in that case the capacities at k are the displayed expressions with $n(k) = n(k-1) + j_k$. If all prefix inequalities hold, this gives maintenance at every checkpoint. If some prefix fails, take the least such k : the earlier checkpoints are maintained with the displayed capacities, and the actual successor at k is either unrepaired or repaired with zero capacity. The induction does not assign it a fictitious paid repair, so the sequence is not maintained throughout. $\square$

The test retains every checkpoint. A delivery after a missed checkpoint cannot make the earlier capacity positive. It also retains the helper: successful recipient maintenance alone does not establish that the whole arrangement was maintained.

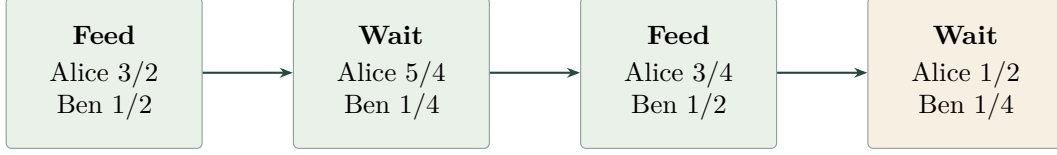
Theorem 17 (The complete four-interval requirement). *In the stated opening, with $H = 2$ and $P = 1/4$, consider any physically valid four-interval history and let j_1, j_2, j_3, j_4 be its numbers of feeds. Both people remain maintained throughout exactly when*

$$j_1 = 1, \quad j_2 + j_3 = 1, \quad j_4 = 0. \quad (18)$$

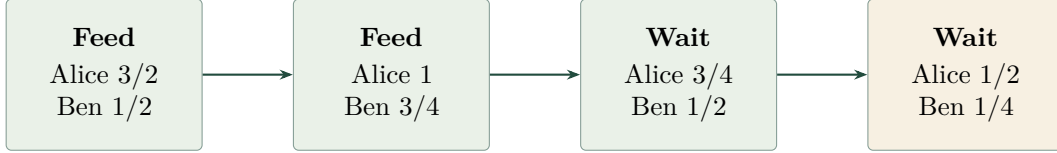
No valid history of five intervals maintains both people under this same opening and work grant.

Proof. Validity gives $j_i \leq 1$ and $\sum_i j_i \leq 2$: each feed uses one work interval, at most one job is permitted per interval, and the grant covers two. Substituting $H = 2$ and $P = 1/4$ into (17) gives the conditions $k + n(k) < 8$ and $k < 1 + 2n(k)$. Ben's first checkpoint requires $1 < 1 + 2n(1)$, so $j_1 \geq 1$ and hence $j_1 = 1$. His third requires $3 < 1 + 2n(3)$, so $n(3) \geq 2$; the grant then forces $j_2 + j_3 = 1$ and $j_4 = 0$. Conversely, these equalities give $n(1) = 1$, $n(2) \in \{1, 2\}$, $n(3) = n(4) = 2$, which satisfy $k < 1 + 2n(k)$ at $k = 1, 2, 3, 4$ and $k + n(k) \leq 6 < 8$ throughout. At a fifth checkpoint, $n(5) \leq 2$ makes $5 < 1 + 2n(5)$ impossible. The feed-wait-feed-wait schedule of Figure 12 is a valid maintained history, so the classified set is not empty. $\square$

Timing A: feed, wait, feed, wait



Timing B: feed, feed, wait, wait



Both timings start at $(2, 1/4)$ and keep both ratios equal to one.
The first feed is required immediately. The second can occur in interval 2 or 3.

Figure 12: The two timings that maintain both people. Each card gives the capacity after that interval's response. Both timings consume two of the three meals and use both work intervals of the grant.

Figure 12 shows the two schedules that (18) allows. Feeding in the first interval is a necessary means of performing this undertaking among the available choices. The second delivery has some timing freedom: interval two or interval three. At each feed, any meal still in stock can be used. Each is consumed once. Extending care to a fifth maintained interval requires a change to the available arrangement. The theorem classifies performance of the stated care requirement; whether an undertaking binds also depends on its authority and the history, taken up below.

Searching every legal history. The classification came from the prefix conditions by hand. It can also be checked by enumeration, because this application's actions are few. A legal action here is one that passes the physical validity tests: the named meal is in stock, care authority and work permission are present, the worker has the capacity the work requires, each unit is accounted once, and the action meets the conditions in Appendix A. Beyond idling and feeding a named meal, a legal action's record can differ only in annotations, such as naming the work grant without drawing on it. Normalization erases unused grant annotations and keeps the grant reference exactly on the action that performs work. In this application every legal action normalizes to one of four menu actions: idle, or feed with meal 1, 2 or 3. The normalized action is legal from the same state and produces the same successor in every component: resources, allocations, work used, prior breaches and both people's response states. Applying normalization interval by interval therefore preserves validity and every checkpoint.

Theorem 18 (Finite menu search for maintained histories). *In this application, fix the opening state, a finite length N and any set S of the represented people. A legal history of N intervals that keeps every person in S maintained at every positive checkpoint exists if and only if one exists whose every action is a menu action. A finite search can therefore decide existence by checking the menu histories for legality and maintenance.*

Proof. Every legal maintained menu history is already a legal maintained history. Conversely, let $a_1, \dots, a_N$ be a legal history from the opening state s_0 with successor states $s_1, \dots, s_N$. Legality of

an action is determined by the action and the state it acts from. Replace a_1 by its normal form: it is legal at s_0 and its successor is exactly s_1 . Since a_2 is legal at s_1 , it can be replaced by its normal form with successor exactly s_2 , and so on through a_N . The normalized history is legal, consists of menu actions, and passes through the identical states $s_1, \dots, s_N$. Being maintained at a checkpoint is a property of the state at that checkpoint, so every person maintained by the original history is maintained by the normalized one. At fixed finite length there are finitely many menu histories, each checked against validity and the maintenance condition. $\square$

This search retains adverse legal outcomes, including histories that leave Ben unmaintained. Legality alone does not establish a duty judgment. An implementation may certify existence as soon as it finds a valid maintained witness; a negative answer requires complete search or a proof excluding every legal history. If a search stops without either, the answer remains unverified. The menu belongs to this application: another recipe, stock or permission system requires its own completeness argument.

The history determines whether these accepted terms still apply. An authorized release removes their future requirement. Alice's unilateral statement that she is released does not. A release after an already missed checkpoint leaves that missed performance in the history. None of these events supplies permission to administer food. The same physical choice can consequently have different contractual consequences in different histories.

The helper's condition can make the requirements incompatible. If Alice instead starts with capacity $1/2$, the first feed costs her $1/4$ in work and $1/4$ in repair, leaving zero; in (17) the helper's inequality at $k = 1$ reads $1/2 - 2/4 > 0$ and fails. Ben meets his care condition, but Alice loses positive capacity. Waiting protects Alice's capacity while failing Ben's accepted condition. In this menu no choice both fulfills the accepted condition for Ben and preserves the stipulated continuation target for Alice. The account identifies whose requirement each alternative fails; adding the capacities would conceal the conflict.

9.3 A choice can remove the means of finishing

An arrangement can lose its remaining care possibilities before either person loses positive capacity. Keep the same food, recipe, work grant and care authority, but start Alice with capacity 1 and Ben with capacity 2. Ask for three maintained checkpoints. Waiting at all three is a legal solution: both ratios stay balanced, and the final capacities are $1/4$ and $5/4$.

Feeding at the first interval is also legal. It leaves Alice at $1/2$ and Ben at $9/4$, both maintained. Yet Alice cannot remain maintained through the two remaining intervals. Their repairs alone use her remaining $1/2$; each further food job costs her more work. The first feed destroys a completion that was available at the same opening. This example concerns three maintenance checkpoints, separately from the four-checkpoint undertaking above.

The distinction can be stated for the general renewed transition, with any specified participants and finite horizon. Write $T(s, a)$ for the full successor of state s under action a , and $K(s)$ when every selected participant has a balanced ratio and positive capacity. Let $A_n(s)$ mean that a legal sequence of n actions maintains those participants at every successor checkpoint. The empty sequence gives $A_0(s)$ for every s . Splitting a nonempty history at its first action gives

$$A_{n+1}(s) \iff \exists a [\text{Legal}(s, a) \wedge K(T(s, a)) \wedge A_n(T(s, a))]. \quad (19)$$

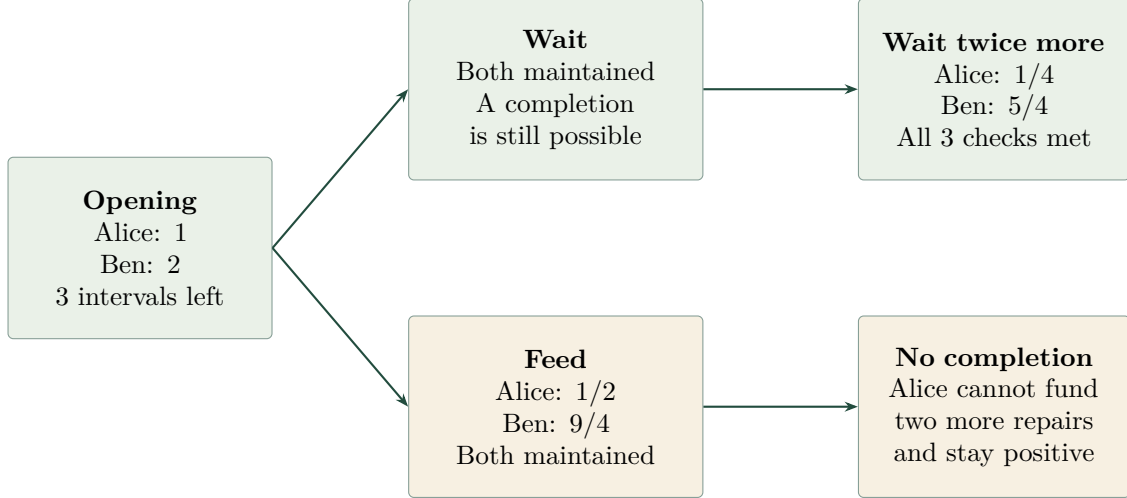


Figure 13: The immediate checkpoint alone does not reveal the lost option. Both first actions preserve both people’s present maintenance. Their remaining physical possibilities differ. The lower branch exhausts Alice’s capacity even if no more work is requested.

An action retains the remaining completion when it satisfies the conjunction on the right. It removes an available completion when $A_{n+1}(s)$ holds, the action is legal, and that conjunction fails for it.

Proposition 19 (Locating a created loss of completion). *Suppose a legal history of length N starts at a state satisfying A_N but fails to maintain the selected participants throughout. Along that history there is an action, preceded by a legal maintained prefix, that removes an available completion. At the state before that action, a different legal action retains completion for the same remaining horizon.*

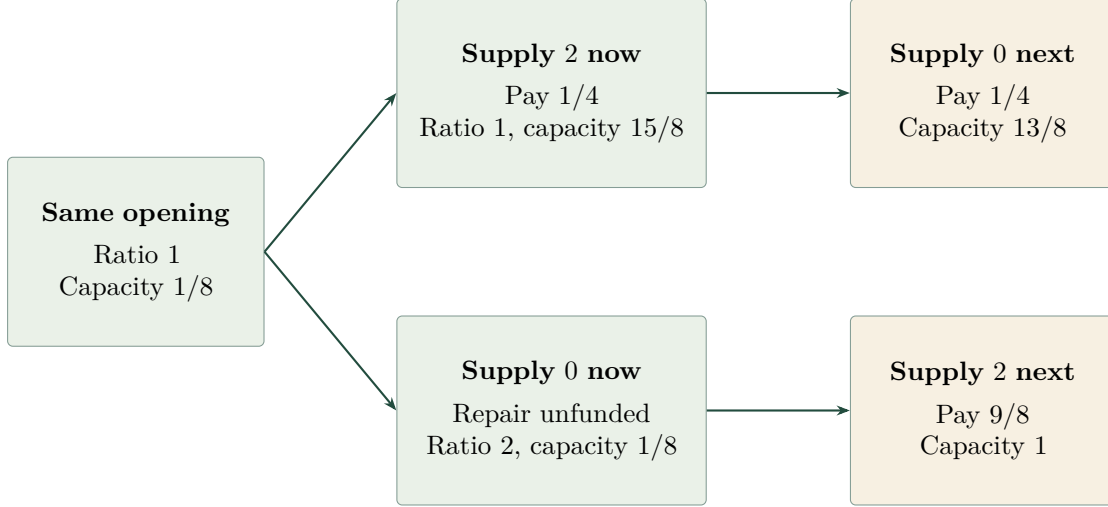
Proof. At the opening, Equation (19) supplies a retaining action. If the history’s first action does not retain completion, it is the required action. Otherwise its successor is maintained and has an available completion for $N - 1$ steps. Apply the same argument to the tail. The procedure must stop: retaining completion at every step would maintain the entire given history. At the stopping state, Equation (19) supplies a retaining alternative. It differs from the chosen action because the chosen action does not retain completion. $\square$

The economic account can therefore distinguish an inability present at entry from an inability created by a particular choice. The complete menu above preserves this distinction because it preserves the entire successor. Attributing responsibility further requires the actor’s control and knowledge, the authority of the undertaking and any competing requirements. Loss of a possible completion supplies a physical fact for that judgment; it does not by itself assign blame.

9.4 Recovery retains the cost of waiting

Now compare two intervals for one recipient, starting from $(x_0, E_0) = (1, 1/8)$. One history supplies two net capacity units immediately and zero next; the other supplies zero first and two next. Both receive the same total and finish with ratio one. Their remaining capacities differ.

The first history pays $1/2$ in total; the second pays $9/8$. The extra $5/8$ is the cost of delay in this example. Its calculation uses the following general account. Let $p_t = c_t$ when repair is funded and



Both finish at ratio 1. Here delaying provision uses 5/8 more capacity.

Figure 14: Recovery does not refund expenditure. The lower path carries its unrepaired ratio into the second exposure, which takes it to four and costs $J(4) = 9/8$ to repair.

$p_t = 0$ otherwise. Equation (15) gives

$$E_{t+1} + p_t = E_t + q_t, \quad (20)$$

$$J(x_{t+1}) + p_t = J(2x_t). \quad (21)$$

Summing yields

$$E_N + \sum_{t < N} p_t = E_0 + \sum_{t < N} q_t, \quad (22)$$

$$J(x_N) + \sum_{t < N} p_t = J(x_0) + \sum_{t < N} [J(2x_t) - J(x_t)]. \quad (23)$$

For two histories with the same opening, total net provision and final incident cost, the difference in final capacity is therefore exactly the opposite of the difference in repair payments. It also equals the opposite of the difference in accumulated exposure increments. An account retaining only the recovered condition loses both the earlier burden and what recovery used.

Burden comparisons retain the same person's opening, every completed checkpoint and each matched exposed state. On $[1, \infty)$, strict monotonicity gives

$$J(2x_t) \leq J(2y_t) \iff J(x_t) \leq J(y_t).$$

Thus a cost ordering at every completed checkpoint, including the opening and final checkpoint, also gives the ordering at the exposed states. This recovers those specific phases under the doubling law. Capacity remains a separate comparison, and other intermediate times require their own physical description.

9.5 Burden present and payment made

The physical moral framework's finite-supply argument counts costs against a reserve that actually pays them. A cost reading and a payment coincide here when repair succeeds. Failed repair separates

them.

To test whether one cost reading per interval can stand in for expenditure, define

$$S_N = \sum_{t < N} J(2x_t), \quad P_N = \sum_{t < N} p_t, \quad U_N = \sum_{t < N} J(x_{t+1}).$$

U_N retains costs left after each response, including costs that a later repair removes. Summing (21) and using (22) gives

$$S_N = P_N + U_N, \quad E_N + S_N = E_0 + \sum_{t < N} q_t + U_N. \quad (24)$$

Proposition 20 (When the sampled sum equals payment). *For (15) with $x_0 \geq 1$, the bound*

$$E_N \leq E_0 + \sum_{t < N} q_t - S_N$$

holds exactly when every repair through interval $N - 1$ is funded.

Proof. Every later ratio is at least one. Each term of U_N is nonnegative and is zero for funded repair. An unfunded repair leaves ratio $2x_t \geq 2$, so its term is strictly positive. Hence $U_N = 0$ exactly when all repairs are funded. Equation (24) makes the stated bound equivalent to $U_N \leq 0$, and therefore to $U_N = 0$. $\square$

For the delayed history in Figure 14, $S_2 = 11/8$, $P_2 = 9/8$ and $U_2 = 1/4$. The reserve paid for the later repair; it did not pay the earlier reading as a separate charge. Treating all $11/8$ as already spent would understate the remaining capacity by $1/4$.

S_N reads the burden once per interval at its exposed state. P_N records what the reserve actually paid. They agree exactly when every repair is funded. When repair fails, the cost reading remains in S_N but no payment leaves the reserve; U_N retains the difference. Identifying that reading with a recurring physical expense requires the actual clock, paying reserve and transfers of capacity. The reading alone creates neither another debit nor a debt to another person. A complete duty judgment must connect these histories to actual authorship, permission, obligations and every affected subject under the governing moral law.

10 Information can be a necessary input

Suppose two possible histories have the same current stock. In history w_1 , delivery undertaking q_1 is already fulfilled and q_2 remains due. In w_2 , the reverse holds. One remaining physical unit must be delivered and credited to the outstanding undertaking. Each history has a feasible plan. If the decision-maker sees no difference, choosing the correct credit in both is impossible.

A world w specifies the physical model, history, observation protocol and initial state y_w . A nonempty set $\mathcal{W}$ specifies the uncertainty. The meanings of persons and actions are shared across worlds. Security for n intervals means

$$\exists \pi \forall w \in \mathcal{W}, \quad \text{Ctrl}_n^w(\pi, \emptyset, y_w). \quad (25)$$

The same policy occurs before the universal quantifier. A different policy selected with knowledge of each hidden world would answer a different question.

For a controller turn in world w , define the necessary first-action set

$$N_w = \{a \in L_c^w(\emptyset, y_w) : \exists \psi, \text{Ctrl}_{n-1}^w(\psi, h_w^+, F_{H_w}(y_w, a))\}.$$

The tail policy ψ may depend on the world, so membership is necessary but not sufficient for a common policy.

Theorem 21 (Indistinguishable histories can exclude security). *Let $n \geq 1$. Suppose two worlds $w_1, w_2 \in \mathcal{W}$ begin on controller c 's turn with the same observation, and $N_{w_1} \cap N_{w_2} = \emptyset$. No policy satisfies (25).*

Proof. If a common policy existed, equal initial history and observation would give the same full first action a in both worlds. The controller branch of (12) makes a legal and supplies a controlled tail in each world. Taking the common policy itself as ψ gives $a \in N_{w_1} \cap N_{w_2}$, a contradiction. $\square$

The action includes performance credits and work-grant choices. Identical physical routes can therefore conceal incompatible actions. The theorem establishes an information obstruction even when the material is sufficient.

The conclusion depends on the available action language. If a trusted mechanism can execute “credit the outstanding undertaking” using its own access to the old record, that conditional operation can remove this particular obstruction for Alice. The mechanism’s observation, authority and physical cost then belong in the model. The explicit-credit example puts that choice with Alice; it does not establish that every institution needs a manual inspection.



Figure 15: Paid inspection changes what the controller can distinguish. It must leave enough time, permitted work and reserve for the subsequent delivery.

In the executable meal instance, Alice has two turns before Ben’s uptake turn. Inspection occupies the first; delivery and the correct credit occupy the second. The policy succeeds when the information arrives in time, Alice has sufficient reserve and work allowance, and the recipient’s response menu supports continuation. With initial worker reserve 5, inspection followed by delivery exhausts the worker by the final checkpoint. A one-unit work grant also excludes the two required work intervals. Allowing Ben’s fatal waiting defeats the nourishment guarantee even after successful inspection.

Proposition 22 (Consistency under forgotten information). *This checks a stipulated refinement of the permission rule. Suppose refined observations $\tilde{o}_p(y)$ have a common forgetting map f with $f(\tilde{o}_p(y)) = o_p(y)$ in every world. Keep the physical model, action ownership and turn schedule fixed. Define refined permissions by applying the original permission rule to f of the current observation and to f of every remembered observation. Then any securing policy π remains securing under*

$$\tilde{\pi}(\tilde{h}, \tilde{o}) = \pi(f[\tilde{h}], f(\tilde{o})).$$

Proof. For corresponding histories, refined legality equals original legality: ownership and the physical step are unchanged, and permissions were defined by forgetting. The lifted policy chooses the original action. Remembering then forgetting equals forgetting then remembering. Induction on (12) therefore preserves both the controller branch and the nonempty, universally quantified response branch. The same f lifts one common policy across all worlds. $\square$

This proposition holds the physical problem fixed. An actual inspection adds an action and its costs, as in Figure 15; its affordability requires a separate calculation.

10.1 Checking a finite policy

For a finite set of worlds and actions, a search can construct a policy by trying each available action and checking every allowed response. It must make the same choice whenever the controller has the same private history and observation. If a trial fails, that choice is rejected and the next is tried. A successful policy is replayed from the initial state.

The meal examples were checked on 22 instances and compared with 96 separate enumerations of complete plans. The two calculations share the physical transition rules but use different searches. A success establishes a policy on the declared action menus. Exhaustive failure establishes failure on those menus under the universal response condition of (12). It says nothing about omitted actions or a narrower assumption about other people’s policies. A computation that reaches a time or other limit reports unknown. One failing branch is an example, not proof that every policy fails.

11 One decision joining provision and repair

Alice must deliver a meal to Ben. Two accepted delivery undertakings exist, each for one meal. One was fulfilled earlier, but Alice’s current information does not identify which. One fresh meal remains. Crediting it to an already fulfilled undertaking cannot discharge the other. The two possible histories have identical current physical states.

Four intervals remain. Alice can inspect the performance records, deliver the meal with the correct credit, and then either repair a shared entrance or wait. Ben owns the last action: consume the meal or decline. Inspection, delivery and repair each require one interval of Alice’s authorized work. The actual work grant permits all three; it requires none merely by being available. No service promise requires the entrance repair. Inputs, credit, permission and reserve all pass through the same transitions defined above.

The planning target is precise. One policy using Alice’s own information must reach a handoff where the same policy for Ben, consuming the delivered meal, secures joint continuation. Both of Ben’s legal responses must preserve the fulfilled deliveries and Alice’s positive reserve. This target preserves Ben’s viable option. It is weaker than Alice’s guarantee in (12), which remains checked separately against both responses.

Proposition 23 (A common policy can preserve an actor’s own option). *Fix distinct persons h, r , a nonempty set of worlds, and one policy for each person, common to all worlds. Suppose the helper’s policy supplies a legal k -interval prefix in every world, with Good at each checkpoint, and every prefix turn belongs to h . Retain the actual ending state and each person’s observation history. If the next turn belongs to r and the fixed recipient policy satisfies Ctrl_1^r there in every world, the*

two policies together supply a legal $(k + 1)$ -interval continuation in every world. No prefix action contains r 's labor or autonomous intake.

Proof. Use one policy profile, assigning the recipient's fixed policy to r and the helper's fixed policy to h . At the handoff, recipient control supplies a legal own action and a final state satisfying Good. Concatenate that step to the prefix at its actual ending state and remembered observations. Formally, induction on the prefix length proves trace concatenation: the empty prefix leaves the tail unchanged, and each longer prefix retains its first legal step before applying the induction hypothesis. Projection through the same physical transition gives continuation for $k + 1$ intervals. Ownership of each helper action excludes every other person's labor and autonomous intake. Both policies were fixed before quantifying over worlds, so no hidden choice of policy has been introduced. $\square$

The conclusion is joint continuation through the recipient's own action. If a different legal recipient response fails Good, the universal response clause of (12) refutes the helper's one-interval guarantee at that same handoff. The viable option and the absence of that guarantee can therefore hold together. A generic prefix containing other actors' turns would need evidence of their cooperation; the proposition explicitly confines the prefix to the helper.

Maintenance costs each person one reserve unit per interval. Each interval of inspection, delivery or repair costs Alice one further reserve unit. Alice starts with 8; Ben starts with 4. Consumption supplies Ben two reserve units. Inspection leaves their reserves at $(6, 3)$ and reveals which undertaking is still due. Delivery with the correct credit leaves $(4, 2)$. In the third interval, waiting leaves $(3, 1)$ and repair leaves $(2, 1)$. After Ben's final interval, the alternatives have reserves

	Ben consumes	Ben declines
Deliver, then wait	$(2, 2)$	$(2, 0)$
Deliver, then repair	$(1, 2)$	$(1, 0)$.

Inspection and delivery are included in both rows. The repaired entrance is available to both people during the final interval. Under the declared service interpretation, repair lowers each service burden by $1/4$; waiting leaves it unchanged. Both policies preserve the stated option, so the strict first comparison in (9) selects repair among these two candidates.

Now give Alice an initial reserve of 7. Inspection, delivery and repair would leave her at zero after Ben's interval. Repair is excluded even if Ben consumes. Inspection, delivery and waiting leave Alice at 1 and preserve Ben's viable option, so waiting is selected. If the grant permits only two work intervals, or repair lacks authorization, the same repair is excluded for a different reason. If inspection is unauthorized, or the records remain hidden after inspection, neither supplied policy succeeds. Separate complete enumeration confirms that no common worker policy from the declared menus preserves the option in the hidden-record case.

Ben's decline remains legal in all these comparisons. With initial reserve 4, decline defeats Alice's full-survival guarantee while leaving both delivery credits intact. With initial reserve 5, consumption and decline instead leave Ben at 3 and 1. The same worker policy then secures survival across both responses and both histories. The change is in physical reserve, not in the recipient's permissions.

The joined example was checked on nine cases and twelve comparisons with a separate enumeration of complete plans. Delivery credits and work use were also reconstructed from what the actions actually did. These checks concern the stated service interpretation. The example shows how past performance, costly information, worker capacity and the recipient's own choice can change the decision together.

The reserve examples leave a physical question: what keeps a person's supply running between checkpoints? A balanced inflow and outflow may still be too weak to provide the required service. The next model separates a sufficient rate from sufficient stored material. The maintenance calculation then determines how much can be transferred while both supplies continue. The response calculation accounts for the physical comparisons used to observe an answer. These are distinct realizations of the same cost law J , with their physical meanings stated separately.

12 A balanced flow may still provide too little

A sensor needs enough signal to distinguish two possible requests. A perfectly balanced supply can still feed it too weakly to overcome measurement error. Material balance answers whether material accumulates; useful service also requires a sufficient rate. Consider two compartments with amounts A and X , an outlet held at reference level 1, and rate constant $k > 0$. With source input $2k\alpha$, their balance equations are

$$\dot{A} = 2k\alpha - k(A - X), \quad \dot{X} = k(A - X) - 2k(X - 1).$$

Suppose the device permits settings $0 < \alpha \leq 1$, so its maximum source rate is $2k$. For any $\alpha > 0$, the state $A = 1 + 3\alpha$, $X = 1 + \alpha$ is stationary: inflow and outflow both equal $2k\alpha$. Substitution makes both derivatives zero. Consequently $J(\text{inflow/outflow}) = J(1) = 0$ at every positive setting, however small the delivered flow.

Suppose the output carries a fresh binary request as a signal of amplitude gq , where $q = 2k\alpha$ and $g > 0$ is the measured gain. An observation is $y = \pm gq + n$, with unknown error $|n| \leq E$. The possible intervals are disjoint exactly when $E < gq$. If this inequality holds, the sign of y decodes the bit. If $E \geq gq$, both requests can produce $y = 0$, so no decoder can always distinguish them. A remembered earlier bit cannot identify a fresh request.

For example, take a horizon $T = 1$, put $k = g = E = 1$, and give the source stock $F = 2$. The settings $\alpha = 1/4$ and $\alpha = 3/4$ both have positive balanced flows and enough stock for the entire interval: qT is respectively $1/2$ and $3/2$, both at most $F = 2$. Their signal amplitudes are $1/2$ and $3/2$. Only the second separates the requests. Zero cost of the flow ratio is therefore compatible with two different service capabilities. It is not the complete moral reading of either system.

Proposition 24 (Rate and stock are separate requirements). *Suppose the available stationary settings allow any constant rate $0 < q \leq 2k$, with its corresponding initial state already present. Let $T > 0$, $g, k > 0$, $E \geq 0$. A setting can preserve nonnegative source stock through T and distinguish every fresh request if and only if*

$$E < 2gk \quad \text{and} \quad ET < gF.$$

Proof. Source stock at time t is $F - qt$. Its minimum on $[0, T]$ occurs at T , so funding is equivalent to $qT \leq F$. Successful decoding requires $q > E/g$. An allowed rate therefore exists exactly when

$$E/g < \min(2k, F/T).$$

When this holds, the midpoint between E/g and that minimum is a witness. Multiplying by the positive g and T gives the stated conditions. $\square$

The inequalities are strict because overlapping observation intervals fail at equality. Extra stock cannot overcome an inadequate maximum rate. These stationary alternatives do not let an interrupted system jump to equilibrium: recovery requires the actual intervening trajectory and its inputs, as developed in the companion recovery calculation [2].

13 Provision with a protected source

A provision duty must identify what sustains the recipient, who controls that supply, and what the same act does to the source. Consider two maintained regimes, each requiring two units of material over the next unit of time. If their feed tanks contain five and one units, respectively, transferring between one and three units preserves both regimes. Transferring nothing interrupts the recipient; transferring four units leaves the source unable to continue. The following model derives these bounds while keeping control, personal effects, and available material distinct.

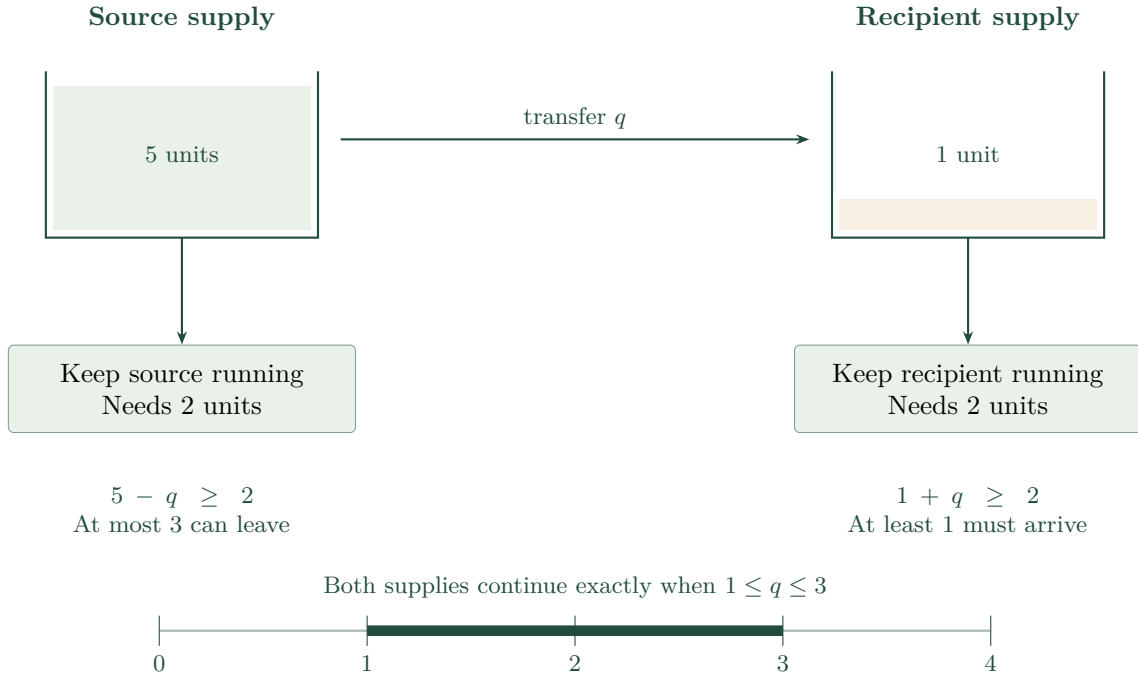


Figure 16: The amount of help has a lower and an upper bound. These are the feed tanks for the controlled maintenance example, with $k_h = k_r = T = 1$. Authorized transfers at time zero in the marked interval preserve both regimes; a later arrival must also meet the deadline in (32). The person's costs are read from the maintained compartments; the tanks determine how long those compartments can be sustained.

13.1 A supply that must keep running

The feed tank supplies material to two connected compartments. Maintaining their reference levels requires a continuous feed. Closing the valve saves tank stock while the compartments depart from those levels. The model records both facts. Each regime has an inlet compartment A , an interior compartment X , a feed tank F , and cumulative removed material R . The outlet is held at $B = 1$ by removal. For conductance $k > 0$ and valve input u , the laws are

$$\dot{A} = u - k(A - X), \quad \dot{X} = k(A - X) - 2k(X - 1), \quad \dot{F} = -u, \quad \dot{R} = 2k(X - 1). \quad (26)$$

The initial state is $(A, X, F, R) = (4, 2, f, 0)$. A maintained valve supplies $u = 2k$; an interrupted valve supplies $u = 0$ from time zero. Their actual paths are

$$M_k(f, t) = (4, 2, f - 2kt, 2kt), \quad (27)$$

$$I_k(f, t) = \left(1 + \frac{3 + 2\rho}{2}s + \frac{3 - 2\rho}{2}v, 1 + \frac{1 + \rho}{2}s + \frac{1 - \rho}{2}v, f, 6 - A(t) - X(t)\right), \quad (28)$$

where $\rho = \sqrt{2}$, $s = e^{(\rho-2)kt}$, and $v = e^{(-\rho-2)kt}$. Substitution verifies the initial state and (26). Both paths conserve $F + A + X + R = f + 6$. On the interrupted path, $A > 1$; also $X - 1 = s + (\rho - 1)(s - v)/2 > 0$, because $s \geq v > 0$ for $t \geq 0$. Hence $\dot{R} \geq 0$ and $R(0) = 0$ give $R \geq 0$. For $t > 0$, $A < 4$: both coefficients of $A - 1$ are positive and both exponentials lie below one.

Write h for the source regime and r for the recipient. A transfer $q \geq 0$ at time zero changes their feed tanks from (F_h, F_r) to $(F_h - q, F_r + q)$. The four offered valve paths are (M_h, M_r) , (M_h, I_r) , (I_h, M_r) , and (I_h, I_r) , with the appropriate transferred initial stocks. Physical eligibility requires nonnegative tanks and removed material and positive compartments throughout $[0, T]$, where $T > 0$. Separate authorization specifies which transfers and valve operations the controller may perform.

13.2 Whose condition does the supply support?

Here a patient is a subject whose represented condition depends on a continuing supply. The supplied compartments model that condition, and the tank holds the material needed to maintain it. Three persons appear: a controller c who operates both valves, a source patient h , and a recipient patient r . Two bonds join c to each patient. The RS cost law is fixed; the physical interpretation specified here reads patient i 's bond ratios as $A_i/4$ and $X_i/2$. Thus the reference regime $(4, 2)$ reads as ratio one on both. Applying the cost law to this physical interpretation gives

$$J(x) = \frac{x + x^{-1}}{2} - 1 = \frac{(x - 1)^2}{2x}, \quad C_i = J(A_i/4) + J(X_i/2), \quad C_c = C_h + C_r. \quad (29)$$

These are the full incident costs of this specified set of relations. All three costs initially vanish. Harm to person j is the change in C_j from that actual baseline. The feed tanks appear in no bond. On every maintained path all ratios stay at one, even for a transfer that empties the source tank exactly at T . Protection of its continuing supply therefore enters through physical eligibility over $[0, T]$. This chart gives no preference for leaving extra stock after that horizon. The admission conditions of Section 5 require the stated author, nonincrease of $\Sigma = \sum_e |\log x_e|$, and nonpositive harm for each protected person. All three persons are included; all changed bonds are incident to the controller. The controller's incidence does not confer ownership of either patient's resources. Authorization remains a separate requirement. Because $C_c = C_h + C_r$ and both patient costs start at zero, protecting the controller already excludes either patient's interruption in this example. The separate patient inequalities retain who bears each effect. The tank constraint has a different role: it determines whether the admitted maintained path can actually continue for the required duration.

13.3 How much must move, and by when?

Let $k_h, k_r, T > 0$ and $F_h, F_r \geq 0$. Evaluate harm at the terminal state T relative to the actual initial state. Among these four paths, admission is equivalent to maintaining both valves. An authorized transfer q makes that path physically eligible exactly when

$$\max\{0, 2k_r T - F_r\} \leq q \leq F_h - 2k_h T. \quad (30)$$

The interval is nonempty exactly when

$$F_h \geq 2k_h T, \quad F_h + F_r \geq 2(k_h + k_r)T. \quad (31)$$

If the recipient has $F_r < 2k_r T$, every eligible admitted plan supplies a strictly positive transfer. If total stock is below $2(k_h + k_r)T$, no offered path is both eligible and admitted.

Proof. Maintaining leaves every multiplier equal to one, hence leaves every person's cost and Σ unchanged. Interrupting either arm gives $A_i(T)/4 \neq 1$, so $J(A_i(T)/4) > 0$. Both that patient's incident cost and the controller's full incident cost rise. The controller is among the protected persons, so its positive harm alone excludes each of the other three paths. For the remaining path, the feed tanks are $F_h - q - 2k_h t$ and $F_r + q - 2k_r t$. They decrease in t , so nonnegativity throughout the interval is equivalent to their terminal inequalities. Together with $q \geq 0$, these give (30); the other physical conditions follow from (27). Its upper endpoint is at least its lower endpoint exactly under (31). When these hold, $q_* = \max\{0, 2k_r T - F_r\}$ constructs a feasible transfer. A deficient recipient makes $q_* > 0$. Adding the two terminal stock inequalities proves the shortage conclusion. $\square$

For $k_h = k_r = T = 1$ and $(F_h, F_r) = (5, 1)$, the interval is $[1, 3]$. Both $q = 1$ and $q = 2$ satisfy every condition when authorized. For the same rates and $(F_h, F_r) = (2, 1)$, the required interval is $[1, 0]$ and total stock is three against a requirement of four. Shortage empties the eligible admitted set. It supplies no admission for an interrupted patient.

This conclusion also controls intermediate times: the admitted maintained path has the reference multipliers at every time, and both tanks remain nonnegative at every prefix. If transfer instead arrives at $\tau \in [0, T]$ while both regimes remain maintained, one additional condition is necessary and sufficient:

$$2k_r \tau \leq F_r, \quad \text{equivalently} \quad \tau \leq F_r / (2k_r). \quad (32)$$

Before arrival the recipient has $F_r - 2k_r t$; after arrival it has $F_r + q - 2k_r t$. The terminal source bound already protects its earlier stock. Thus the funded example permits arrival at $\tau = 1/2$, but not $\tau = 3/4$. An adequate eventual transfer cannot replace timely delivery.

From admission to selected restoration. For a finite offered menu, rank only authorized, physically eligible, admitted plans. The selection rule from Section 5 first minimizes greatest personal harm, then maximizes the stated welfare, then minimizes absolute strain. Equivalently, minimize lexicographically

$$\left(\max_j \{C_j(p) - C_j(0)\}, \quad -\sum_j (1 - e^{C_j(p)}), \quad \Sigma(p) \right). \quad (33)$$

The sums and maximum use the declared person list. In the two-regime menu, all admitted funded plans have the same costs and absolute strain. Consequently every offered authorized transfer in (30) is selected; the ranking fixes the valve behavior but leaves those transfer quantities tied.

The following additional result addresses menus containing admitted plans with genuinely different personal effects. The maintenance menu just analyzed has no such strict comparison: its admitted plans all preserve the same reference. A recovery model or a fuller physical cost chart must establish the strict-improvement premise before this result can require restoration.

More generally, let restoration be a specified physical property of plans. Suppose at least one eligible admitted plan exists, and every eligible admitted nonrestoring plan p has an eligible admitted restoring alternative z satisfying $C_j(z) \leq C_j(p)$ for every listed person, strictly for at least one.

Then a selected plan exists and every selected plan restores. Indeed, the inequalities weakly improve the maximum harm and strictly lower the negative-welfare coordinate in (33), by strict monotonicity of the exponential. Either the first score improves or its tie is broken by the second. Thus p cannot be selected; finiteness gives a best eligible admitted plan. This argument includes the helper and affected third parties in the same comparison. For actions on the controller’s own incident bonds, including every changed bond endpoint in the person list protects everyone represented in the ledger: anyone outside that list has unchanged incident bonds. For a recovery path, admission must also hold against each successive actual state. A favorable terminal comparison alone does not supply that fact.

Composition with several recipients. The resource result extends constructively to m distinct recipient regimes with separate pairs of incident bonds and one protected source. Assume the same dynamics and initial reference in every arm, and simultaneous transfers $q_i \geq 0$ exclusively from the source to recipients. The exact material condition for maintaining all arms is

$$\boxed{\sum_{i=1}^m \max\{0, 2k_i T - F_i\} \leq F_h - 2k_h T.} \quad (34)$$

Proof. Each recipient requires $q_i \geq \max\{0, 2k_i T - F_i\}$, while the source requires $\sum_i q_i \leq F_h - 2k_h T$. Summing proves necessity. Setting each q_i equal to its lower bound proves sufficiency at every time, because each tank decreases linearly. The preceding personal-harm argument forces every valve to remain maintained. $\square$

This condition respects the transfer topology. With $T = k_h = k_1 = k_2 = 1$ and stocks $(F_h, F_1, F_2) = (2, 0, 4)$, total stock meets total demand, yet the source has no surplus and recipient 1 lacks two units. A permitted route from recipient 2 would change the feasible set; pooling totals alone does not. Attaching any finite set of unchanged surrounding bonds preserves the admission conclusion: their contributions to each person’s cost and to Σ are constant and cancel from every change. This embeds the result in a larger ledger without assigning new personal weights.

The application to actual human provision requires identifying its physical variables, incident personal effects, and genuine controller authority. The present chart assigns feed stocks to material feasibility and the two compartment ratios to personal cost. Labor, depletion consequences beyond T , and any changing third-party bonds require their own actual incidence and paths. Under the componentwise-improvement condition stated above, every selected plan restores. This is a consequence of the specified decision rule. Establishing a duty of the controller to provide that restoration requires an interpersonal argument: the continuation result of Section 2.3 concerns the agent’s own continuation. The model supplies the physical alternatives and their represented effects to that argument.

14 What the response record costs

Suppose a service must keep a recorded yes-or-no answer current. A late answer and a prompt answer can end at the same correct value. Their records differ in how long the wrong value remained. To measure that difference, compare the output with the request at each stated observation time. For one fixed request b and output p_t , assign positive codes $c(b)$ and $c(p_t)$. The comparison cost at

boundary t is

$$j_t = J\left(\frac{c(p_t)}{c(b)}\right).$$

In the two-endpoint allocation used here, each endpoint receives half of that standing charge. Over N equally spaced recorded boundaries its accumulated charge is

$$\Gamma_i(N) = \frac{1}{2} \sum_{t=0}^{N-1} j_t, \quad i = 0, 1. \quad (35)$$

This is a sum over the declared observation schedule. Changing that schedule changes the quantity; a continuous-time integral would need a time measure. This accumulated charge is not the incident cost or its signed change used in admission and selection; Γ_i enters neither condition here.

Let an initially wrong output become correct at boundary d and remain correct. Then

$$\Gamma_i(N) = \frac{\min(N, d)}{2} J\left(\frac{c(p_0)}{c(b)}\right).$$

There are exactly $\min(N, d)$ wrong states in the sum; all later terms vanish. By contrast, signed changes $j_{t+1} - j_t$ telescope to $j_N - j_0$. They cannot recover the waiting history. Neither expression changes the accepted obligations or erases a missed deadline in Section 4.

14.1 An actual comparison has two outputs

To give the comparison a resource account, specify a physical operation. For positive real operands a, b , put $v = (a, b)$ in a two-dimensional signal space. Let P project onto the equal-coordinate line and let $Q = I - P$. The exchange

$$(v, d) \mapsto (Pv + Qd, Qv + Pd)$$

conserves $u(\|v\|^2 + \|d\|^2)$ for a calibrated $u > 0$, because the P and Q components are orthogonal. With a blank second input $d = 0$, the two outputs contain

$$U_{\text{retained}} = \frac{u}{2}(a+b)^2, \quad U_{\text{exported}} = \frac{u}{2}(a-b)^2 = uabJ(a/b). \quad (36)$$

Their sum is the initial preparation energy $u(a^2 + b^2)$. The word “exported” refers to the second output. The energy still exists. Treating it as unavailable requires an actual access restriction; both outputs together permit the inverse exchange. This construction and its calibration are developed in [1].

Consider an independently prepared bank of executed binary comparisons, all with the same codes $c_0, c_1 > 0$ and calibration u . Matching operands contribute zero export. For a mismatch, their product is $c_0 c_1$. Summing (36) therefore gives

$$U_{\text{exported}} = 2uc_0 c_1 \Gamma_i = uc_0 c_1 (\Gamma_0 + \Gamma_1) \leq U_{\text{initial}}. \quad (37)$$

Here Γ_i sums the charges at the *executed comparisons*. It equals (35) only when an actual comparison was executed at every indexed boundary. A stored record alone does not establish that execution. The two endpoint shares account for one export, not two copies of the physical energy. They do not by themselves establish either person’s full incident moral burden.

Proposition 25 (Less export can require more preparation). *Encode false as 1 and true as 2. The fixed request is true; the initial output is false. Execute three independently prepared comparisons, at boundaries 0, 1, 2. Compare completion at boundary 1 with completion at boundary 2. Both records agree at boundary 3, but their accounts are*

	complete at 1	complete at 2
output codes at 0, 1, 2	(1, 2, 2)	(1, 1, 2)
Γ_i	1/8	1/4
U_{exported}	$u/2$	u
U_{initial}	$21u$	$18u$

Consequently a preparation budget $18u \leq B < 21u$ admits the later bank and excludes the earlier bank, although the earlier bank exports less.

Proof. An incorrect comparison has $J(1/2) = 1/4$ and each endpoint's share is $1/8$. A correct comparison has zero cost. Equation (37) then gives the exports. A preparation with operands (1, 2) requires $5u$; one with (2, 2) requires $8u$. The sums are $5u + 8u + 8u = 21u$ and $5u + 5u + 8u = 18u$. The budget conclusion follows directly. $\square$

This is a result about the specified bank, whose inputs are separately prepared without reuse. It is not a lower bound on every possible device or a reason to prefer delaying an answer. It shows why a resource filter must use the actual preparation and reuse protocol. Export, source expenditure, retained history and provision to a person are different facts.

A separate calculation connects prepared comparisons to the food example under bounded measurement error. It checks 26 comparison histories and 12 food continuations, including a comparison prepared but never executed. Ambiguous or contradictory readings remain unresolved. These checks establish the stated device calculations; a full moral assessment still requires the affected person's actual relations and condition.

Comparison time	0	1	2	
Earlier response	1 : 2	2 : 2	2 : 2	Prepared: $21u$ Exported: $u/2$
Later response	1 : 2	1 : 2	2 : 2	Prepared: $18u$ Exported: u

Each pair is a separately prepared physical comparison.
Under budget $18u \leq B < 21u$, only the later-response bank fits.

Figure 17: Less mismatch can require more prepared energy. The request is 2 throughout; the output changes from 1 to 2 after one or two intervals. Each cell shows output:request. In this device, preparing (2, 2) costs $8u$, while preparing (1, 2) costs $5u$. The reversal depends on these codes and separately prepared comparisons: interchanging the codes reverses the preparation comparison. Exported mismatch energy alone cannot decide which whole bank fits a budget.

15 Individual effects in 505 deliveries

The following routing calculation illustrates why the economic account keeps individual effects. It establishes feasibility and those effects under supplied operational rules; it does not instantiate every moral relation defined above. The Grubhub meal-delivery library and its routing study [6, 12] supply an operationally derived setting. The complete configuration `0o100t100s1p100` contains 505 requests, 116 restaurants and 113 courier records. It is constructed from historical operations with sampled orders and perturbed geography. These records supply routes and times; their labels do not authenticate persons, consent, physiological need or actual wages.

Two rolling dispatchers receive the same placed requests and exact readiness information, future shifts, travel law and service times. Neither receives future unplaced requests. Both enumerate the same complete candidate menu at each dispatch. The greedy method favors larger bundles, then lower summed delivery time. The integer optimizer maximizes the number served at that dispatch, then minimizes summed delivery time. All 602 dispatch optimizations across the two optimizer runs returned optimal statuses. These are dispatch optima, not a full-day optimum.

Every scenario imposes a modeled 90-minute delivery limit. The original work rule permits delivery after shift end when pickup begins in time. A second, stricter rule requires the whole assignment to finish within shift. The stricter rule is a hypothetical work limit, not an inference about historical permission.

Rule and method	Served	Mean time	Active work minutes	Beyond shift
Original, greedy	505	31.8	12,032	392
Original, optimizer	505	31.0	12,535	414
Strict, greedy	500	33.4	11,718	0
Strict, optimizer	499	32.6	11,981	0

Table 1: Complete-instance modeled outcomes. Mean times are rounded to a tenth of a minute and include served requests only. All work figures aggregate courier records, whose individual outcomes are retained separately.

Under the original rule, the optimizer’s lower mean accompanies 53 later deliveries and lower modeled payments for 33 courier records. Active work rises by $12,535 - 12,032 = 503$ minutes and work beyond shifts by $414 - 392 = 22$ minutes. The aggregate improvement leaves these effects true. Comparing the strict optimizer with the strict greedy schedule, two requests become served while three previously served requests become unserved. The served-only means also compare different populations.

15.1 A complete repair and who bears its cost

A retrospective schedule repair inserts the five strict-greedy misses and retimes affected routes. It retains every previous courier assignment and each courier’s old bundle order. The resulting complete schedule serves all 505 requests within the delivery limit and strict shifts. Independent event replay checks readiness, occupied service time, route continuity, distinct deliveries and shift endings. The upstream evaluator provides corroboration under its weaker conditions.

Eleven existing deliveries become later, by 1 to 32 minutes, totaling 185 minutes. Five courier records carry 130 additional active minutes and 85 fewer waiting minutes, a net increase of 45

occupied minutes. Two finish later, by 22 and 2 minutes, within shift. Modeled payments increase by 40 units in total across affected couriers, with no decrease.

The schedule is a witness that all requests can fit these physical limits. It uses the completed day's information and therefore does not establish an equal-information online improvement. Its physical feasibility also leaves the historical question intact: were these later deliveries and added work allowed by the actual accepted terms? Constructed histories permit the same schedule under one set of terms and exclude it under another.

This application checks route timing, service, distinct delivery and shifts. Its 505 records do not supply calibrated reserves, intake or the complete production-and-history interpretation of Sections 3–4. The demonstrated capability is to find and check a repair while exposing its individual consequences. Evidence of improved lives requires prospective observation.

There is a precise partial connection to the general model. Requiring the entire assignment to finish within shift has the same interval-containment form as a work grant. A unique recorded delivery can supply a performance identity. Treating either as an accepted undertaking requires the corresponding authorship and terms, which the source data do not contain. The constructed-history checks supply those assumptions explicitly; a proof that the entire routing adapter instantiates the enriched transition remains to be established.

16 What must be established at an actual site

A deployment needs an identified boundary. Which actual people and assets are represented? Which inputs enter from outside? What does a resource slot mean? Are readiness and expiry measured or estimated? Which records establish an agreement, which show performance, and which remain disputed? The answers fix the model that a guarantee concerns.

For food provision, distinguish availability, delivery and intake. Calibrate work and reserve against the actual task, including the producer. For a shared entrance, establish access and continuous availability, including outages during work. A service available to two people can remain one physical asset; consumable performance requires a different allocation rule. The two settings share persons and histories while differing in how use consumes resources.

Uncertainty should retain those distinctions. Missing history is different from a known breach. Insufficient computing time is different from physical impossibility. A plan that depends on uptake should name that dependency. If a forecast or observation excludes an actual response, the guarantee covers the smaller stated model and must be revised when that exclusion is unsupported.

The prospective protocol begins with site mapping and rehearsal, then ten shadow days using the site's full workload. Recommendations are recorded without directing participants. A representative deadline test measures whether the analysis finishes in time; an interrupted analysis returns unknown and leaves the existing authorized process in place. A later assisted phase requires the site's and participants' actual authorization. Delivery, person-specific work, accepted changes, unmet requirements and earlier failures are recorded separately. No site has been enrolled and no human intervention is reported here.

The repair and maintenance examples establish choices that improve provision under their stated conditions. The maintenance model derives an adequate transfer interval while protecting the source. The assisted-care construction follows provision through separately funded responses. Repeated care derives the necessary funding at each checkpoint, classifies the successful timings, and

proves completeness of the finite action menu. A separate example locates the loss of an available completion even while both participants remain maintained. Conserving Love selects the least-cost endpoint compatible with each participant’s renewal budget. Accepted undertakings and earlier failures remain attached to the histories that establish them. These results concern the represented compartments, stocks, bonds and authorized actions.

Applying that result to people requires physical identifications: which relations carry their needs and burdens, how capacity and effects are measured, and how actual authority and history enter the action. Continued capacity alone leaves moral distinctions unresolved. False agreement, unpaid performance and an authorized release can differ even when every party remains able to act.

Two questions remain decisive for a complete duty account. When does available capacity make assistance required, including responsibility for the consequences of withholding it? What is required when every feasible course leaves someone’s need unmet? An empty set of eligible, admitted plans identifies a limitation of the represented choice; it neither makes physical moral law indeterminate nor erases the person who bears the loss. A prospective application must test whether the completed account improves provision while retaining the costs and obligations of everyone affected.

Computational evidence. The [computational materials](#) supply the food-policy programs, comparison calculations and routing procedures, with pinned inputs and reproduction instructions. The accompanying [verification record](#) reports the completed checks. These support the computational claims in the information, comparison and delivery examples. A separate [care calculation](#) reproduces the funded responses and capacity boundaries in Section 8. The [care histories and comparisons](#) extend those calculations through exhaustion, preserving each person’s effects and the distinction between physical reach and authority. The [repeated-care calculations](#) retain intake, work, accepted care and every represented person through renewed responses, phase comparisons and the separate accounts of repair payment and remaining burden.

A The complete action conditions

The rules below make four checks explicit: an item really crossed, it has not already been credited elsewhere, work stayed within its accepted terms, and promised service remained available. The enriched state and transition in Section 4 use the following finite data. Let U be canonical physical units and Q undertaking identities. Each resource slot has an owner $o(r) \in U \cup \{\perp\}$. Ownership here identifies the physical unit represented by a slot; permission to use it is the separate predicate $A_H(s, a)$. Accepted delivery terms specify quantity q^{qty} and deadline q^{due} . Accepted work terms specify worker, start, finish, maximum active work and a finite set of break intervals. Accepted service terms specify a person, an asset slot and a service interval. An absent active undertaking supplies no permission to work and imposes no contractual delivery or service requirement through this history reader. Physical need and duties arising through other relations remain questions for the complete moral account.

A.1 Crossing and allocating a unit

A located unit has a path $v_u : \mathbb{N} \rightarrow \mathbb{Z}^3$ which stays put or moves one lattice edge per tick. Its cube address at t is the coordinatewise integer quotient $\alpha_u(t) = \lfloor v_u(t)/2 \rfloor$. For undertaking q , let α_q, β_q be source and destination addresses and let r_{uq}^-, r_{uq}^+ be its source and destination slots. The crossing

condition is

$$\begin{aligned} X_{uq}(s, a) \iff & \alpha_q \neq \beta_q, \quad \alpha_u(t) = \alpha_q, \quad \alpha_u(t+1) = \beta_q, \\ & r_{uq}^- \neq r_{uq}^+, \quad o(r_{uq}^-) = o(r_{uq}^+) = u, \\ & I_{r_{uq}^-} = O_{r_{uq}^+} = x_{r_{uq}^-} = x'_{r_{uq}^+} = 1. \end{aligned}$$

Here x' is the physical update, including expiry. Identifying these locations and slots with an observed crossing is application data.

Write $N_q(\rho) = |\{u : \rho(u) = q\}|$. Process the action's credit requests $(u_1, q_1), \dots, (u_m, q_m)$ in their given order, starting at $\rho_0 = \rho$. Request i is valid exactly when

$$X_{u_i q_i}(s, a), \quad \rho_{i-1}(u_i) = \perp, \quad q_i \text{ has active delivery terms}, \quad N_{q_i}(\rho_{i-1}) < q_i^{\text{qty}}.$$

For a valid request, set $\rho_i(u_i) = q_i$ and leave all other allocations unchanged. Every request must be valid; the resulting allocation is ρ_m . The crossing is tested against the same physical batch, while unused status and outstanding quantity are tested against the sequentially updated allocation.

For each unit u , write $x[u] = \sum_{r: o(r)=u} x_r$, and use the same notation for inputs, outputs and ending stock. Canonical-unit soundness requires

$$x[u] \leq 1, \quad x'[u] \leq 1, \quad I[u] \leq 1, \quad O[u] \leq I[u].$$

These checks apply to named units. Ordinary production slots can have owner $\perp$; allocating an identity to a newly produced item requires a corresponding extension of the unit representation.

A.2 Work, service and due requirements

Let $\kappa(p)$ be the work grant chosen by the action and put

$$\Delta_q(a) = \sum_{p: \kappa(p)=q} \ell_p.$$

For every p with $\ell_p > 0$, there must be an active grant $q = \kappa(p)$ whose terms satisfy

$$\begin{aligned} q^{\text{worker}} &= p, & q^{\text{start}} &\leq t, & t+1 &\leq q^{\text{finish}}, \\ \ell_p &\leq 1, & g_q + \Delta_q(a) &\leq q^{\text{max}}, \end{aligned}$$

and, for every break $[b_0, b_1)$ of that grant, $t+1 \leq b_0$ or $b_1 \leq t$. Work use updates to $g'_q = g_q + \Delta_q(a)$.

For every active service undertaking covering $[q^{\text{start}}, q^{\text{finish}})$, if $q^{\text{start}} \leq t < q^{\text{finish}}$, require access for its named person and

$$x_{q^{\text{asset}}} - I_{q^{\text{asset}}} > 0.$$

Thus an asset withdrawn at the opening is unavailable during that interval. Access is an independently supplied relation between time, person and asset.

At every checkpoint, require the application's physical conditions and, for every active delivery undertaking,

$$q^{\text{due}} \leq t \implies q^{\text{qty}} \leq N_q(\rho).$$

Finally, $\text{Step}(H, y, a)$ is the conjunction of physical enabledness, $A_H(s, a)$, all service conditions, all positive-work conditions, canonical-unit soundness and validity of every sequential credit. The next state is

$$F_H(y, a) = (T(s, a), \rho_m, g + \Delta(a), B).$$

These are the action conditions used in the projection result. Positivity of reserves and due requirements are separate checkpoint conditions. The finite policy recursion checks both kinds of condition through the entire chosen horizon.

B The routing calculation

Dispatch occurs at each integer minute. An idle, on-duty courier stays at its actual location until assigned. Each assignment visits one restaurant and delivers an ordered bundle of already placed requests. The courier finishes that assignment before receiving another; there is no prepositioning or reassignment in transit. Time arithmetic uses integer half-minutes.

For locations a, b in meters, travel time in minutes is

$$T_{ab} = \left\lceil \frac{\|a - b\|_2}{320} \right\rceil.$$

Pickup and delivery each occupy four minutes, split evenly before and after physical handover. For an assignment at t from location a to restaurant r , with bundle ready times r_o , pickup is

$$t_{\text{pick}} = \max \left\{ \max_{o \text{ in bundle}} r_o, t + T_{ar} + 2 \right\}.$$

Departure is $t_{\text{pick}} + 2$. Each delivery is the next arrival plus two minutes, followed by another two minutes before departure. Every delivery must occur within 90 minutes of placement. The 40-minute target is a reported metric. Modeled pay for courier c is $\max(10N_c, 15H_c)$, where N_c is the number delivered and H_c the listed shift length in hours.

At a dispatch, enumerate all ordered bundles at each restaurant for each eligible courier. Reject a partial sequence only when extension cannot repair its violated constraint. There is no fixed bundle-size cap. Let $\mathcal{T}$ be the resulting feasible assignments, $c(j)$ the courier of assignment j , $O(j)$ its request set, and d_j its summed placement-to-delivery time. Binary decisions z_j satisfy

$$\sum_{j:c(j)=c} z_j \leq 1 \quad (\text{each courier } c), \quad \sum_{j:o \in O(j)} z_j \leq 1 \quad (\text{each request } o).$$

The first optimization maximizes $\sum_j |O(j)|z_j$. The second fixes that attained maximum and minimizes $\sum_j d_j z_j$. The greedy comparison sorts by decreasing bundle size, increasing summed delivery time, increasing activity, then courier and request identities; it accepts each assignment that conflicts with no accepted one.

Each complete rolling run has a 240-second computation limit and each menu a limit of 200,000 feasible sequences. Each integer optimization stage has two seconds, one solver thread and zero relative gap tolerance. A limit on a complete run yields unresolved work. A stage without a valid first incumbent falls back to the same-menu greedy rule; a failed second stage retains the first valid choice. The reported runs used no fallback and every optimization stage had an optimal status. Computation time was recorded separately from virtual route time, so this experiment supplies no real-time deployment guarantee.

The retrospective repair processes missed requests in increasing deadline order, breaking ties by request identity. For each request it considers each courier and each insertion position in that courier’s route. It inserts a singleton trip and reschedules that trip and the old suffix at their earliest feasible integer-minute departures, preserving the old bundles and their order. Infeasible suffixes are rejected. Among feasible insertions it minimizes, in order: the number of currently assigned requests delayed, their total added delay, their largest added delay, added active work, final release time, courier identity and insertion position. These are deterministic search choices, not moral weights.

After all insertions, independent replay recomputes the full event sequence and person-record differences. A successful complete trace proves feasibility under this model. Failure of the insertion heuristic would establish no global impossibility. The computational supplement supplies input hashes, dispatch and repair programs, and procedures that regenerate the full traces and checks. The public input configuration is specified in [6].

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