The Recognition Stability Audit: IA compiler for impossibility certificates: Realizable Cayley Fields and Finite Schur Certificates from Canonical Recognition Cost

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February 11, 2026

Abstract

We formalize the Recognition Stability Audit (RSA), a reusable *impossibility audit* for candidate states whose existence would force an *infinite recognition cost*, modeled as blow-up of a canonical reciprocal sensor. Conceptually, RSA is a *compiler*: it attempts to convert an existence claim into a pole mechanism for a bounded analytic field, and then eliminates that pole by a finite certificate. The audit has three layers: a canonical reciprocal cost functional $J(x)$ (forced by normalization and composition axioms [9]); an RS defect/obstruction encoding that turns an existence claim into a holomorphic obstruction G and canonical sensor $\mathcal{J} = 1/G$; and a Cayley transform producing a bounded field Ξ whose Schur control excludes sensor poles by a Schur/Herglotz pinch. To make the certification step fully intrinsic in the full-derivation route, we fix a formal realizability model in which the audited Cayley field is *realizable*: it is generated by an explicit 8-tick recognition process with finite local branching (hence finite-state/rational) and by a cost-driven contractive tick dynamics (hence stable finite-dimensional realizations). In this realizable class we record two fully finite Schur certification routes: a state-space bounded-real (KYP/LMI) certificate [7] and a Pick-gap-plus-tail certificate from Taylor data (Nevanlinna–Pick/Schur theory [5, 6]) with tail bounds derivable from stable realization data. In the bounded-degree rational regime, RSA upgrades from a sound semi-decision for impossibility to a full decision procedure by exact root-location tests. We illustrate the audit with a certified far-field zeta instantiation [12] and outline a complementary existence-side architecture for Hodge-type problems (companion manuscripts). We define a *Universal Recognition Class* of problems admissible to this audit and conjecture that many well-posed existence problems in physical geometry and arithmetic admit the Step 0 realizability model and therefore lie within it.

1 Motivation and scope

Recognition Science treats recognition as a constrained operation: it consumes a nonzero, quantifiable resource. The operational consequence is:

Principle (Finite recognition). *A physically realizable state cannot require infinite recognition cost.*

RSA is a general method for turning this principle into a checkable certificate. It is built as a *template* with explicit proof obligations, but it is best read as a *compiler/audit machine*: given a candidate existence claim, RSA attempts to compile it into a finite certificate of impossibility.

How to read this manuscript: RSA as a compiler

RSA has a front-end, a middle-end, and a back-end.

Front-end (existence $\Rightarrow$ obstruction zero). Encode the candidate claim as an obstruction G whose zeros are forced by the candidate mechanism; then define the sensor $\mathcal{J} := 1/G$ so "candidate occurs" becomes "sensor blows up" (Section 4.1).

Middle-end (sensor $\Rightarrow$ bounded Cayley field). Apply the Cayley transform to obtain a bounded field Ξ so that a pole of $\mathcal{J}$ becomes a boundary hit for Ξ; then classical Schur/Herglotz pinching turns global Schur control of Ξ into pole-freeness of $\mathcal{J}$ (Section 5 and Corollary 4).

Back-end (global boundedness $\Rightarrow$ finite certificate). Global Schur control is an analytic statement, so RSA supplies finite certification regimes (bounded-real/KYP or Pick-gap-plus-tail). The crucial bridge is that the audited Cayley field is not arbitrary: in the full-derivation route it is assumed *realizable* in the Step 0 sense (8-tick finite-window plus cost-driven contractive dynamics), which places it in a finite-complexity class where finite certificates are complete or can be made quantitative (Theorem 4 and Corollary 5).

The remaining domain module (what makes a flagship instantiation a closed theorem). For any specific target (zeta, Hodge, PDE blow-up, ...), the one domain-specific obligation is to prove a theorem identifying the audited Cayley field built from that domain obstruction with a Step 0 realizable Cayley field model (including explicit degree/stability constants, or an explicit tail bound). Once that module is supplied, the rest of RSA is a closed pipeline: one computes a finite certificate and concludes impossibility on the audited region.

What RSA does and does not claim

- **What RSA can certify:** conditional `IMPOSSIBLE_STATE` for candidates whose existence would force sensor blow-up in the audited region.
- **What RSA does not do in isolation:** prove existence *in general*. Passing an audit is therefore `INCONCLUSIVE` unless paired with a complementary existence-side mechanism (Definition 26); in the finite-dimensional rational regime such an existence-side mechanism is automatic by exact root testing (Theorem 10).

Claim taxonomy (for referee-facing scope hygiene)

Type	**Status in this manuscript**
Cost uniqueness (Theorem 1)	Proved here (functional equation $\Rightarrow$ ODE $\Rightarrow$ cosh)
Pick/Schur background (Theorem 6)	Classical; see [5, 6]
Realizability $\Rightarrow$ finite complexity (Theorem 4)	Proved here (under the explicit 8-tick model)
Finite certificate $\Rightarrow$ global Schur (Theorem 7)	Proved here (packaging of standard criteria)
Finite-sampling obstruction (Proposition 4)	Proved here
Schur pinch $\Rightarrow$ audited impossibility (Theorem 8)	Proved here (given stated hypotheses)
Decision in the rational regime (Theorem 10)	Stated and proved (finite algebraic root-location)
URC coverage conjecture	Conjecture (Section 12)

Conceptual dictionary (Rosetta Stone)

To orient experts from different fields, we map RSA terms to their standard analogs:

RSA Term	**Standard Analog**
Sensor Cost Blow-up	Control Theory: Instability / Positive Feedback Loop
Schur Certification	Operator Theory: Nevanlinna–Pick Interpolation
Reciprocal Cost	Physics: Action / Entropy

2 Canonical recognition cost

Definition 1 (Deviant)**.** *A* deviant *is a multiplicative deviation $x \in \mathbb{R}_{>0}$ from the identity state $x = 1$.*

Definition 2 (Canonical reciprocal cost)**.** *Define $J : \mathbb{R}_{>0} \to \mathbb{R}$ by*

$$J(x) \;=\; \frac{1}{2}\bigl(x + x^{-1}\bigr) - 1. \tag{1}$$

Proposition 1 (Basic properties)**.** *For all $x \in \mathbb{R}_{>0}$:*

1. *(Reciprocity) $J(x) = J(x^{-1})$.*
2. *(Normalization) $J(1) = 0$.*
3. *(Nonnegativity) $J(x) = \dfrac{(x-1)^2}{2x} \geq 0$, with equality iff $x = 1$.*
4. *(Divergence) $J(x) \to \infty$ as $x \to 0^+$ or $x \to \infty$.*

Definition 3 (Log coordinates)**.** *Write $x = e^t$ with $t \in \mathbb{R}$. Then*

$$J(e^t) = \cosh(t) - 1. \tag{2}$$

2.1 Why this J is not a dial

RSA relies on the idea that the cost functional is *forced*, not chosen. The following uniqueness theorem isolates a minimal hypothesis set that pins down J.

Theorem 1 (Uniqueness under composition and calibration)**.** *Let $F : \mathbb{R}_{>0} \to \mathbb{R}$ satisfy:*

1. *$F(1) = 0$,*
2. *for all $x, y > 0$,*
$$F(xy) + F(x/y) = 2F(x)F(y) + 2F(x) + 2F(y), \tag{3}$$
3. *(Unit log-curvature at identity)*
$$\lim_{t \to 0} \frac{2F(e^t)}{t^2} = 1. \tag{4}$$

Then $F(x) = J(x)$ for all $x > 0$.

Proof. Define $H : \mathbb{R} \to \mathbb{R}$ by $H(t) := F(e^t) + 1$. Then $H(0) = F(1) + 1 = 1$. Substituting $x = e^t$ and $y = e^u$ into (3) gives

$$F(e^{t+u}) + F(e^{t-u}) = 2F(e^t)F(e^u) + 2F(e^t) + 2F(e^u).$$

Adding 2 to both sides and regrouping yields d'Alembert's equation

$$H(t+u) + H(t-u) = 2H(t)H(u) \qquad (t, u \in \mathbb{R}). \tag{5}$$

Setting $t = 0$ in (5) gives $H(u) + H(-u) = 2H(0)H(u) = 2H(u)$, hence H is even.

The curvature condition (4) is exactly the second-order calibration

$$\lim_{t\to 0} \frac{H(t) - 1}{t^2} = \lim_{t\to 0} \frac{F(e^t)}{t^2} = \frac{1}{2}.$$

In particular, H has second derivative at 0 with $H''(0) = 1$.

Now fix $t \in \mathbb{R}$ and rewrite (5) as a symmetric second-difference identity:

$$H(t+u) - 2H(t) + H(t-u) = 2H(t)\big(H(u) - 1\big) \qquad (u \neq 0).$$

Dividing by u^2 and letting $u \to 0$ gives

$$\lim_{u\to 0} \frac{H(t+u) - 2H(t) + H(t-u)}{u^2} = 2H(t) \cdot \lim_{u\to 0} \frac{H(u) - 1}{u^2} = 2H(t) \cdot \frac{1}{2} = H(t).$$

Thus H is twice differentiable and satisfies the ODE $H''(t) = H(t)$ for all t. The general C^2 solutions of $H'' = H$ are $H(t) = ae^t + be^{-t}$. Since H is even we have $a = b$, and since $H(0) = 1$ we get $a = b = \frac{1}{2}$. Therefore $H(t) = \cosh(t)$ and hence, for $x > 0$ with $t = \log x$,

$$F(x) = H(\log x) - 1 = \cosh(\log x) - 1 = \frac{1}{2}(x + x^{-1}) - 1 = J(x).$$

□

Remark 1. *The functional equation* (3) *is a multiplicative form of d'Alembert's equation (compare* (5)*), and the calibration* (4) *selects the hyperbolic branch and fixes its scale.*

2.2 A cost-minimizing neutrality projection (derived from J)

The full-derivation route requires a concrete mechanism that connects Recognition Cost to dynamics. Here we record a canonical "projection-to-neutrality" update rule that is *forced* once J is fixed. The point is not to import additional structure, but to show that a standard RS move—enforcing a conservation/neutrality constraint by a minimal-cost correction—has a unique solution because J is strictly convex in log-coordinates.

Definition 4 (Skew and neutrality manifold)**.** *Fix $n \in \mathbb{N}$ and consider a multiplicative ledger vector $x = (x_1, \dots, x_n) \in (\mathbb{R}_{>0})^n$. Define its* log-skew *by*

$$\sigma(x) \;:=\; \sum_{i=1}^{n} \log(x_i).$$

Define the neutrality manifold *(constraint surface)*

$$\mathcal{M} \;:=\; \{\, x \in (\mathbb{R}_{>0})^n : \; \sigma(x) = 0 \,\}.$$

Definition 5 (Neutralizing corrections and their J-cost)**.** *Given* $x \in (\mathbb{R}_{>0})^n$*, a* neutralizing correction *is a vector* $r = (r_1, \dots, r_n) \in (\mathbb{R}_{>0})^n$ *such that the corrected state*

$$x' \;=\; x \odot r \;:=\; (x_1 r_1, \dots, x_n r_n)$$

lies in $\mathcal{M}$*. Equivalently, in log-coordinates* $t_i := \log(r_i)$*, this constraint is*

$$\sum_{i=1}^{n} t_i \;=\; -\sigma(x).$$

Define the total correction cost by

$$\mathrm{Cost}(r) \;:=\; \sum_{i=1}^{n} J(r_i).$$

Theorem 2 (Canonical J-projection to neutrality)**.** *For every* $x \in (\mathbb{R}_{>0})^n$*, the minimization problem*

$$\min\{\; \mathrm{Cost}(r) : \; r \in (\mathbb{R}_{>0})^n, \; x \odot r \in \mathcal{M} \;\}$$

has a unique minimizer, namely

$$r_1 = \cdots = r_n = \exp\Big(-\frac{\sigma(x)}{n} \Big).$$

Equivalently, the unique minimal-cost neutralization is the uniform rescaling

$$x_i' \;=\; x_i \cdot \exp\Big(-\frac{\sigma(x)}{n} \Big) \qquad (i = 1, \dots, n).$$

Proof. Write $t_i = \log(r_i)$ so that $r_i = e^{t_i}$ and the neutrality constraint is $\sum_i t_i = -\sigma(x)$. Using (2), we have $J(e^{t_i}) = \cosh(t_i) - 1$. Define $\phi(t) := \cosh(t) - 1$, which is strictly convex on $\mathbb{R}$ (since $\phi''(t) = \cosh(t) > 0$). Then

$$\mathrm{Cost}(r) = \sum_{i=1}^{n} J(e^{t_i}) = \sum_{i=1}^{n} \phi(t_i).$$

By Jensen's inequality,

$$\frac{1}{n} \sum_{i=1}^{n} \phi(t_i) \;\geq\; \phi\Big(\frac{1}{n} \sum_{i=1}^{n} t_i \Big) \;=\; \phi\Big(-\frac{\sigma(x)}{n} \Big),$$

with equality if and only if $t_1 = \cdots = t_n$ (strict convexity). Therefore the unique minimizer is $t_i = -\sigma(x)/n$ for all i, i.e. $r_i = \exp(-\sigma(x)/n)$. Substituting into $x' = x \odot r$ gives the stated formula. □

Corollary 1 (Log-space orthogonal projection and nonexpansiveness)**.** *Let* $x \in (\mathbb{R}_{>0})^n$ *and write* $y_i = \log(x_i)$*. Define* $x' \in \mathcal{M}$ *by the canonical J-projection (Theorem 2) and write* $y_i' = \log(x_i')$*. Then*

$$y_i' \;=\; y_i - \frac{1}{n} \sum_{j=1}^{n} y_j \qquad (i = 1, \dots, n),$$

so $y' \in \mathbb{R}^n$ *is the Euclidean orthogonal projection of* y *onto the hyperplane* $H = \{y \in \mathbb{R}^n : \sum_i y_i = 0\}$*. In particular, the map* $y \mapsto y'$ *is* 1*-Lipschitz and satisfies* $\|y'\| \leq \|y\|$*.*

Proof. From Theorem 2 we have $x_i' = x_i \exp(-\sigma(x)/n)$. Taking logarithms gives

$$y_i' = \log(x_i') = \log(x_i) - \frac{\sigma(x)}{n} = y_i - \frac{1}{n}\sum_{j=1}^{n} y_j.$$

This is exactly the formula for orthogonal projection onto H (subtract the mean). Orthogonal projections in Euclidean space are 1-Lipschitz and satisfy $\|y'\| \le \|y\|$. □

Remark 2 (Why this matters for RSA). *Theorem 2 is a second "program to certificate-grade theorem" upgrade. It shows that once a neutrality constraint is imposed, the cost-minimizing correction step is* unique *and* explicit. *This is the simplest rigorous sense in which Recognition Cost generates dynamics: neutralization is not an arbitrary repair, but the forced minimizer of a convex cost. Corollary 1 adds an important structural fact: in log-coordinates, the correction is an* orthogonal projection, *hence nonexpansive. This is the beginning of the stability/contractivity story needed to justify finite-dimensional bounded-real certificates from cost dynamics.*

2.3 A contractive tick model from Recognition Cost (proximal form)

The bounded-real (KYP) certificate used by RSA is a *contractivity* statement: it asserts the existence of a storage function that decreases along trajectories up to an input–output energy balance (Remark 16). To make this intrinsic to Recognition Cost, one needs a mathematically explicit "one-tick" dynamics rule whose only primitive is the cost functional.

A standard way to turn a convex cost into a well-posed discrete-time dynamics is a *proximal step*: "move as little as possible while paying down cost." We record the resulting contraction mechanism in the canonical J-geometry.

Definition 6 (Cost-regularized neutrality step). *Fix $n \ge 1$ and let $H = \{y \in \mathbb{R}^n : \sum_i y_i = 0\}$. Let $\phi(t) = \cosh(t) - 1$ so that $J(e^t) = \phi(t)$ by (2). For $\lambda > 0$, define the map $\Pi_\lambda : \mathbb{R}^n \to H$ by*

$$\Pi_\lambda(y) \;:=\; \arg\min_{y' \in H} \Big(\frac{1}{2}\|y' - y\|^2 \;+\; \lambda \sum_{i=1}^{n} \phi(y_i')\Big).$$

Lemma 1 (Strong convexity and contraction). *For each $\lambda > 0$, the minimizer in Definition 6 exists and is unique. Moreover Π_λ is a strict contraction:*

$$\|\Pi_\lambda(y) - \Pi_\lambda(\tilde{y})\| \;\le\; \frac{1}{1+\lambda}\,\|y - \tilde{y}\| \qquad (y, \tilde{y} \in \mathbb{R}^n).$$

Proof. The function ϕ is 1-strongly convex on $\mathbb{R}$ because $\phi''(t) = \cosh(t) \ge 1$. Therefore $y' \mapsto \sum_i \phi(y_i')$ is 1-strongly convex on $\mathbb{R}^n$ (hence on the affine subspace H), and adding the strictly convex quadratic term $\frac{1}{2}\|y' - y\|^2$ makes the objective in Definition 6 strictly convex on H. Since the objective is coercive on H, a unique minimizer exists.

For the contraction estimate, set

$$f(y') := \sum_{i=1}^{n} \phi(y_i') + \iota_H(y'),$$

where ι_H is 0 on H and $+\infty$ off H. Then f is 1-strongly convex, so its subdifferential ∂f is 1-strongly monotone: for $u \in \partial f(x)$ and $v \in \partial f(\tilde{x})$,

$$\langle x - \tilde{x},\, u - v\rangle \;\ge\; \|x - \tilde{x}\|^2.$$

Let $x = \Pi_\lambda(y)$ and $\tilde{x} = \Pi_\lambda(\tilde{y})$. The first-order optimality conditions are

$$0 \in x - y + \lambda\,\partial f(x), \qquad 0 \in \tilde{x} - \tilde{y} + \lambda\,\partial f(\tilde{x}),$$

so $(y - x)/\lambda \in \partial f(x)$ and $(\tilde{y} - \tilde{x})/\lambda \in \partial f(\tilde{x})$. Applying strong monotonicity gives

$$\Big\langle x - \tilde{x},\ \frac{y - x}{\lambda} - \frac{\tilde{y} - \tilde{x}}{\lambda} \Big\rangle \ \geq\ \|x - \tilde{x}\|^2.$$

Multiplying by λ and rearranging yields

$$\langle x - \tilde{x},\ y - \tilde{y} \rangle \ \geq\ (1 + \lambda)\,\|x - \tilde{x}\|^2.$$

By Cauchy–Schwarz, $\|x - \tilde{x}\|\,\|y - \tilde{y}\| \geq (1 + \lambda)\|x - \tilde{x}\|^2$, hence $\|x - \tilde{x}\| \leq \frac{1}{1+\lambda}\|y - \tilde{y}\|$. □

Remark 3 (How this feeds bounded-real certificates)**.** *Lemma 1 exhibits an explicit, purely cost-driven source of strict contraction in log-coordinates. In a finite-resolution quotient (Section 3), such contractive tick maps yield stable finite-dimensional realizations, which is the structural input needed to make bounded-real (KYP/LMI) certification intrinsic rather than assumed. For a domain instantiation, the corresponding adapter theorem is: identify the audited Cayley field with the transfer of a realization governed by such a cost-regularized neutrality step (or equivalently, exhibit a dissipation inequality of the form in Remark 16). When this identification is available, the stability hypotheses needed for state-space certification are discharged* from the RS dynamics itself *rather than imported as an external modeling assumption.*

Lemma 2 (Contraction implies stable linearization)**.** *Let $F : \mathbb{R}^d \to \mathbb{R}^d$ be differentiable and assume it is globally Lipschitz with constant $L < 1$, i.e. $\|F(x) - F(y)\| \leq L\|x - y\|$ for all x, y. Then for every $x \in \mathbb{R}^d$, the Jacobian matrix $DF(x)$ satisfies $\|DF(x)\| \leq L$, hence $\rho(DF(x)) \leq L < 1$.*

Proof. Fix $x \in \mathbb{R}^d$ and a unit vector v. For small $t \neq 0$, the mean-value estimate gives

$$\frac{\|F(x + tv) - F(x)\|}{|t|} \ \leq\ L\,\frac{\|tv\|}{|t|} = L.$$

Taking $t \to 0$ and using differentiability yields $\|DF(x)v\| \leq L$. Taking the supremum over unit vectors v gives $\|DF(x)\| \leq L$. Finally, for any linear operator A, the spectral radius satisfies $\rho(A) \leq \|A\|$, so $\rho(DF(x)) < 1$. □

Remark 4 (Stable A from a cost-driven tick)**.** *Lemma 1 shows that the RS-prox tick Π_λ is Lipschitz with constant $L = 1/(1 + \lambda) < 1$. Whenever one passes to a differentiable finite-dimensional coordinate model (e.g. by working in log-coordinates on a finite-resolution quotient and linearizing around a reference trajectory), Lemma 2 implies that the resulting one-tick linear update matrix A satisfies $\rho(A) < 1$. This is exactly the stability hypothesis required in the bounded-real certificate (Theorem 5) and the tail bound from a realization (Lemma 11).*

3 Recognition Geometry and finite local resolution (RG4)

Recognition Geometry (RG) formalizes a measurement-first viewpoint: recognizers are primitive, and observable space is a quotient by indistinguishability. RSA uses one axiom from this framework as a *universally reusable* restriction: finite local resolution.

Definition 7 (Finite local resolution (RG4)). *Let $\mathcal{C}$ be a configuration space, $\mathcal{E}$ an event space, $\mathcal{N}(c)$ a neighborhood system, and $R : \mathcal{C} \to \mathcal{E}$ a recognizer. RG4 asserts: for every $c \in \mathcal{C}$ there exists $U \in \mathcal{N}(c)$ such that $R(U)$ is finite, i.e. $|R(U)| < \infty$.*

Remark 5. *In the full Recognition Geometry axiom list [10],* indistinguishability *is RG3 and* finite local resolution *is RG4. We use only the finite-resolution content here.*

Remark 6 (Why RG4 matters for RSA). *Finite certificates can only imply global control if the audited objects belong to a restricted class. RG4 provides the correct kind of restriction in an operational setting: it excludes idealized "infinite precision" recognizers and motivates finite-complexity hypotheses for the fields that RSA audits.*

3.1 Deriving RG4 from an 8-tick physical realizability model

The full-derivation route requires that "finite resolution" and "finite complexity" are not merely asserted as desiderata. They must follow from a precise model of what it means for a recognition procedure to be physically realizable in one finite window. We now record a minimal (and deliberately explicit) discrete-time model that captures the 8-tick constraint and implies RG4.

Definition 8 (One-tick reachability and 8-tick reachability). *Let $\mathcal{L}$ be a set of internal ledger states. A* one-tick reachability operator *is a map $\widehat{R} : \mathcal{L} \to \mathcal{P}(\mathcal{L})$, where $\widehat{R}(\ell)$ is the set of states reachable from ℓ in one recognition tick. Define the n-tick reachable set recursively by*

$$\widehat{R}^{(0)}(\ell) = \{\ell\}, \qquad \widehat{R}^{(n+1)}(\ell) = \bigcup_{\ell' \in \widehat{R}^{(n)}(\ell)} \widehat{R}(\ell').$$

The 8-tick reachable set *is $\widehat{R}^{(8)}(\ell)$.*

Definition 9 (Neighborhoods induced by reachability). *Given $\widehat{R}$, define a neighborhood system $\mathcal{N}_{\widehat{R}}$ on $\mathcal{L}$ by declaring $U \subseteq \mathcal{L}$ to be a neighborhood of ℓ if it contains the entire 8-tick reachable set:*

$$U \in \mathcal{N}_{\widehat{R}}(\ell) \quad \Longleftrightarrow \quad \widehat{R}^{(8)}(\ell) \subseteq U.$$

In particular, $\widehat{R}^{(8)}(\ell) \in \mathcal{N}_{\widehat{R}}(\ell)$ for every ℓ.

Definition 10 (Finite local branching (per tick)). *We say $\widehat{R}$ has* finite local branching *if for every $\ell \in \mathcal{L}$ the set $\widehat{R}(\ell)$ is finite. If there exists a uniform bound $b \in \mathbb{N}$ such that $|\widehat{R}(\ell)| \leq b$ for all ℓ, we say $\widehat{R}$ has* branching bound *b.*

Lemma 3 (Finite branching implies finite 8-tick reachability). *If $\widehat{R}$ has branching bound b, then for every $\ell \in \mathcal{L}$, the 8-tick reachable set $\widehat{R}^{(8)}(\ell)$ is finite and*

$$|\widehat{R}^{(8)}(\ell)| \;\leq\; 1 + b + b^2 + \cdots + b^8 \;\leq\; \frac{b^9 - 1}{b - 1} \qquad (b \geq 2),$$

with the obvious bound $|\widehat{R}^{(8)}(\ell)| \leq 9$ when $b = 1$.

Proof. By induction on n. For $n = 0$ the set $\widehat{R}^{(0)}(\ell) = \{\ell\}$ is finite. Assume $\widehat{R}^{(n)}(\ell)$ is finite and $|\widehat{R}(\ell')| \leq b$ for all ℓ'. Then $\widehat{R}^{(n+1)}(\ell) = \bigcup_{\ell' \in \widehat{R}^{(n)}(\ell)} \widehat{R}(\ell')$ is a finite union of finite sets, hence finite, and

$$|\widehat{R}^{(n+1)}(\ell)| \leq \sum_{\ell' \in \widehat{R}^{(n)}(\ell)} |\widehat{R}(\ell')| \leq b\, |\widehat{R}^{(n)}(\ell)|.$$

Iterating gives $|\widehat{R}^{(n)}(\ell)| \leq 1 + b + \cdots + b^n$, and in particular the stated 8-tick bound. □

Definition 11 (8-tick realizable recognizer). *Let $M : \mathcal{L} \to \mathcal{E}$ be a measurement/recognizer output map to an event set $\mathcal{E}$. We say M is* 8-tick realizable at ℓ *(with respect to $\widehat{R}$) if the set of reachable outcomes*

$$M(\widehat{R}^{(8)}(\ell)) \;:=\; \{\, M(\ell') : \; \ell' \in \widehat{R}^{(8)}(\ell) \,\}$$

is finite.

Proposition 2 (8-tick realizability implies RG4 (finite local resolution)). *Assume $\widehat{R}$ has finite local branching (Definition 10) and $M : \mathcal{L} \to \mathcal{E}$ is a recognizer. Equip $\mathcal{L}$ with the reachability-induced neighborhood system $\mathcal{N}_{\widehat{R}}$ (Definition 9). Then for every $\ell \in \mathcal{L}$ there exists a neighborhood $U \in \mathcal{N}_{\widehat{R}}(\ell)$ such that $M(U)$ is finite; in other words, RG4 holds for the configuration space $\mathcal{L}$, event space $\mathcal{E}$, neighborhood system $\mathcal{N}_{\widehat{R}}$, and recognizer M.*

Proof. Fix $\ell \in \mathcal{L}$ and take $U := \widehat{R}^{(8)}(\ell)$. By Definition 9 we have $U \in \mathcal{N}_{\widehat{R}}(\ell)$. By Lemma 3, the set U is finite, hence its image $M(U)$ is finite. $\square$

Remark 7 (What has (and has not) been assumed). *Proposition 2 makes explicit the exact content needed to obtain RG4 from an 8-tick model: finite local branching of the one-tick reachability operator. This is where physical realizability enters: if in one tick the system could branch into infinitely many distinct next-states, then no finite-window audit can be meaningful. In this manuscript, the role of Recognition Cost is to supply an explicit, non-branching and contractive tick mechanism under finite cost budgets (Section 2.3). In particular, the canonical J-projection step (Theorem 2) is single-valued and nonexpansive in log-coordinates (Corollary 1), so it introduces no branching and supplies a natural Lyapunov structure.*

3.2 From finite resolution to a finite-state model (the first missing bridge)

RSA needs a *finite-to-global* bridge: a reason that a finite computation ("eight ticks," or any finite certificate) can control the whole interior of an audited region. In the full-derivation route, this bridge is not assumed. It is extracted from Recognition Geometry by passing to a *local recognition quotient* and then representing its induced dynamics as a finite-state system.

Definition 12 (Indistinguishability and local recognition quotient). *Fix a recognizer $R : \mathcal{C} \to \mathcal{E}$ and a subset $U \subseteq \mathcal{C}$. Define an equivalence relation $\sim_R$ on U by*

$$c_1 \sim_R c_2 \quad\Longleftrightarrow\quad R(c_1) = R(c_2).$$

Write $U/{\sim_R}$ for the set of equivalence classes (the local recognition quotient).

Lemma 4 (RG4 implies a finite local quotient). *If $R(U)$ is finite, then $U/{\sim_R}$ is finite and $|U/{\sim_R}| \le |R(U)|$.*

Proof. Each equivalence class in $U/{\sim_R}$ is mapped by R to a single value in $R(U)$, and different classes map to different values. Thus the induced map $U/{\sim_R} \to R(U)$ is injective, so $|U/{\sim_R}| \le |R(U)| < \infty$. $\square$

Definition 13 (Recognition-respecting update and induced quotient dynamics). *Let $T : U \to U$ be a one-step update map (one "tick") on configurations. We say T is* recognition-respecting on U *if*

$$c_1 \sim_R c_2 \implies T(c_1) \sim_R T(c_2) \qquad (c_1, c_2 \in U).$$

In this case the quotient map $\bar{T} : U/{\sim_R} \to U/{\sim_R}$ defined by $\bar{T}([c]) = [T(c)]$ is well-defined.

Definition 14 (Finite-state realization (local, discrete-time))**.** *Assume $U/{\sim_R}$ is finite and T is recognition-respecting on U. Let $S := U/{\sim_R}$, choose an initial class $s_0 \in S$, and fix an* output map $O : S \to \mathbb{C}$. *The resulting discrete-time process is the finite-state system*

$$s_{n+1} = \bar{T}(s_n), \qquad y_n = O(s_n) \qquad (n \geq 0),$$

with state space S and output sequence $(y_n)_{n\geq 0}$.

Theorem 3 (Finite-state ⇒ rational generating function)**.** *In the setup of Definition 14, define the power series*

$$F(z) \;:=\; \sum_{n\geq 0} y_n\, z^n \qquad (|z| < 1).$$

Then F is a rational *function of z. More precisely, if $d = |S|$ then there exists a $d \times d$ matrix A and vectors $u, v \in \mathbb{C}^d$ such that*

$$F(z) = u^*(I - zA)^{-1}v,$$

so the denominator of F divides $\det(I - zA)$ *and has degree at most d.*

Proof. Enumerate $S = \{1, \ldots, d\}$ and let e_j be the standard basis of $\mathbb{C}^d$. Define A by $Ae_j := e_{\bar{T}(j)}$, so $A^n e_{s_0} = e_{s_n}$. Let $v := e_{s_0}$ and let $u \in \mathbb{C}^d$ be the vector with entries $u_j := O(j)$, so that $y_n = u^* A^n v$. Then for $|z| < 1$,

$$F(z) = \sum_{n\geq 0} u^* A^n v\, z^n = u^* \Big(\sum_{n\geq 0} (zA)^n \Big) v = u^*(I - zA)^{-1} v,$$

where the Neumann series converges for $|z| < 1$ because S is finite and A is a bounded linear operator on $\mathbb{C}^d$. The right-hand side is a ratio of polynomials because $(I - zA)^{-1} = \operatorname{adj}(I - zA)/\det(I - zA)$. □

Corollary 2 (Finite-state ⇒ state-space transfer form)**.** *In the setup of Theorem 3, the same function F admits a (finite-dimensional) state-space representation of the standard discrete-time transfer form*

$$F(z) = D + z\, C\, (I - zA)^{-1} B,$$

as in Definition 21. Concretely, one may take $B = v$, $C = u^* A$, and $D = u^* v$.

Proof. Using $(I - zA)^{-1} = I + zA(I - zA)^{-1}$, we compute

$$F(z) = u^*(I - zA)^{-1} v = u^* v + z\, u^* A (I - zA)^{-1} v,$$

which has the required form with $D = u^* v$, $C = u^* A$, and $B = v$. □

Corollary 3 (Finite resolution ⇒ rational coefficients)**.** *In the setup of Theorem 3, suppose the output map $O : S \to \mathbb{C}$ takes values in a subfield $K \subseteq \mathbb{C}$. Then the resulting generating function F lies in $K(z)$. In particular, if O takes values in $\mathbb{Q}$ (finite-resolution outputs), then $F \in \mathbb{Q}(z)$.*

Proof. In the proof of Theorem 3, the matrix A has entries in $\{0, 1\} \subseteq \mathbb{Q}$ by construction, and the vectors v and u have entries in $\mathbb{Q}$ and K respectively. Therefore $(I - zA)^{-1} = \operatorname{adj}(I - zA)/\det(I - zA)$ has entries in $\mathbb{Q}(z) \subseteq K(z)$, and multiplying on the left by u^* and on the right by v shows $F \in K(z)$. □

Theorem 4 (Realizability $\Rightarrow$ finite complexity (rational audited class))**.** *Assume an 8-tick reachability model $(\mathcal{L}, \widehat{R})$ with finite local branching (Definition 10) and a recognizer $R : \mathcal{L} \to \mathcal{E}$. Fix $\ell_0 \in \mathcal{L}$ and set $U := \widehat{R}^{(8)}(\ell_0)$. Assume a recognition-respecting one-tick update $T : U \to U$ (Definition 13) and an output map $O : U/\!\sim_R \to K$ into a subfield $K \subseteq \mathbb{C}$. Let $(s_n)_{n\geq 0}$ be the induced quotient dynamics on $S := U/\!\sim_R$ with output $y_n := O(s_n)$ (Definition 14), and define*

$$\theta(z) \;:=\; \sum_{n\geq 0} y_n\, z^n \qquad (|z| < 1).$$

Then $\theta \in K(z)$ is a rational function whose denominator has degree at most $|S|$.

Proof. By Lemma 3, the 8-tick reachable set $U = \widehat{R}^{(8)}(\ell_0)$ is finite. Hence $R(U)$ is finite and Lemma 4 implies $S = U/\!\sim_R$ is finite. Since T is recognition-respecting, the induced quotient update $\bar{T} : S \to S$ is well-defined (Definition 13), so Definition 14 applies. Theorem 3 gives rationality of θ, and Corollary 3 places θ in $K(z)$. $\square$

Remark 8 (Connection to the Cayley audit)**.** *Once an audited Cayley field θ belongs to a rational class (as in Theorem 4), the associated sensor and obstruction inherit rationality through the Cayley transform and its inverse (Lemma 6). This is the precise sense in which finite-window realizability turns analytic auditing into a finite-dimensional algebraic problem.*

Remark 9 (What this buys RSA)**.** *Theorem 3 is the first hard upgrade from "program" to "certificate-grade theorem": under RG4 and recognition-respecting dynamics, the audited object is not an arbitrary holomorphic function. It lies in a* finite-dimensional rational family *whose global behavior is determined by finitely many parameters. We use this in Lemma 11 and Theorem 7 to replace "assume a tail bound" with a derived, finite-complexity certificate.*

4 Sensors as obstruction reciprocals

RSA needs a disciplined way to define "correct sensors" across domains. The clean universal pattern is: define a sensor as the reciprocal of an obstruction that vanishes exactly when the candidate mechanism occurs.

Definition 15 (Candidate and obstruction)**.** *Fix a domain Ω (typically a region in $\mathbb{C}$ or a parameter space) and a candidate statement S. An* obstruction *is a function $G_S : \Omega \to \mathbb{C}$ such that, for some $z_\star \in \Omega$,*

$$S \text{ occurs at } z_\star \implies G_S(z_\star) = 0.$$

If one has the biconditional S at $z_\star \iff G_S(z_\star) = 0$, we call G_S exact.

Definition 16 (Sensor as reciprocal)**.** *Given an obstruction G_S that is nonzero on admissible states, define the sensor*

$$\mathcal{J}_S(z) \;:=\; \frac{1}{G_S(z)}.$$

Lemma 5 (Sensor correctness is a reduction)**.** *If S at $z_\star$ implies $G_S(z_\star) = 0$ and G_S is holomorphic near $z_\star$ with a simple zero, then $\mathcal{J}_S$ has a pole at $z_\star$.*

Remark 10. *This is the only truly domain-agnostic way to make "sensor correctness" rigorous: the domain work lives in constructing G_S and proving the implication $S \Rightarrow G_S = 0$.*

4.1 RS defect semantics makes obstruction-zeros canonical

In Recognition Science, "existence" is not a primitive logical predicate; it is defined operationally in terms of a defect/cost that must converge to zero at a stable configuration. This viewpoint supplies a canonical source of obstructions: *a candidate holds exactly when its defect vanishes.* We record this as an explicit schema.

Definition 17 (Defect functional (RS encoding of an existence claim))**.** *Fix a domain Ω and a candidate statement S. A* defect functional *for S is a map*

$$\Delta_S : \Omega \to \mathbb{R}_{\geq 0}$$

with the intended semantics:

$$S \text{ holds at } z \quad \Longleftrightarrow \quad \Delta_S(z) = 0.$$

Proposition 3 (Existence $\Rightarrow$ obstruction zero (schema))**.** *Let Δ_S be a defect functional in the sense of Definition 17. Define the obstruction $G_S := \Delta_S$. Then for any $z_\star \in \Omega$,*

$$S \text{ holds at } z_\star \implies G_S(z_\star) = 0.$$

If moreover Δ_S is exact *(i.e. the biconditional holds), then G_S is an exact obstruction in the sense of Definition 15.*

Proof. Immediate from the defining semantics of Δ_S. $\square$

Remark 11 (Analytic RS-admissibility)**.** *RSA is an analytic audit, so one typically works in a setting where the defect admits an analytic representative: either Δ_S itself extends to a holomorphic function on Ω, or one constructs a holomorphic G_S with the same zero set as Δ_S. Under this analytic admissibility, the reciprocal sensor $\mathcal{J}_S = 1/G_S$ is a canonical blow-up detector for the candidate.*

5 Bounded Cayley fields and the Schur/Herglotz pinch

5.1 The Cayley transform

Definition 18 (Bounded transform)**.** *Define the bounded field Ξ_S by*

$$\Xi_S(z) \;=\; \frac{2\mathcal{J}_S(z) - 1}{2\mathcal{J}_S(z) + 1}. \tag{6}$$

Lemma 6 (Cayley transform preserves rationality)**.** *If $\mathcal{J}_S$ is a rational function of z, then Ξ_S defined by* (6) *is rational. Conversely, if Ξ_S is rational and not identically* 1*, then the Cayley inverse*

$$2\mathcal{J}_S(z) \;=\; \frac{1 + \Xi_S(z)}{1 - \Xi_S(z)} \tag{7}$$

is rational on its domain of definition, hence $\mathcal{J}_S$ is rational.

Proof. This is immediate from algebra: the Cayley transform (6) and its inverse (7) are obtained by a finite number of additions, multiplications, and divisions. These operations preserve rationality. The nontriviality condition $\Xi_S \not\equiv 1$ ensures the inverse expression is not identically undefined. $\square$

Lemma 7 (Pole-to-boundary behavior)**.** *If $\mathcal{J}_S(z) \to \infty$ along an approach to $z = z_\star$, then $\Xi_S(z) \to 1$ along that approach.*

Proof. Write

$$\Xi_S(z) - 1 \;=\; \frac{2\mathcal{J}_S(z) - 1}{2\mathcal{J}_S(z) + 1} - 1 \;=\; \frac{-2}{2\mathcal{J}_S(z) + 1}.$$

If $\mathcal{J}_S(z) \to \infty$ along an approach, then the denominator tends to ∞ and hence $\Xi_S(z) - 1 \to 0$. $\square$

Lemma 8 (Right half-plane to unit disk)**.** *If $\Re(\mathcal{J}_S(z)) > 0$ on a region, then $|\Xi_S(z)| < 1$ on that region.*

Proof. Let $w = \mathcal{J}_S(z)$ and $\Xi = \frac{2w-1}{2w+1}$. Then $|\Xi| < 1$ is equivalent to $|2w - 1| < |2w + 1|$. Squaring both sides gives

$$|2w - 1|^2 < |2w + 1|^2 \iff (2w - 1)(2\overline{w} - 1) < (2w + 1)(2\overline{w} + 1) \iff -2(w + \overline{w}) < 2(w + \overline{w}),$$

which is equivalent to $\Re(w) > 0$. $\square$

5.2 Why a Schur bound prevents poles (removability pinch)

Lemma 9 (Removable singularity under a Schur bound)**.** *Let $D \subset \mathbb{C}$ be a disc centered at ρ. If Ξ is holomorphic on $D \setminus \{\rho\}$ and satisfies $|\Xi| < 1$ there, then Ξ extends holomorphically to D.*

Proof. Since Ξ is bounded on the punctured disc, Riemann's removable singularity theorem applies; see, e.g., [8]. $\square$

Corollary 4 (Schur bound prevents poles of the Cayley inverse)**.** *Let $U \subset \Omega$ be a domain. Assume Ξ is meromorphic on U and satisfies $|\Xi| \leq 1$ on U away from its poles. Then Ξ extends holomorphically to U, and the Cayley inverse*

$$2\mathcal{J} \;=\; \frac{1 + \Xi}{1 - \Xi}$$

extends holomorphically to U; in particular $\mathcal{J}$ has no poles in U.

Proof. The poles of Ξ form a discrete subset of U. On a punctured disc around any pole, $|\Xi| \leq 1$ implies Ξ is bounded, hence removable by Lemma 9. Thus Ξ extends holomorphically across all its poles. If Ξ is not identically 1 on U, then $\Xi \neq 1$ on U (otherwise $|\Xi|$ would achieve its maximum 1 at an interior point and the Maximum Modulus Principle would force $\Xi \equiv 1$). Therefore the Cayley inverse $(1 + \Xi)/(1 - \Xi)$ is holomorphic on U and equals $2\mathcal{J}$ wherever originally defined, so $\mathcal{J}$ has no poles in U. $\square$

Remark 12 (Degenerate case $\Xi \equiv 1$)**.** *If $\Xi(s_0) = 1$ at an interior point and $|\Xi| \leq 1$ in a connected region, then $\Xi \equiv 1$ by the Maximum Modulus Principle. In RSA applications one excludes this by a normalization (e.g. a limit at "infinity") or by an explicit nontriviality check.*

6 Why finite sampling needs a complexity bound

The following elementary fact is the key "referee objection" RSA must address.

Proposition 4 (No finite sampling can rule out poles without extra structure)**.** *Fix distinct sample points $z_1, \ldots, z_n \in \mathbb{C}$ and target values $w_1, \ldots, w_n \in \mathbb{C}$. For any point $a \in \mathbb{C} \setminus \{z_1, \ldots, z_n\}$ there exists a meromorphic function f on $\mathbb{C}$ such that $f(z_k) = w_k$ for all k and f has a pole at a.*

Proof. Let p be the Lagrange interpolating polynomial with $p(z_k) = w_k$. Define

$$f(z) \;:=\; p(z) \;+\; \frac{(z-z_1)\cdots(z-z_n)}{z-a}.$$

Then $f(z_k) = p(z_k) = w_k$ for all k, and f has a pole at $z = a$. □

Remark 13 (Tail risk and the Pick-gap-plus-tail guardrail)**.** *Therefore RSA cannot honestly claim that "8 samples" alone give global pole-freeness. Any finite sampling regime is vulnerable to* tail risk*: without additional structure, the true audited field can "wiggle" between sample points (or develop an interior singularity) while still matching the sampled values. RSA therefore never treats finite samples as a global certificate by themselves.*

To make a finite computation global one needs a finite-complexity control statement*: either a finite-dimensional restriction (finite state/degree, as derived from 8-tick realizability in Theorem 4), or a proved tail bound in a norm controlling the interior. The coefficient* Pick-gap-plus-tail *mechanism (Proposition 6) is the explicit guardrail: it converts a finite spectral gap together with a quantitative tail bound into positivity of the infinite Pick operator, hence a global Schur conclusion. In the stable realization regime, such tail bounds can be derived from stability data (Lemma 11).*

7 Eight-tick discretization (operational sampling form)

Recognition Science motivates an eight-phase cadence (an "eight-tick window"). In the full-derivation route, this cadence is not merely philosophical: it is the operational reason that *finite-window certificates* are the only admissible kind of audit. Section 3 made this explicit by modeling one-tick reachability $\widehat{R}$ and proving that finite local branching implies finite 8-tick outcome sets (Proposition 2), i.e. a concrete route to finite local resolution.

Once an audited domain Ω is normalized to the unit disk $\mathbb{D}$ by a chart $\psi : \Omega \to \mathbb{D}$, one natural way to represent an eight-tick audit is to allocate one sample per tick on a symmetric sampling pattern on $\mathbb{D}$. We record the standard Nevanlinna–Pick matrix for *values* at these eight points, and we emphasize exactly what such a finite value certificate does (and does not) certify without additional finite-complexity control.

Definition 19 (Eight-tick sampling set)**.** *Fix $r \in (0, 1)$ and define the eight roots of unity $\omega_k = e^{2\pi i k/8}$, $k = 0, \ldots, 7$. The eight-tick sampling points are*

$$z_k \;=\; r\,\omega_k \in \mathbb{D}.$$

Definition 20 (Value-sample Pick matrix)**.** *Let $\theta : \mathbb{D} \to \mathbb{C}$ be holomorphic and set $\xi_k = \theta(z_k)$ at the points in Definition 19. Define the* 8×8 value-sample Pick matrix

$$P^{\mathrm{val}} \;=\; \left[\frac{1-\xi_i\overline{\xi_j}}{1-z_i\overline{z_j}}\right]_{i,j=0}^{7}.$$

Proposition 5 (Schur ⇒ value-sample Pick positivity)**.** *If θ is Schur on $\mathbb{D}$, then $P^{\mathrm{val}} \succeq 0$.*

Remark 14 (What finite Pick positivity actually means)**.** *If $P^{\mathrm{val}} \succeq 0$, the Pick criterion implies that there exists* some *Schur function $\widetilde{\theta}$ with $\widetilde{\theta}(z_k) = \theta(z_k)$ for $k = 0, \ldots, 7$. This does* not *by itself imply that the specific function θ is Schur (Proposition 4). To use value-sample positivity as a global certificate for this specific function θ, one must also control uniqueness: either θ is known to lie in a finite-dimensional class where finitely many data determine θ (e.g. the finite-state/rational regime of Theorem 4), or one proves a quantitative deviation bound (a tail bound) controlling $\theta - \widetilde{\theta}$ in the interior.*

8 Finite certificates for global Schur control

RSA reduces impossibility claims to the absence of poles of a sensor, via a Schur bound for the associated Cayley field. To make this *algorithmic*, we need a finite certificate that implies a global Schur bound on $\mathbb{D}$. We record two complementary routes: (i) a *state-space bounded-real* certificate that is exact when a finite-dimensional realization is available, and (ii) a *Pick-gap-plus-tail* certificate that is robust when working from Taylor data with rigorous error bars.

8.1 State-space bounded-real certificate (exact in the finite-dimensional class)

Definition 21 (State-space realization). *A (discrete-time) state-space realization of a scalar holomorphic function $\theta : \mathbb{D} \to \mathbb{C}$ is a quadruple (A, B, C, D) with $A \in \mathbb{C}^{d\times d}$, $B \in \mathbb{C}^{d\times 1}$, $C \in \mathbb{C}^{1\times d}$, $D \in \mathbb{C}$ such that*

$$\theta(z) \;=\; D \;+\; z\,C\,(I - zA)^{-1}B. \tag{8}$$

Theorem 5 (Discrete-time bounded-real lemma (scalar form)). *Assume θ admits a realization (A, B, C, D) with $\rho(A) < 1$ (spectral radius strictly less than 1), so that (8) is holomorphic on $\mathbb{D}$. Then θ is Schur on $\mathbb{D}$ (i.e. $|\theta(z)| \le 1$ for all $z \in \mathbb{D}$) if and only if there exists a Hermitian matrix $P \succ 0$ such that*

$$\begin{pmatrix} P - A^*PA - C^*C & -A^*PB - C^*D \\ -B^*PA - D^*C & 1 - B^*PB - D^*D \end{pmatrix} \;\succeq\; 0. \tag{9}$$

This is the standard discrete-time bounded-real (KYP) lemma; see, e.g., [7].

Remark 15 (Why this matters for the full-derivation route). *Theorem 5 turns Schur certification into a finite-dimensional semidefinite feasibility problem. In the full-derivation route, RG4 yields finite-dimensionality (Section 3, Theorem 3). Recognition Cost supplies a canonical source of contraction in log-coordinates via the proximal tick model (Section 2.3, Lemma 1); and strict contraction implies a stable linearization (Lemma 2), which is the structural origin of the stability hypothesis $\rho(A) < 1$ in state-space certification. This route is* exact *on the finite-dimensional class: there is no tail bound to assume.*

Remark 16 (KYP/LMI as a cost (dissipation) inequality). *Control-theoretically, the matrix P is a* storage *(Lyapunov) certificate for the state x_k of the realization: one interprets $V(x) = x^*Px$ as an "internal cost" and the LMI (9) as the existence of a quadratic dissipation inequality of the form*

$$V(x_{k+1}) - V(x_k) \;\le\; \|u_k\|^2 - \|y_k\|^2$$

along all trajectories of the state-space system. This is precisely the formal meaning of "contractive/passive dynamics" for a finite-dimensional recognition process. Thus, in the full-derivation route, the key RS-facing obligation is not "assume bounded-real" but rather: exhibit a cost-driven mechanism that yields such a storage inequality (perhaps after passing to log-coordinates and a finite-resolution quotient), thereby making the bounded-real certificate intrinsic. Corollary 5 records one explicit route in a cost-contracting regime.

Corollary 5 (Intrinsic certification in the cost-contracting regime). *Suppose the pulled-back Cayley field θ admits a finite-dimensional realization (8) whose one-tick state update $x \mapsto Ax$ arises as the linearization of a differentiable strict contraction map with Lipschitz constant $L < 1$ (for example, a differentiable coordinate model of the RS proximal tick in Section 2.3). Then $\rho(A) < 1$ (Lemma 2), so θ is holomorphic on $\mathbb{D}$ and both certification regimes become fully finite without additional stability or tail assumptions:*

- *the bounded-real (KYP/LMI) test applies in state space (Theorem 5);*
- *the coefficient Pick-gap certificate can use a derived tail bound from realization data (Lemma 11), hence implies a global Schur bound by Theorem 7.*

8.2 Schur class and Pick kernels on the disk

Definition 22 (Schur kernel)**.** *Let $\theta : \mathbb{D} \to \mathbb{C}$ be holomorphic. Define its Pick kernel*

$$K_\theta(z, w) \;:=\; \frac{1 - \theta(z)\overline{\theta(w)}}{1 - z\overline{w}}.$$

Theorem 6 (Pick criterion, functional form)**.** *A holomorphic $\theta : \mathbb{D} \to \mathbb{C}$ is Schur (i.e. $|\theta| \le 1$ on $\mathbb{D}$) if and only if K_θ is a positive semidefinite kernel on $\mathbb{D}$ (equivalently: every finite Pick matrix $[K_\theta(z_i, z_j)]$ is positive semidefinite).*

Remark 17. *This is the classical Nevanlinna–Pick/Schur criterion; see [5, Ch. 2] or [6].*

8.3 Coefficient Pick matrix and a stability lemma

Definition 23 (Coefficient Pick matrix)**.** *Write $\theta(z) = \sum_{n\ge 0} a_n z^n$ and expand*

$$K_\theta(z, w) \;=\; \sum_{i,j\ge 0} P_{ij}\, z^i \overline{w}^{\,j}.$$

The infinite Hermitian matrix $P = [P_{ij}]_{i,j\ge 0}$ is the coefficient Pick matrix *of θ.*

Lemma 10 (Coefficient formula)**.** *With notation above, for all $i, j \ge 0$,*

$$P_{ij} \;=\; \delta_{ij} \;-\; \sum_{k=0}^{\min\{i,j\}} a_{i-k}\, \overline{a_{j-k}}.$$

Equivalently $P = I - AA^*$ where A is the lower-triangular Toeplitz matrix $A_{ij} = a_{i-j}$ for $i \ge j$ and 0 for $i < j$.

Definition 24 (Weighted tail)**.** *For $N \ge 1$, define the Dirichlet-type tail size*

$$\varepsilon_N^2 \;:=\; \sum_{n\ge N} (n+1)\, |a_n|^2.$$

Lemma 11 (Tail bound from a contractive realization)**.** *Assume θ admits a state-space realization (8) and write its Taylor coefficients as $\theta(z) = \sum_{n\ge 0} a_n z^n$. If $\|A\| \le \rho < 1$ (operator norm on $\mathbb{C}^d$), then for every $N \ge 1$,*

$$\varepsilon_N^2 \;=\; \sum_{n\ge N} (n+1)\, |a_n|^2 \;\le\; \|C\|^2\, \|B\|^2 \left(\frac{(N+1)\rho^{2(N-1)}}{1-\rho^2} + \frac{\rho^{2N}}{(1-\rho^2)^2} \right).$$

Proof. Expanding (8) as a power series gives $a_0 = D$ and, for $n \ge 1$,

$$a_n \;=\; C\, A^{n-1} B.$$

Therefore $|a_n| \le \|C\| \, \|A\|^{n-1} \, \|B\| \le \|C\| \, \rho^{n-1} \, \|B\|$ for all $n \ge 1$. Hence, for any $N \ge 1$,

$$\varepsilon_N^2 \;=\; \sum_{n \ge N} (n+1) \, |a_n|^2 \;\le\; \|C\|^2 \, \|B\|^2 \sum_{n \ge N} (n+1) \rho^{2(n-1)}.$$

Let $r = \rho^2 \in (0,1)$. A standard geometric-sum computation yields

$$\sum_{n \ge N} (n+1) r^{n-1} \;=\; \frac{(N+1) r^{N-1}}{1-r} + \frac{r^N}{(1-r)^2}.$$

Substituting $r = \rho^2$ gives the stated bound. $\square$

Lemma 12 (Tail-to-infinite stability)**.** *There is an absolute constant $C \le 2$ such that, for each $N \ge 1$, the operator difference $P(\theta) - P(\theta^{(\le N-1)})$ has norm at most $C \, \varepsilon_N$, where $\theta^{(\le N-1)}(z) = \sum_{n=0}^{N-1} a_n z^n$.*

Proposition 6 (Finite gap + tail bound $\Rightarrow$ Schur)**.** *Let θ be holomorphic on $\mathbb{D}$ with coefficient Pick matrix $P(\theta)$. Fix $N \ge 1$ and assume:*

1. *(finite gap) the $N \times N$ principal minor satisfies $P_N(\theta) \succeq \delta I_N$ for some $\delta > 0$;*
2. *(tail bound) $C \, \varepsilon_N < \delta$ with C as in Lemma 12.*

Then $P(\theta) \succeq 0$ as an operator on $\ell^2(\mathbb{N}_0)$, hence θ is Schur on $\mathbb{D}$.

Theorem 7 (Finite certificate $\Rightarrow$ global Schur bound)**.** *Let $\theta : \mathbb{D} \to \mathbb{C}$ be holomorphic. Suppose one of the following holds:*

1. (State-space bounded-real certificate) *θ admits a stable realization (A, B, C, D) with $\rho(A) < 1$ and there exists $P \succ 0$ satisfying the bounded-real LMI* (9) *(Theorem 5);*
2. (Coefficient Pick gap + tail certificate) *there exist $N \ge 1$ and $\delta > 0$ such that $P_N(\theta) \succeq \delta I_N$ and $C\varepsilon_N < \delta$ (Proposition 6).*

Then θ is Schur on $\mathbb{D}$.

Proof. In case (1), Theorem 5 implies that θ is Schur. In case (2), Proposition 6 implies that θ is Schur. $\square$

Remark 18 (Algorithmic certificate)**.** *In applications, one computes $a_0, \dots, a_{N-1}$ with rigorous error bars, computes a certified spectral gap lower bound δ, and proves an explicit tail bound ε_N. The strict inequality $C\varepsilon_N < \delta$ converts the finite computation into a global Schur bound.* In the finite-dimensional realization regime, *a tail bound can be obtained directly from contractivity/stability data (Lemma 11), so the only remaining numerical work is to certify the finite gap.*

9 The Recognition Stability Audit (RSA)

Algorithm 1 (Recognition Stability Audit (RSA), certificate form)**.** ***Input:*** *A candidate state S and an audited region Ω.*

Output: *`IMPOSSIBLE_STATE` or `INCONCLUSIVE`.*

Step 0 (Structures to fix). *Fix:*

- *an RS-style defect/obstruction encoding of the candidate on Ω: either a defect functional Δ_S with semantics S at $z \Leftrightarrow \Delta_S(z) = 0$ (Definition 17), or directly a holomorphic obstruction G_S whose zero set encodes S (Definition 15). In either case, define the sensor by $\mathcal{J}_S := 1/G_S$ (Definition 16 and Section 4.1);*

- *(analytic admissibility) G_S is holomorphic on the chart domain and has a simple zero at any candidate point (so that $S \Rightarrow$ pole of $\mathcal{J}_S$ is automatic by Lemma 5);*

- *a domain normalization $\psi : \Omega \to \mathbb{D}$ on which $\mathcal{J}_S$ is holomorphic away from the candidate poles;*

- *a* realizable audited Cayley field model *for the pulled-back Cayley field $\theta(z) := \Xi_S(\psi^{-1}(z))$ in the full-derivation route:*

  - finite-window (8-tick) realizability: *an explicit 8-tick reachability model $(\mathcal{L}, \widehat{R})$ with finite local branching and a recognition-respecting induced quotient dynamics whose output generating function is θ; in particular θ is finite-state/rational (Theorem 4);*
  - cost-driven realizability: *a cost-governed one-tick dynamics (e.g. the RS proximal neutrality step in log-coordinates, Section 2.3) together with a differentiable coordinate model so that the induced tick is a strict contraction and its linearization yields a stable state update matrix A (Lemma 2); consequently θ admits a stable finite-dimensional realization and lies in the intrinsic finite certification regime (Corollary 5).*

Step 1 (Bounded field). *Form Ξ_S by* (6) *and pull back $\theta(z) = \Xi_S(\psi^{-1}(z))$.*
Step 2 (Finite Schur certification). *Choose a certification regime:*

- (State-space regime) *when a finite-dimensional realization of θ is available, certify Schur contractivity by the bounded-real LMI (Theorem 5).*

- (Coefficient regime) *compute Taylor data for θ and certify a finite Pick gap plus tail bound (Proposition 6).*

- (Point-sample regime) *sample θ at finitely many points and certify positivity together with the derived finite-complexity control (e.g. a rational/finite-state realization as in Theorem 3) that makes finite interpolation global.*

Any successful certificate implies that θ is Schur on $\mathbb{D}$ (Theorem 7).
Step 3 (Pinch). *If the Schur certificate succeeds on $\mathbb{D}$ (hence on Ω), conclude that $\mathcal{J}_S$ has no poles in Ω (Corollary 4).*
Step 4 (Decision). *If the candidate mechanism requires such a pole in Ω, return `IMPOSSIBLE_STATE`. Otherwise return `INCONCLUSIVE`. In the finite-dimensional rational regime, this can be upgraded to a full decision procedure by an existence-side root test (Algorithm 2).*

Implementation checklist (what an application must prove)

To instantiate RSA in a new domain, the following items are the usual proof obligations:

- **(S1) RS defect/obstruction encoding:** provide a defect functional Δ_S with semantics $S \Leftrightarrow \Delta_S = 0$ (Definition 17) and an analytic representative G_S with the same zeros (Section 4.1); then $\mathcal{J}_S := 1/G_S$ is the canonical sensor (Definition 16) and $S \Rightarrow$ blow-up is a reduction lemma (Lemma 5).

- **(S2) Analytic class:** the pulled-back Cayley field belongs to a class where global control is meaningful (e.g. holomorphic on the chart domain).
- **(S3) Finite-to-global bridge:** a derived finite-complexity restriction (finite-state/rational realization) sufficient to globalize a finite certificate (Section 3, Theorem 3); in practice one may discharge this via an explicit realization, or by a certified tail bound when working in coefficient form.
- **(S4) Nontriviality:** exclude the degenerate case $\Xi \equiv 1$ on the audited region.
- **(S5) Audit artifacts:** record the finite certificate (matrix gap, tail bound) in a reproducible form.

10 Correctness theorem (what RSA actually proves)

Theorem 8 (Schur pinch $\Rightarrow$ impossibility in the audited region)**.** *Assume:*

1. *(Correct sensor reduction) the candidate S is encoded by an RS defect/obstruction pair on Ω (Section 4.1) with a holomorphic representative G_S having a simple zero at any candidate point, and $\mathcal{J}_S := 1/G_S$;*
2. *(Schur bound) the Cayley field Ξ_S is Schur on Ω (i.e. $|\Xi_S| \le 1$ on Ω);*
3. *(Nontriviality) $\Xi_S \not\equiv 1$ on Ω.*

Then the candidate S does not hold at any point of Ω.

Proof. Suppose for contradiction that S holds at some point $z_\star \in \Omega$. By hypothesis (Schur bound) and (Nontriviality), Corollary 4 applies and implies that the Cayley inverse

$$2\mathcal{J}_S = \frac{1+\Xi_S}{1-\Xi_S}$$

extends holomorphically across Ω; in particular, $\mathcal{J}_S$ has *no poles* in Ω.

On the other hand, by hypothesis (Correct sensor reduction), S at $z_\star$ forces $G_S(z_\star) = 0$ and $\mathcal{J}_S := 1/G_S$. Since G_S is holomorphic near $z_\star$ with a simple zero, Lemma 5 implies that $\mathcal{J}_S = 1/G_S$ has a pole at $z_\star$. This contradicts the pole-freeness of $\mathcal{J}_S$ on Ω established above. □

Corollary 6 (RSA soundness in certificate form)**.** *In the setting of Algorithm 1, if RSA succeeds in Step 2 in certifying a Schur bound for Ξ_S on Ω and $\Xi_S \not\equiv 1$, then RSA's output `IMPOSSIBLE_STATE` is correct: the candidate does not occur in Ω.*

Proof. Under the stated hypotheses, Theorem 8 applies directly. □

11 Flagship Instantiations: Arithmetic and Geometry

We demonstrate the universality of the RSA template by summarizing two implementations: one in analytic number theory (Riemann Hypothesis) and one in complex geometry (Hodge Conjecture). These case studies show that the abstract "sensor" and "finite check" concepts map to concrete, standard mathematical objects.

11.1 Case I (Arithmetic): The Riemann Zeta Zero-Free Region

Remark 19 (Admissibility tier for Case I). *This instantiation is presented as a* recognition-admissible impossibility certificate *(Definition 25): a correct sensor reduction and a finite Schur certificate yield a terminating* `IMPOSSIBLE_STATE` *conclusion on the audited far-field region. It is not used here as a full decision procedure in the sense of Theorem 10; the rational-regime decision mechanism applies when the pulled-back obstruction is a bounded-degree rational function.*

Target: Rule out zeros of $\zeta(s)$ in the far-field half-plane $\Re s \geq 0.6$.

- **Monster:** A zero ρ of $\zeta(s)$ with $\Re\rho \geq 0.6$.
- **Sensor:** An arithmetic ratio $\mathcal{J}(s)$ whose poles in $\Re s > 1/2$ encode zeros of ζ (e.g. $\mathcal{J}(s) = \frac{\det_2(I-A(s))}{\zeta(s)} \cdot \frac{s}{s-1}$ in a raw gauge; see [12]).
- **Mechanism:** If $\zeta(\rho) = 0$, the denominator vanishes, forcing $\mathcal{J}(\rho)$ to have a pole.
- **Bounded Field:** $\Theta(s) = \frac{2\mathcal{J}(s)-1}{2\mathcal{J}(s)+1}$. Pole $\Rightarrow \Theta(\rho) \to 1$.
- **Finite Check (Pick gap + tail).** In the audited far-field implementation [12], a 16×16 coefficient Pick minor is certified SPD at $\sigma_0 = 0.7$ with spectral gap $\delta_{\mathrm{cert}} = 0.6273368612$, and the coefficient tail is certified as $\sum_{n\geq 16}(n+1)|a_n(0.7)|^2 \leq 0.0127$, hence $\varepsilon_{16} \leq 0.113$. With a perturbation constant $C \leq 2$, one checks $C\varepsilon_{16} \leq 0.226 < 0.627 = \delta_{\mathrm{cert}}$, yielding a global Schur bound on $\{\Re s > 0.7\}$.
- **Hybrid cover to $\Re s \geq 0.6$.** A certified rectangle bound on $[0.6, 0.7] \times [0, 20]$ plus an explicit far-$|t|$ bound extends the Schur property to $\{\Re s > 0.6\}$ [12].
- **Result:** The Schur/Herglotz pinch removes poles of $\mathcal{J}$ in the audited region, hence excludes zeros of ζ there: `IMPOSSIBLE_STATE` for a far-field counterexample.

(See [12] for a fully auditable implementation with certified artifacts.)

11.2 Case II (Geometry): The Hodge Conjecture

Remark 20 (Role of Case II). *Case II is explicitly an* existence-side *architecture sketch intended to complement RSA in geometric settings (Definition 26). It is labeled as an architecture sketch because the full domain adapter theorems (that identify the audited Cayley field with a cost-contracting realizable class, and that provide the needed defect/obstruction encoding with analytic admissibility) are substantial geometric proofs and are recorded in companion manuscripts.*

Target (architecture sketch with quantitative modules): Record how a Hodge-type existence question can be organized into finite-resolution modules with explicit tail controls, suitable as an *existence-side* complement to RSA (Definition 26). **Reframed as Impossibility:** Rule out a persistent "non-algebraic gap" (a strictly positive lower bound on a calibration/cohomological defect) by constructing a refinement sequence whose defect is forced to vanish.

- **Monster:** A rational Hodge class γ that cannot be realized by an algebraic cycle.
- **Finite-resolution direction labels (dictionary module).** A stable dictionary fit produces per-cell weights and robust discrete labels (winner-take-all away from ties) for strongly positive (p, p) data; see [13].

- **Holomorphic manufacturing (analytic module).** On a projective Kähler manifold, large tensor powers admit holomorphic complete intersections whose local sheets match prescribed tangent templates with uniform C^1 control on the Bergman scale $m_{\text{hol}}^{-1/2}$; see [14].
- **Deterministic interface geometry (corner-exit module).** Corner-exit slivers force deterministic face incidence under small C^1 perturbations and yield per-face boundary mass control $\simeq v^{(k-1)/k}$; see [15, Prop. (G1)–(G2)].
- **Combinatorial coherence (prefix module).** A prefix-template selection rule confines mismatch across a face to a terminal tail, and under slow variation the unmatched tail has $O(h)$ relative size; see [16].
- **Tail control via flat norm (gluing module).** A global weighted estimate bounds the residual boundary in flat norm,

  $$\mathcal{F}(\partial T^{\text{raw}}) \;\lesssim\; \varrho\, h^2 \sum m_{Q,a}^{(k-1)/k},$$

  and implies a vanishing-mass correction when $\mathcal{F}(\partial T^{\text{raw}}) \to 0$; see [17].
- **Exact homology via period locking (quantization module).** Fixed-dimension discrepancy rounding controls all periods to $< 1/4$, and a tiny boundary correction then forces exact integrality by lattice locking; see [18].
- **Validation on toy models.** As a sanity check, this calibrated-current machinery recovers the standard (p,p)-cohomology generators on projective space $\mathbb{CP}^n$ by strictly minimizing the comass norm, reproducing known results via the variational pathway.

Together these modules supply a plausible *existence-side certificate* architecture for Hodge-type problems: finite-resolution recognition produces coherent local pieces, a quantified tail bound forces vanishing boundary defect under refinement, and integrality locks the global class.

12 The Universal Recognition Class

These examples suggest a generalization. RSA is not merely a heuristic: on a well-defined admissible class it gives a reproducible *impossibility certificate*. Upgrading to a full *decision* procedure is automatic in the finite-dimensional rational regime (Theorem 10); outside that regime it requires additional completeness hypotheses (i.e. a complementary existence witness mechanism in the sense of Definition 26).

Definition 25 (Recognition-Admissible (impossibility form))**.** *A mathematical existence problem P is* recognition-admissible for impossibility *if it can be transformed into data $(\Omega, \Delta_P, G_P, \mathcal{C}_{fin})$ where:*

1. *Ω is a domain with a valid complex-analytic or geometric structure.*
2. *$\Delta_P : \Omega \to \mathbb{R}_{\geq 0}$ is a defect functional with semantics P at $z \Leftrightarrow \Delta_P(z) = 0$ (Definition 17).*
3. *G_P is a holomorphic representative on Ω with the same zero set as Δ_P (analytic admissibility; Section 4.1); define the canonical sensor $\mathcal{J}_P := 1/G_P$ (Definition 16). Assume that at any candidate point the zero of G_P is simple, so that $P \Rightarrow$ pole of $\mathcal{J}_P$ is automatic (Lemma 5).*
4. *$\mathcal{C}_{fin}$ is a finite-complexity control regime (finite-state/rational/tail-bounded) sufficient to convert a finite certificate into global Schur control on Ω.*

Definition 26 (Recognition-Admissible (decidable form)). *A problem P is* recognition-admissible for decision *if, in addition to Definition 25, there exists a complementary* existence-side *mechanism that yields a terminating certificate of P when P holds (e.g. a constructive witness, or an exact analytic reconstruction of G_P from finite data).*

Theorem 9 (A proved URC subclass: realizable audited Cayley fields). *Let P be a mathematical existence problem on a domain Ω. Assume P admits the Step 0 structures of Algorithm 1, including: (i) an RS defect/obstruction encoding with analytic admissibility and simple zeros, and (ii) the* realizable audited Cayley field model *(finite-window 8-tick realizability and cost-driven realizability) for the pulled-back Cayley field θ. Then P is recognition-admissible for impossibility (Definition 25).*

If, in addition, after normalization $\psi : \Omega \to \mathbb{D}$ the pulled-back obstruction $\widetilde{G}_P(z) = G_P(\psi^{-1}(z))$ is a bounded-degree rational function over a computable number field (as in Theorem 10), then P is recognition-admissible for decision (Definition 26).

Proof. By Step 0, P comes with a defect/obstruction encoding, a holomorphic representative G_P with simple zeros at candidate points, and a normalization to the disk, so the analytic items of Definition 25 are satisfied. The Step 0 realizability model supplies an explicit finite-complexity control regime $\mathcal{C}_{\text{fin}}$: finite-window realizability yields a finite-state/rational audited class for the Cayley field (Theorem 4), and cost-driven realizability places θ in the intrinsic finite certification regime (Corollary 5). Thus P is recognition-admissible for impossibility.

If the additional rational/degree/computability hypothesis holds, Theorem 10 furnishes a terminating existence-side mechanism (root witness vs. certified no-root), hence Definition 26 holds. □

Theorem 10 (Decision in the finite-dimensional rational class). *Assume P is recognition-admissible for impossibility (Definition 25) and that, after a domain normalization $\psi : \Omega \to \mathbb{D}$, the pulled-back obstruction*

$$\widetilde{G}_P(z) \ := \ G_P(\psi^{-1}(z))$$

is a rational *function of z with coefficients in a computable number field and with a known degree bound. Then there is a terminating decision procedure that either:*

- *produces a witness point $z_\star \in \mathbb{D}$ with $\widetilde{G}_P(z_\star) = 0$ (hence P holds at $\psi^{-1}(z_\star)$), or*
- *certifies that $\widetilde{G}_P$ has no zeros in $\mathbb{D}$ (hence P does not hold anywhere in Ω).*

Proof. Write $\widetilde{G}_P(z) = p(z)/q(z)$ with coprime polynomials p, q over the coefficient field; then $\widetilde{G}_P(z_\star) = 0$ if and only if $p(z_\star) = 0$. Since p has bounded degree and computable coefficients, there are standard terminating exact procedures to decide whether p has a root in the open unit disk (e.g. Schur–Cohn/Jury-type root-location tests, or algebraic root isolation over number fields). If such a root exists, the procedure returns one (as an algebraic number, hence an explicit witness); otherwise it certifies that no such root exists. □

Remark 21 (Why the rational decision hypothesis is natural in the full-derivation route). *In the full-derivation route, finite local resolution plus recognition-respecting dynamics yields a finite-state/rational class for the audited Cayley field (Theorem 3). Rationality is preserved under the Cayley transform and its inverse (Lemma 6), so whenever the Cayley field is rational, the associated sensor $\mathcal{J}_P$ and obstruction $G_P = 1/\mathcal{J}_P$ are rational as well (away from the degenerate case $\Xi \equiv 1$). Thus Theorem 10 captures the most literal "finite description $\Rightarrow$ finite decision" regime.*

Corollary 7 (Rational regime implies decidable admissibility). *Under the hypotheses of Theorem 10, the problem P is recognition-admissible for decision (Definition 26).*

Proof. Theorem 10 provides a terminating existence-side mechanism (a witness root when P holds, or a certified no-root conclusion when P fails). $\square$

Algorithm 2 (Decision procedure in the finite-dimensional rational class)**.** ***Input:*** *A problem P in the admissibility class of Definition 25, together with a normalization $\psi : \Omega \to \mathbb{D}$ and a rational representation (with degree bound) of the pulled-back obstruction $\widetilde{G}_P(z) = G_P(\psi^{-1}(z))$.*

Output: *`WITNESS` (with $z_\star$) or `IMPOSSIBLE_STATE`.*

Step 1 (Optional nonexistence certificate). *Attempt a finite Schur certificate for the pulled-back Cayley field θ (the field certified in Step 2 of Algorithm 1). If a Schur bound is certified on $\mathbb{D}$ and $\theta \not\equiv 1$, return `IMPOSSIBLE_STATE` by Corollary 6.*
Step 2 (Exact root test). *Write $\widetilde{G}_P(z) = p(z)/q(z)$ with coprime polynomials over the coefficient field. Use an exact terminating root-location procedure to decide whether p has a root in $\mathbb{D}$ (Theorem 10).*
Step 3 (Decision). *If a root $z_\star \in \mathbb{D}$ exists, output `WITNESS` with $z_\star$ (so P holds at $\psi^{-1}(z_\star)$). Otherwise output `IMPOSSIBLE_STATE`.*

Proposition 7 (RSA is a semi-decision procedure for impossibility)**.** *For any problem in the admissibility class of Definition 25, RSA is a terminating certificate procedure for `IMPOSSIBLE_STATE` whenever the Schur certificate succeeds on the audited region.*

Proof. Fix P in the admissibility class of Definition 25. By construction there is a holomorphic obstruction G_P whose zero set encodes P, and the canonical sensor is $\mathcal{J}_P := 1/G_P$. By the simple-zero hypothesis, if P held at some $z_\star \in \Omega$ then $\mathcal{J}_P$ would have a pole at $z_\star$ (Lemma 5).

RSA forms the Cayley field and runs a finite Schur certification procedure in a finite-complexity regime. When this Schur certificate succeeds on the audited region, the Schur pinch (Corollary 4) implies that $\mathcal{J}_P$ has no poles in that region. Therefore P cannot hold there, and RSA returns `IMPOSSIBLE_STATE`. $\square$

Remark 22 (Decision requires an existence-side certificate)**.** *RSA by itself is not an existence prover; therefore Definition 26 makes explicit that a full decision procedure requires an additional mechanism beyond the Schur/pole-exclusion audit.*

Remark 23 (RSA + existence modules (how URC_dec is intended to be used))**.** *In practice, a URC_dec claim is typically assembled from two complementary pieces: (i) an RSA-style* impossibility *certificate that rules out candidates by excluding sensor blow-up, and (ii) a domain-specific* existence *architecture that constructs (or certifies) admissible witnesses when the object exists. Case II outlines one such existence architecture (dictionary $\to$ manufacturing $\to$ deterministic interfaces $\to$ flat-norm tail control $\to$ period locking), intended as the complement to RSA in geometric settings.*

13 Gödel/undecidability objections and scope

Claims of "universal algorithms" in mathematics are immediately met with Gödelian and Turing-style objections. This manuscript avoids contradiction by an explicit scope restriction: RSA is an *audit* for stability of *configurations* in a cost-theoretic ontology. It does not claim a decision procedure for arbitrary formal sentences, and it does not claim to settle Gödel-style questions about provability in arithmetic.

Gödel's setup vs. RSA's setup

Gödel's incompleteness theorems concern effectively axiomatized formal proof systems capable of encoding arithmetic and an internal provability predicate; truth is evaluated externally (e.g. in the standard model) [1, 2]. Recognition Science instead treats "truth" as stabilization/cost-minimization and "existence" as convergence to zero defect; this is a different target [11, 10].

Self-reference and "non-configurations"

A key mechanism behind incompleteness is self-reference. In the RS ontology, the direct analog is a *self-referential stabilization query* (a configuration asserting its own non-stabilization). The Gödel dissolution manuscript argues that such objects have no fixed point under coercive dynamics and are therefore *outside the ontology* ("non-configurations") [11]. For the purposes of this manuscript, the key point is the scope consequence: self-referential "this does not stabilize" objects are excluded from the recognition-admissible input class, so Gödel-style diagonal statements do not arise as configurations that RSA is meant to audit.

Example: RSA failure on the Halting Problem

To demonstrate that RSA respects undecidability (and to make the boundary concrete), consider applying it to the Halting Problem. If one attempts to construct a sensor $\mathcal{J}_H$ that blows up exactly when a Turing machine M halts, the attempt fails at Step 0: either the sensor is not computable (evaluation requires an unbounded simulation), or the Step 0 realizability model needed to globalize a finite Schur certificate cannot be discharged without already deciding whether M halts. In either case the audit cannot produce a terminating certificate, and it correctly returns `INCONCLUSIVE` (or the instantiation fails to meet the admissibility requirements), reflecting that RSA cannot decide undecidable problems.

What this does and does not buy

This resolves the *logical* objection "does RSA contradict incompleteness?" by clarifying that RSA does not target unrestricted arithmetic truth. It does *not* by itself prove that all mathematical problems admit the Step 0 realizability model (and hence lie in the admissible class of Section 12); that remains a substantive coverage conjecture requiring domain-by-domain reductions and finite-complexity control.

Conjecture: Coverage of the realizable Cayley model

We conjecture that *all well-posed existence problems in physical geometry and arithmetic* can be reduced into the Step 0 framework of Algorithm 1 in its *full-derivation* form: namely, each such problem admits an analytically admissible RS defect/obstruction encoding and a realizable audited Cayley field model (finite-window 8-tick realizability plus cost-driven contractive dynamics) so that the proved mechanism of Theorem 9 applies. Concretely:

- **Number theory:** zeros of L-functions are encoded as obstruction zeros (hence sensor poles) and the audited Cayley field arises from a finite-window, certified recognition ratio (as in Case I).
- **Geometry:** existence of special cycles/metrics is encoded by coercive defect functionals with holomorphic representatives, and the audited Cayley field is governed by cost-driven coercive dynamics (as suggested by the Case II architecture).

- **Physics:** mass gaps and regularity questions are encoded by finite-cost recognition constraints whose induced tick dynamics is contractive under the canonical cost.

Under this conjecture, the open content is not the audit mechanism (proved in this manuscript) but the *coverage*: showing that the domains of interest admit the Step 0 realizability model.

Remark 24 (Relationship to "solve all math")**.** *Any claim approaching "all mathematical problems are solvable by a set algorithm" must be interpreted as a claim about the scope of the admissible class (Section 12) and/or about an RS-internal semantics of truth/existence. Without such a scope change, classical undecidability results apply.*

14 Limitations and open problems

- **Existence-side certificates outside the rational class.** RSA is designed for impossibility. In the finite-dimensional rational regime, an existence-side decision mechanism is available by exact root testing (Theorem 10). Beyond that regime (general holomorphic obstructions without a degree bound), existence-side certificates are genuinely domain-dependent.

- **Finite-complexity and stability justification.** RSA needs a finite-to-global bridge. Section 3 shows one explicit route: an 8-tick reachability model with finite local branching yields RG4 and a finite-state/rational class (Theorem 3). On the stability side, Recognition Cost supplies a canonical strict contraction mechanism in log-coordinates (Section 2.3, Lemma 1), and strict contraction implies stable linearization (Lemma 2); in stable realization regimes this also yields an explicit Taylor tail bound (Lemma 11). What remains open in broad generality is to connect the *domain-specific* audited Cayley field to a realization governed by such cost-driven dynamics, so that the stability hypotheses are discharged intrinsically rather than by external modeling.

- **URC coverage conjecture.** The claim that a very broad class of important problems admits the Step 0 realizability model (and hence falls in URC) is conjectural and should be tested domain-by-domain.

15 Conclusion

RSA packages a standard analytic mechanism (Cayley transform + Schur/Herglotz pinch) together with an explicit finite certification step that globalizes control on the unit disk. Two complementary certification regimes are recorded: a state-space bounded-real (KYP/LMI) certificate (exact in the finite-dimensional realization class) and a Pick-gap-plus-tail certificate from Taylor data (with tail bounds derivable from stable realization data; Lemma 11).

Its universality is the universality of a pipeline:

$$\text{candidate} \Rightarrow \text{obstruction zero} \Rightarrow \text{sensor pole} \Rightarrow \Xi \to 1.$$

When a strict Schur certificate succeeds (and the degenerate case $\Xi \equiv 1$ is excluded), the pinch removes poles of the sensor on the audited region; Theorem 8 then implies that the candidate does not occur there. In the finite-dimensional rational regime, an existence-side mechanism is also available by exact root-location tests (Theorem 10), yielding a full decision procedure (Algorithm 2).

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