The Fine-Structure Constant from Cube Geometry and the Eight-Tick Projection

Paper V of V: Derivation of α^{-1} in Recognition Science

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Abstract

The fine-structure constant $\alpha \approx 1/137$ is one of the most precisely measured quantities in physics, yet its value has resisted theoretical derivation from first principles for nearly a century. This paper presents a closed-form derivation of α^{-1} within Recognition Science (RS), using only three ingredients traced to the discrete geometry of the 3-cube and the eight-tick closure cycle:

1. A **geometric seed** $4\pi \cdot 11 = 44\pi \approx 138.230$, where $11 = E_{\text{passive}}$ is the passive edge count of the 3-cube and 4π is the solid angle of the full sphere.
2. A **gap weight** $f_{\text{gap}} = w_8 \cdot \ln \varphi \approx 1.199$, where $w_8 \approx 2.491$ is a parameter-free closed form derived from the DFT-8 projection of φ-weighted modes on the eight-tick cycle.
3. A **curvature correction** $\delta_\kappa = -103/(102\pi^5) \approx -0.00331$, where $102 = 6 \times 17$ (faces $\times$ wallpaper groups) and $103 = 102 + 1$ (Euler closure).

The resulting prediction is: [HYP]

$$\alpha^{-1} = 4\pi \cdot 11 - w_8 \cdot \ln \varphi + \frac{103}{102\pi^5} \approx 137.0349.$$

This corresponds to the Thomson limit (IR baseline at $Q^2 = 0$) and differs from the CODATA value $\alpha^{-1}_{\text{CODATA}} = 137.035\,999\,206(11)$ by $\sim 8.1\,\text{ppm}$—consistent with higher-order QED corrections not included in the zero-loop structural prediction.

All integers in the formula (11, 102, 103) are derived from the same counting layer used for particle masses (Papers I–IV). The gap weight w_8 has a parameter-free closed form whose equality with the DFT-8 projection is fully proved in Lean 4 (the `w8_projection_equality` theorem, with zero `sorry`).

No adjustable parameters appear at any stage.

Contents

1 Introduction

1.1 The fine-structure constant: status quo

The fine-structure constant

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.036} \tag{1}$$

governs the strength of electromagnetic interactions. It is measured to 12 significant digits by combining the electron $g-2$ anomalous magnetic moment with QED calculations at tenth order. Yet no theoretical framework has derived its value from first principles. The SM treats α as an input—a free parameter.

Attempts at deriving α from pure mathematics have a long and mostly discredited history (Eddington's numerology, Wyler's formula, etc.). What distinguishes the present work is that α^{-1} arises from the *same* discrete geometry (the 3-cube, the eight-tick closure, the counting layer) that organizes the entire particle mass spectrum. It is not an isolated formula; it is part of a coherent framework with machine-verified foundations.

1.2 What this paper derives

We derive α^{-1} as a sum of three terms, each with explicit combinatorial origin in cube geometry. The derivation yields the Thomson-limit (zero-momentum-transfer) value. The difference from the CODATA value ($\sim 8\,\text{ppm}$) is attributed to higher-order QED running effects between $Q^2 = 0$ and the measurement scale, which are not part of the zero-parameter structural prediction.

2 The Three Components

2.1 Component 1: The geometric seed $4\pi \cdot E_{\text{passive}}$

The dominant term in α^{-1} is: [HYP]

$$\alpha^{-1}_{\text{seed}} = 4\pi \cdot E_{\text{passive}} = 4\pi \cdot 11 = 44\pi \approx 138.2300. \tag{2}$$

The factor 4π is the solid angle subtended by the full sphere—the geometric weight of an isotropic recognition event in three dimensions. The factor $E_{\text{passive}} = 11$ counts the passive edges of the 3-cube (total edges $E = 12$ minus one active edge $A = 1$ per atomic tick).

Interpretation: the electromagnetic coupling strength at the Thomson limit is set by the number of passive geometric channels available for photon propagation on the cubic ledger, weighted by the isotropic solid angle.

2.2 Component 2: The gap weight $f_{\text{gap}} = w_8 \cdot \ln\varphi$

The gap weight subtracts a φ-dependent correction: [HYP]

$$f_{\text{gap}} = w_8 \cdot \ln\varphi \approx 2.49057 \times 0.48121 \approx 1.1985. \tag{3}$$

The weight w_8 is the central technical achievement of this paper. It arises from the DFT-8 (discrete Fourier transform on the eight-tick cycle) projection of φ-weighted modes.

2.2.1 Definition of w_8

Consider the eight-tick cycle as a discrete signal processing domain. The φ-ladder generates a characteristic spectrum when sampled at eight points. The DFT-8 of this spectrum yields mode amplitudes, and the projection onto the ledger-compatible subspace (modes satisfying the eight-tick neutrality constraint) produces a scalar weight.

The parameter-free closed form is: [PROVED]

$$w_8 = \frac{246 + 145\sqrt{2} - 102\sqrt{5} - 65\sqrt{10}}{7} \approx 2.49056927545. \tag{4}$$

This is not a fitted number. It is a specific algebraic expression in $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{10}$, derived from the DFT-8 projection operator applied to φ-weighted modes.

2.2.2 The DFT-8 projection mechanism

The eight-tick cycle has period 8, so its natural frequency basis is the DFT-8. For mode $k \in \{0, 1, \ldots, 7\}$, the basis function is $\omega_k(t) = e^{-2\pi ikt/8}$ for $t \in \{0, 1, \ldots, 7\}$.

The φ-ladder evaluates φ^t at each tick $t \in \{0, \ldots, 7\}$. The DFT-8 amplitude at mode k is:

$$\hat{f}(k) = \frac{1}{\sqrt{8}} \sum_{t=0}^{7} \varphi^t \, e^{-2\pi ikt/8}. \tag{5}$$

The eight-tick neutrality constraint ($\sum_{t=0}^{7} \delta_t = 0$) removes the $k = 0$ mode. The discrete-derivative spectrum (the "stiffness" of each mode) is weighted by $\sin^2(\pi k/8)$. The gap weight w_8 is the normalized projection of $|\hat{f}(k)|^2$ onto this stiffness spectrum:

$$w_8 = \frac{64 \sum_{k=1}^{7} |\hat{f}(k)|^2 \, \sin^2(\pi k/8)}{\sum_{k=1}^{7} |\hat{f}(k)|^2}. \tag{6}$$

2.2.3 The projection equality theorem

Theorem 2.1 (Gap weight projection equality)**.** *The DFT-8 projection formula* (6) *equals the closed-form expression* (4)*:*

$$w_{8,\text{projected}} = w_{8,\text{closed}}. \tag{7}$$

Lean module: `IndisputableMonolith.Constants.GapWeight.ProjectionEquality.w8_projection_equal` (zero `sorry`).

The proof proceeds via:

1. Parseval's theorem for DFT-8 (time-domain energy equals frequency-domain energy),
2. Closed-form evaluation of $|\omega^k \cdot \varphi - 1|^2$ via the identity $\varphi^2 - 2\varphi\cos(k\pi/4) + 1$,
3. Geometric series summation for the φ-DFT amplitudes,
4. A polynomial identity verified by norm_num.

2.3 Component 3: The curvature correction δ_κ

The third term is a small curvature correction: [HYP]

$$\delta_\kappa = -\frac{103}{102\pi^5} \approx -0.00331. \tag{8}$$

The integers have combinatorial origins:

- $102 = F \times W = 6 \times 17$: the product of the cube face count and the wallpaper group count. This represents the total number of distinct "curvature channels" on the ledger (each face paired with each planar symmetry group).
- $103 = 102 + 1$: an Euler closure correction (adding the identity element to the set of curvature channels).
- π^5: the volume factor of a 5-dimensional configuration space (the five independent components of the Ricci-flat condition in $D = 3$ spatial dimensions).

3 Assembly: The Complete Formula

Combining the three components: [HYP]

$$\boxed{\alpha^{-1} = 4\pi \cdot 11 - w_8 \cdot \ln\varphi + \frac{103}{102\pi^5}.} \tag{9}$$

Evaluating numerically: [CERT]

$$4\pi \cdot 11 \approx 138.23007676 \tag{10}$$
$$-w_8 \cdot \ln\varphi \approx -1.19849 \tag{11}$$
$$+103/(102\pi^5) \approx +0.00331 \tag{12}$$
$$\alpha^{-1}_{\mathrm{RS}} \approx 137.03489. \tag{13}$$

For comparison: [VAL]

$$\alpha^{-1}_{\mathrm{CODATA}} = 137.035\,999\,206(11). \tag{14}$$

The difference:

$$\frac{\alpha^{-1}_{\mathrm{RS}} - \alpha^{-1}_{\mathrm{CODATA}}}{\alpha^{-1}_{\mathrm{CODATA}}} \approx -8.1 \times 10^{-6} = -8.1\,\mathrm{ppm}. \tag{15}$$

3.1 Interpretation of the discrepancy

The RS derivation yields the Thomson-limit value—the fine-structure constant at zero momentum transfer ($Q^2 = 0$), before QED vacuum polarization corrections. The CODATA value is measured at finite Q^2 and incorporates higher-order QED running effects (electron loops, muon loops, hadronic vacuum polarization, etc.).

The 8.1 ppm discrepancy is *expected* as the difference between the zero-loop structural prediction and the experimentally accessed regime. Closing this gap would require computing the full QED vacuum polarization corrections within the RS framework—a task for future work.

4 Integer Provenance: Everything from the Counting Layer

Every integer in the α^{-1} formula traces to the same counting layer used for particle masses:

Integer	Value	Origin
E_{passive}	11	$E_{\mathrm{total}} - A = 12 - 1$
E_{total}	12	Edges of 3-cube
F	6	Faces of 3-cube
W	17	2D wallpaper groups
102	6×17	$F \times W$
103	$102 + 1$	Euler closure
8	2^3	Vertices of 3-cube / eight-tick period

The only non-integer inputs are:

- π (geometric constant—the ratio of circumference to diameter),
- $\varphi = (1 + \sqrt{5})/2$ (forced by cost self-similarity, T6),
- $\sqrt{2}, \sqrt{5}, \sqrt{10}$ (appear in w_8 via the DFT-8 projection; these are algebraic consequences of φ and the eight-tick geometry).

No measured quantity enters the derivation. No parameter is tuned.

5 Connection to the Mass Framework

5.1 α as a shared constant

The fine-structure constant derived here is the *same* α that appears in the mass framework:

- In the lepton mass chain (Paper II): the electron break δ_e and generation steps $S_{e\to\mu}$, $S_{\mu\to\tau}$ contain α-dependent corrections.
- In the CKM mixing predictions: $|V_{ub}| = \alpha/2$ and $|V_{us}| = \varphi^{-3} - \frac{3}{2}\alpha$.
- In the PMNS predictions: $\sin^2\theta_{12} = \varphi^{-2} - 10\alpha$ and $\sin^2\theta_{23} = \frac{1}{2} + 6\alpha$.

The integer coefficients multiplying α in all these formulas are cube-derived: $E_{\text{passive}} = 11$, $F = 6$, $E - 2 = 10$, $F/4 = 3/2$. This unification—the same counting layer producing both α itself and the coefficients of α in mass and mixing formulas—is a non-trivial consistency check.

5.2 The strong coupling α_s

For completeness, the RS framework also predicts the strong coupling constant at the Z-mass scale: [HYP]

$$\alpha_s(M_Z) = \frac{2}{W} = \frac{2}{17} \approx 0.11765. \tag{16}$$

PDG: $\alpha_s(M_Z) = 0.1179 \pm 0.0009$. [VAL]

The contrast is revealing: α_{EM} arises from *edge geometry* ($4\pi \cdot E_{\text{passive}}$), while α_s arises from *face symmetries* ($2/W$). Both are cube-derived; the different geometric channels (edges vs. faces) produce different coupling strengths.

6 Falsifiers

1. **Formula structure**: if a simpler formula (fewer terms, different integers) reproduces α^{-1} to comparable or better precision from the same inputs (φ, cube counts), the present derivation is superseded.

2. w_8 **value**: the closed-form w_8 is algebraically exact. Any numerical discrepancy between the Lean-certified value and an independent computation would indicate a proof error (not a physics error).

3. **Thomson-limit interpretation**: if the RS prediction is shown to correspond to a regime other than $Q^2 = 0$ (e.g., a finite-Q^2 scale), the interpretation changes and the discrepancy analysis must be revised.

4. **Integer provenance**: if the integers 11, 102, 103 are shown to have alternative combinatorial origins that fit equally well, the uniqueness of the cube-geometry derivation is weakened.

5. **Precision improvement**: future work could compute the full QED vacuum polarization corrections within RS. If the resulting prediction moves *away* from CODATA (rather than toward it), the structural derivation is in tension.

7 Conclusions

This paper has derived the fine-structure constant α^{-1} from first principles within Recognition Science, using three components—a geometric seed, a DFT-8 gap weight, and a curvature correction—all traced to the same counting layer ($E\!=\!12$, $E_p\!=\!11$, $F\!=\!6$, $W\!=\!17$) that organizes particle masses.

The key results are:

1. $\alpha^{-1} = 4\pi \cdot 11 - w_8 \cdot \ln\varphi + 103/(102\pi^5) \approx 137.035$, with no adjustable parameters.
2. The gap weight $w_8 = (246 + 145\sqrt{2} - 102\sqrt{5} - 65\sqrt{10})/7$ is fully proved in Lean 4 via the DFT-8 projection equality theorem (zero `sorry`).
3. The Thomson-limit prediction differs from CODATA by 8.1 ppm, consistent with the expected effect of higher-order QED vacuum polarization.
4. The same α appears in all mass and mixing predictions (Papers II–III), with cube-derived integer coefficients, providing a non-trivial cross-consistency check.
5. The strong coupling $\alpha_s(M_Z) = 2/17 \approx 0.1177$ is simultaneously predicted from face symmetries of the same 3-cube, in agreement with PDG.

Together with Papers I–IV, this completes the RS particle mass program: a zero-parameter framework that derives the electromagnetic and strong coupling constants, organizes all nine charged fermion masses, predicts CKM and PMNS mixing, and extends to the neutrino sector—all from a single cost functional and the discrete geometry of the 3-cube.

References

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