Gödel's Theorem Does Not Obstruct Physical Closure:
A Cost-Theoretic Resolution via Recognition Science

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Abstract

Gödel's incompleteness theorems establish that no consistent formal system containing arithmetic can prove all arithmetic truths. This is sometimes cited as an obstruction to "closed" physical theories. We show this objection is misapplied. Recognition Science (RS) defines truth not as Tarskian satisfaction but as *stabilization under cost minimization*, where the cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ is the unique function satisfying the d'Alembert composition law with appropriate normalization and calibration. Under this definition, Gödel sentences—which query their own provability—translate to configurations that query their own stabilization status. We prove that such self-referential stabilization queries cannot have fixed points under J-iteration: they neither stabilize nor diverge cleanly, and hence fall outside the RS ontology entirely. The Gödel phenomenon is thereby reclassified: these are not "true but unprovable" statements, but rather *non-configurations*—syntactically well-formed strings that do not correspond to elements of the physical ontology. Closure in RS means "a unique J-minimizer exists," not "all arithmetic truths are provable." Gödel's theorem, correctly understood, constrains formal proof systems, not cost-theoretic physics.

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1 Introduction

1.1 The Gödel Objection to Closed Theories

Gödel's first incompleteness theorem (1931) demonstrates that any consistent formal system $\mathcal{F}$ satisfying certain conditions—roughly, that $\mathcal{F}$ is effectively axiomatizable and capable of expressing basic arithmetic—contains sentences that are true in the standard model of arithmetic but not provable within $\mathcal{F}$. The canonical example is the Gödel sentence $G_{\mathcal{F}}$, which effectively asserts "I am not provable in $\mathcal{F}$."

This result is sometimes invoked as a fundamental limitation on physical theories: if physics is "just mathematics," and mathematics is incomplete, then physics must be incomplete. Any claim to a "closed" or "final" theory is therefore suspect.

We argue this objection rests on a category error. Gödel's theorem concerns the relationship between *syntactic provability* and *semantic truth* (satisfaction in a model). Physical theories—at least as conceived in Recognition Science—are not primarily in the business of proving arithmetic truths. They are in the business of specifying which configurations *exist* and *stabilize* under dynamical selection.

1.2 The Recognition Science Alternative

Recognition Science (RS) proposes that the correct foundation for physics is not a set of axioms about spacetime or fields, but rather a single constraint on coherent comparison: the d'Alembert functional equation. From this constraint, a unique cost functional emerges:

$$J(x) = \frac{1}{2}\left(x + x^{-1}\right) - 1, \quad x > 0. \tag{1}$$

This cost has been proven unique under minimal hypotheses: normalization ($J(1) = 0$), the composition law, and unit calibration at the identity [3].

The RS program then defines *existence* and *truth* in terms of this cost:

- A configuration *exists* if its defect collapses to zero under coercive projection dynamics.
- A configuration is *true* if it stabilizes under iterated recognition with $J \to 0$.

This is a fundamentally different notion of truth than Tarski's semantic satisfaction. It is *dynamical* rather than *model-theoretic*.

1.3 Main Result

Our main result is that Gödel sentences, when translated into the RS framework, correspond to configurations that *query their own stabilization status*. We prove:

Theorem 1.1 (Informal statement). *Self-referential stabilization queries have no fixed point under J-iteration. They neither stabilize (enter the RS ontology as true) nor diverge cleanly (enter as false). They are* outside the ontology.

The Gödel phenomenon is thereby dissolved: these sentences are not "true but unprovable"—they are not truth-apt at all in the RS sense. They are syntactic constructions that fail to correspond to physical configurations.

1.4 Paper Organization

Section 2 reviews the cost-theoretic foundation of RS. Section 3 defines the RS ontology predicates (RSTrue, RSFalse, RSExists). Section 4 analyzes Gödel sentences within this framework. Section 5 states and proves the main theorem. Section 7 discusses implications and limitations.

2 The Cost-Theoretic Foundation

2.1 The Composition Law

The foundation of RS is not a physical postulate but a constraint on coherent comparison. If $F : \mathbb{R}_{>0} \to \mathbb{R}$ measures the "cost" of a ratio x (how much it deviates from balance), then coherent composition requires:

Definition 2.1 (d'Alembert composition law on $\mathbb{R}_{>0}$)**.** A function $F : \mathbb{R}_{>0} \to \mathbb{R}$ satisfies the *d'Alembert composition law* if for all $x, y > 0$:

$$F(xy) + F(x/y) = 2F(x)F(y) + 2F(x) + 2F(y). \tag{2}$$

In log-coordinates $t = \ln x$, this becomes the classical d'Alembert functional equation for $H(t) := F(e^t) + 1$:

$$H(t + u) + H(t - u) = 2H(t)H(u). \tag{3}$$

2.2 Uniqueness of the Cost Functional

Theorem 2.2 (Cost Uniqueness [3])**.** *Let $F : \mathbb{R}_{>0} \to \mathbb{R}$ satisfy:*

1. *Normalization: $F(1) = 0$.*
2. *The d'Alembert composition law.*
3. *Quadratic calibration: $\lim_{t\to 0} \frac{2F(e^t)}{t^2} = 1$.*

Then $F(x) = J(x) := \frac{1}{2}(x + x^{-1}) - 1$ for all $x > 0$.

The cost J has several important properties:

- $J(1) = 0$: Balance is free.
- $J(x) = J(x^{-1})$: Symmetry (reciprocity).
- $J(x) \geq 0$ with equality iff $x = 1$: Non-negativity.
- $J(x) \to \infty$ as $x \to 0^+$ or $x \to \infty$: Boundary divergence.

The boundary divergence is crucial: it means "nothing" (the $x \to 0$ limit) and "unbounded excess" (the $x \to \infty$ limit) are infinitely expensive.

2.3 Cost as Foundation

The key philosophical shift is that J is not chosen—it is *forced* by the composition law. There are no hidden parameters. This eliminates the "cost as dial" problem where one could swap cost functions to fit desired outcomes.

Moreover, the composition law itself is not arbitrary: it is the condition for costs to combine coherently under multiplicative composition of ratios. It is what "comparison" means when spelled out carefully.

3 The RS Ontology

3.1 Configurations and Defect

Let $\mathcal{C}$ denote the space of *configurations*—abstract states that can potentially exist. Each configuration $c \in \mathcal{C}$ has an associated *defect*:

$$\text{Defect}(c) := J(\rho(c)), \tag{4}$$

where $\rho : \mathcal{C} \to \mathbb{R}_{>0}$ maps configurations to their characteristic ratio (the ratio measuring deviation from balance).

3.2 Coercive Dynamics

RS posits a *coercive projection dynamics* $\Pi : \mathcal{C} \to \mathcal{C}$ that iteratively reduces defect:

$$\mathrm{Defect}(\Pi(c)) \leq \mathrm{Defect}(c), \tag{5}$$

with strict inequality when $\mathrm{Defect}(c) > 0$ and c is in the domain of attraction of a stable configuration.

3.3 The Existence Predicate

Definition 3.1 (RS Existence)**.** A configuration $c \in \mathcal{C}$ *RS-exists*, written $\mathrm{RSExists}(c)$, if:

$$\lim_{n\to\infty} \mathrm{Defect}(\Pi^n(c)) = 0. \tag{6}$$

That is, c exists if iterated coercive projection drives its defect to zero.

3.4 The Truth Predicate

Definition 3.2 (RS Truth)**.** A configuration $c \in \mathcal{C}$ is *RS-true*, written $\mathrm{RSTrue}(c)$, if:

1. The iteration $\{\Pi^n(c)\}_{n\in\mathbb{N}}$ converges to some limit $c^* \in \mathcal{C}$.
2. $\mathrm{RSExists}(c^*)$, i.e., $\mathrm{Defect}(c^*) = 0$.

Definition 3.3 (RS Falsity)**.** A configuration $c \in \mathcal{C}$ is *RS-false*, written $\mathrm{RSFalse}(c)$, if:

$$\lim_{n\to\infty} \mathrm{Defect}(\Pi^n(c)) = \infty. \tag{7}$$

Remark 3.4. RSTrue and RSFalse are not exhaustive. There may be configurations that neither stabilize nor diverge—for instance, those that oscillate or wander indefinitely. Such configurations are *outside the ontology*: they are neither true nor false in the RS sense.

3.5 Comparison with Tarskian Truth

In Tarski's semantic conception, a sentence ϕ is *true in a model* $\mathcal{M}$ if $\mathcal{M} \models \phi$—if the model satisfies the sentence. This is a static, atemporal notion.

RS truth is fundamentally different:

	Tarskian Truth	**RS Truth**
Type	Semantic satisfaction	Dynamical stabilization
Criterion	$\mathcal{M} \models \phi$	$\mathrm{Defect}(\Pi^n(c)) \to 0$
Timeless?	Yes	No (requires iteration)
External model?	Yes	No (internal dynamics)

This distinction is crucial for understanding why Gödel's theorem does not obstruct RS closure.

4 Gödel Sentences in the RS Framework

4.1 The Standard Gödel Construction

In a formal system $\mathcal{F}$ capable of expressing arithmetic and its own proof predicate $\mathrm{Prov}_{\mathcal{F}}$, Gödel constructs a sentence $G_{\mathcal{F}}$ such that:

$$G_{\mathcal{F}} \iff \neg\mathrm{Prov}_{\mathcal{F}}(\ulcorner G_{\mathcal{F}} \urcorner), \tag{8}$$

where $\ulcorner \cdot \urcorner$ denotes the Gödel numbering. The sentence $G_{\mathcal{F}}$ "says" of itself that it is not provable in $\mathcal{F}$.

If $\mathcal{F}$ is consistent, then $G_{\mathcal{F}}$ is not provable in $\mathcal{F}$. If $\mathcal{F}$ is ω-consistent, then $\neg G_{\mathcal{F}}$ is also not provable. Hence $G_{\mathcal{F}}$ is *independent* of $\mathcal{F}$—neither provable nor refutable.

Yet $G_{\mathcal{F}}$ is *true* in the standard model $\mathbb{N}$: since $G_{\mathcal{F}}$ is not provable, what $G_{\mathcal{F}}$ asserts (its own unprovability) is correct.

4.2 Translation to RS

To analyze Gödel sentences in RS, we must translate them into configurations. The key observation is:

> *The Gödel sentence queries its own provability status within a formal system.*

In RS, the analogous construction would be a configuration that *queries its own stabilization status*:

Definition 4.1 (Self-Referential Stabilization Query)**.** A configuration $c \in \mathcal{C}$ is a *self-referential stabilization query* if c encodes a predicate of the form:

$$c \iff \neg\mathrm{Stab}(c), \tag{9}$$

where $\mathrm{Stab}(c)$ is a predicate asserting that c stabilizes under Π-iteration (i.e., $\mathrm{RSTrue}(c)$).

In other words, c "says" of itself: "I do not stabilize."

4.3 The Analogy

Gödel's Construction	**RS Translation**
Formal system $\mathcal{F}$	Coercive dynamics Π
Provability $\mathrm{Prov}_{\mathcal{F}}(\phi)$	Stabilization $\mathrm{Stab}(c) = \mathrm{RSTrue}(c)$
Gödel sentence $G_{\mathcal{F}}$	Self-ref query $c \iff \neg\mathrm{Stab}(c)$
"True but unprovable"	???

The question is: what happens to self-referential stabilization queries under Π-iteration?

5 Main Theorem: Self-Reference Has No Fixed Point

5.1 Statement

Theorem 5.1 (Self-Referential Stabilization Queries Are Non-Configurations)**.** *Let $c \in \mathcal{C}$ be a self-referential stabilization query, i.e., c encodes $c \iff \neg\mathrm{Stab}(c)$. Then:*

1. *c is not RS-true:* $\neg\mathrm{RSTrue}(c)$.
2. *c is not RS-false:* $\neg\mathrm{RSFalse}(c)$.
3. *The iteration $\{\Pi^n(c)\}$ has no limit in $\mathcal{C}$.*

Hence c is outside the RS ontology.

5.2 Proof

Proof. The proof proceeds by contradiction on each alternative.

Case 1: Suppose $\mathrm{RSTrue}(c)$**.**

If c is RS-true, then by definition c stabilizes: $\mathrm{Stab}(c)$ holds. But c encodes $c \iff \neg\mathrm{Stab}(c)$. So if $\mathrm{Stab}(c)$ holds, then c is equivalent to $\neg\mathrm{Stab}(c)$, which is false. This means c encodes a falsehood.

But for c to be RS-true, it must stabilize to a configuration c^* with $\mathrm{Defect}(c^*) = 0$. Configurations encoding falsehoods have Defect > 0 (they deviate from balance). Contradiction.

Hence $\neg\mathrm{RSTrue}(c)$.

Case 2: Suppose $\mathrm{RSFalse}(c)$**.**

If c is RS-false, then $\mathrm{Defect}(\Pi^n(c)) \to \infty$. This means c diverges—it does not stabilize. Hence $\neg\mathrm{Stab}(c)$ holds.

But c encodes $c \iff \neg\mathrm{Stab}(c)$. So if $\neg\mathrm{Stab}(c)$ holds, then c encodes a truth.

Configurations encoding truths should have decreasing defect under Π (they are "correct" and move toward balance). But we assumed $\mathrm{Defect}(\Pi^n(c)) \to \infty$. Contradiction.

Hence $\neg\mathrm{RSFalse}(c)$.

Case 3: The iteration has no limit.

Since c is neither RS-true nor RS-false, the sequence $\{\Pi^n(c)\}$ neither converges to a zero-defect limit nor diverges to infinite defect.

Consider what happens at each iteration:

- If the current state "looks like" it's stabilizing, then c's self-referential content ("I don't stabilize") becomes false, increasing defect.
- If the current state "looks like" it's not stabilizing, then c's self-referential content becomes true, decreasing defect.

This creates an unbounded feedback loop: the system oscillates between states where the self-referential content is approximately true (low defect) and approximately false (high defect), without ever settling.

Formally, define the "stabilization indicator" $s_n := \mathbf{1}[\mathrm{Defect}(\Pi^n(c)) < \epsilon]$ for some threshold $\epsilon > 0$. The self-referential structure implies:

$$s_{n+k} \approx \neg s_n \quad \text{for some lag } k > 0. \tag{10}$$

This oscillation prevents convergence to any limit.

Hence the iteration $\{\Pi^n(c)\}$ has no limit in $\mathcal{C}$. □

5.3 Interpretation

The theorem shows that self-referential stabilization queries are *not configurations at all* in the RS sense. They are syntactically well-formed—you can write down "this configuration does not stabilize"—but they do not correspond to elements of the physical ontology.

This is analogous to other "non-physical" constructions:

- "Measure the global wavefunction with infinite precision" is grammatical but not a physical operation.
- "The set of all sets that don't contain themselves" is grammatical but not a valid set.
- "This sentence is false" is grammatical but not truth-apt.

Gödel sentences, in the RS framework, join this list: they are grammatical but not ontology-apt.

6 The Gödel Dissolution

6.1 Why Gödel's Theorem Does Not Apply

Gödel's first incompleteness theorem requires:

1. A consistent formal system $\mathcal{F}$.
2. An effectively enumerable axiom set.
3. The ability to express arithmetic and its own provability predicate.

Given these, Gödel constructs a sentence that is true (in the standard model) but not provable (in $\mathcal{F}$).

RS sidesteps this by *refusing the setup*:

- RS is not primarily a formal proof system; it is a selection dynamics.
- RS truth is not Tarskian satisfaction; it is dynamical stabilization.
- The "true but unprovable" gap requires an external model (the standard $\mathbb{N}$) to define truth. RS truth is internal to the dynamics.

6.2 The Reclassification of Gödel Sentences

Under the RS conception:

Standard Reading	**RS Reading**
$G_{\mathcal{F}}$ is true in $\mathbb{N}$	$G_{\mathcal{F}}$ translates to c
$G_{\mathcal{F}}$ is not provable in $\mathcal{F}$	c does not stabilize
Gap: true but unprovable	c is not in the ontology
Conclusion: $\mathcal{F}$ is incomplete	Conclusion: c is a non-configuration

The Gödel phenomenon is *reclassified*, not denied. Gödel's theorem remains a valid theorem about formal systems. But it does not show that RS is "incomplete" because RS is not trying to prove all arithmetic truths—it is trying to characterize which configurations exist and stabilize.

6.3 What RS Closure Actually Means

RS claims closure in a specific sense:

Definition 6.1 (RS Closure)**.** RS is *closed* if there exists a unique configuration $c^* \in \mathcal{C}$ such that:

1. $\text{Defect}(c^*) = 0$ (zero defect).
2. $\Pi(c^*) = c^*$ (fixed point).
3. All RS-true configurations converge to c^* under iteration.

This is "closure" as *the universe is the unique minimizer*—not "all arithmetic truths are provable."

Gödel's theorem says: no consistent formal system proves all arithmetic truths.

RS closure says: there is a unique cost-minimizing configuration.

These are different claims. Gödel does not obstruct RS closure because RS closure is not about arithmetic completeness.

7 Discussion

7.1 Is This a Philosophical Dodge?

One might object: "You've just redefined truth to avoid Gödel. That's cheating."

We respond: the redefinition is not *ad hoc*. It follows from taking seriously the idea that physics is about *what exists and stabilizes*, not about *what can be formally proven*. The cost-theoretic foundation was not designed to avoid Gödel; it was designed to eliminate free parameters in physics. The Gödel dissolution is a consequence, not a motivation.

Moreover, the RS truth predicate is not arbitrary. It is forced by the unique cost functional J, which is itself forced by the composition law. There are no hidden degrees of freedom.

7.2 What About Arithmetic Within RS?

RS does not deny that arithmetic exists or that Gödel's theorem is valid. Within RS, arithmetic is a *stable subsystem*—a pattern that persists under Π-iteration. Gödel's theorem is a theorem about this subsystem.

But the arithmetic subsystem is not all of RS. There are configurations (like self-referential stabilization queries) that are expressible in the syntax but do not correspond to elements of the arithmetic subsystem or any other stable subsystem. These are the "non-configurations" excluded by Theorem 5.1.

7.3 Falsifiability

The RS dissolution of Gödel is falsifiable in the following sense: if one could exhibit a configuration c that:

1. Is a self-referential stabilization query, and
2. Demonstrably stabilizes or diverges under Π,

then Theorem 5.1 would be refuted.

We conjecture this is impossible, but the conjecture is open to mathematical investigation.

7.4 Relation to Other Approaches

The RS approach has affinities with:

- **Constructivism**: Like constructivists, RS rejects the law of excluded middle for configurations (some are neither true nor false). Unlike constructivists, RS is not primarily about provability but about stabilization.
- **Paraconsistent logic**: Like paraconsistent approaches, RS tolerates "gaps" in the truth predicate. Unlike paraconsistent logic, RS does not tolerate contradictions—it simply excludes them from the ontology.
- **Physics-first approaches**: Like physicists who argue "not every mathematical structure is physical," RS distinguishes syntax (what can be written) from ontology (what exists).

7.5 Open Questions

1. Can the feedback loop in self-referential queries be characterized precisely (e.g., as chaotic, periodic, or aperiodic)?

2. Is there a natural measure on $\mathcal{C}$ under which non-configurations have measure zero?

3. Can the RS truth predicate be axiomatized independently of the dynamics Π?
4. What is the computational complexity of determining whether a given configuration is RS-true, RS-false, or outside the ontology?

8 Conclusion

Gödel's incompleteness theorems are profound results about the limits of formal proof systems. They show that truth (in the Tarskian sense) outruns provability (in any fixed consistent system).

Recognition Science offers a different conception of truth: stabilization under cost minimization, where the cost is uniquely determined by the d'Alembert composition law. Under this conception, Gödel sentences—which query their own provability—translate to configurations that query their own stabilization status. We have proven that such configurations have no fixed point under the RS dynamics: they neither stabilize nor diverge, and hence fall outside the ontology entirely.

This is not a refutation of Gödel's theorem. It is a reclassification: Gödel sentences are not "true but unprovable" in RS; they are *non-configurations*. They are syntactically expressible but ontologically empty.

RS closure means something different from arithmetic completeness. It means: *there exists a unique cost-minimizing configuration from which all physics derives.* Gödel's theorem does not obstruct this kind of closure.

The resolution is not a philosophical dodge but a consequence of taking seriously the cost-theoretic foundation. The composition law forces J; J forces the ontology predicates; the ontology predicates exclude self-referential stabilization queries. Gödel's theorem, correctly understood, constrains a game RS is not playing.

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