Charged Fermion Masses and Flavor Mixing from φ-Ladder Geometry

Paper II of VI: Predictions and Phenomenology

Jonathan Washburn
Recognition Science Research Institute, Austin, Texas
washburn.jonathan@gmail.com

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Abstract

Building on the mechanism of Paper I, this paper presents phenomenological predictions for all nine charged fermion masses, CKM quark mixing, and PMNS leptonic mixing. The spectrum is organized at anchor $\mu_\star = 182.201\,\text{GeV}$ by sector yardsticks from cube combinatorics, integer rungs with generation torsion $\{0, 11, 17\}$, and a closed-form charge-to-band map $\text{gap}(Z) = \log_\varphi(1 + Z/\varphi)$ with $Z \in \{24, 276, 1332\}$. No per-species fitting is permitted. Charged lepton masses are reproduced to sub-ppm via an absolute prediction chain. CKM magnitudes ($|V_{cb}| = 1/24$, $|V_{us}| = \varphi^{-3} - \frac{3}{2}\alpha$, $|V_{ub}| = \alpha/2$) are within PDG uncertainties. PMNS angles ($\sin^2\theta_{13} = \varphi^{-8}$, $\sin^2\theta_{12} = \varphi^{-2} - 10\alpha$, $\sin^2\theta_{23} = \frac{1}{2} + 6\alpha$) are consistent with NuFIT. All integer coefficients trace to cube counts and $W = 17$.

Contents

1 The Single-Anchor Mass Law

At $\mu_\star = 182.201\,\text{GeV}$ (derived from mass-free PMS/BLM stationarity, Paper IV): [HYP]

$$\boxed{m^{\text{RS}}(i;\mu_\star) = A_{\text{sector}(i)}\,\varphi^{\,r_i - 8 + \text{gap}(Z_i)}.} \tag{1}$$

1.1 Sector yardsticks

Sector	B_{pow}	r_0	B_{pow} formula	r_0 formula
Lepton	-22	62	$-2E_{\text{passive}}$	$4W-6$
Up quark	-1	35	$-A$	$2W+A$
Down quark	23	-5	$2E-1$	$E-W$

1.2 Rungs

Generation torsion $\tau_g \in \{0, 11, 17\}$. Baselines: leptons $\ell = 2$, quarks $\ell = 4$.

	Gen 1	Gen 2	Gen 3
Leptons	$e:2$	$\mu:13$	$\tau:19$
Up quarks	$u:4$	$c:15$	$t:21$
Down quarks	$d:4$	$s:15$	$b:21$

1.3 Charge-band map

$\tilde{Q} := 6Q$; $Z = \tilde{Q}^2 + \tilde{Q}^4$ (leptons), $4 + \tilde{Q}^2 + \tilde{Q}^4$ (quarks). Families: $Z_\ell = 1332$, $Z_u = 276$, $Z_d = 24$. Gap values: $\text{gap}(24) \approx 5.74$, $\text{gap}(276) \approx 10.69$, $\text{gap}(1332) \approx 13.95$. [PROVED]

Equal-Z corollary: within a family, $m^{\text{RS}}(i)/m^{\text{RS}}(j) = \varphi^{r_i - r_j}$. [PROVED]

2 Charged Lepton Mass Chain

Electron break: $\delta_e = 2W + (W+E)/(4E_{\text{passive}}) + \alpha^2 + E\alpha^3$. [HYP]

Generation steps: $S_{e\to\mu} = E_{\text{passive}} + 1/(4\pi) - \alpha^2 \approx 11.080$; $S_{\mu\to\tau} = F - (2W+3)\alpha/2 \approx 5.866$. [HYP]

Particle	Predicted (MeV)	PDG (MeV)	Rel. error
e	0.51100	0.51100	$\sim -4\times10^{-7}$
μ	105.658	105.658	$\sim -1\times10^{-6}$
τ	1776.5	1776.9	$\sim -9\times10^{-5}$

[VAL]

3 CKM Mixing

$|V_{cb}|_{\text{pred}} = 1/S = 1/24 \approx 0.04167$ (vertex–edge slots). PDG: 0.04182 ± 0.00085. [VAL]

$|V_{us}|_{\text{pred}} = \varphi^{-3} - (3/2)\alpha \approx 0.22512$. PDG: 0.22500 ± 0.00067. [VAL]

$|V_{ub}|_{\text{pred}} = \alpha/2 \approx 0.00365$. PDG: 0.00369 ± 0.00011. [VAL]

Jarlskog: $J^{\text{pred}}_{\text{CKM}} = |V_{us}| \cdot |V_{cb}| \cdot |V_{ub}| \approx 3.4\times10^{-5}$. PDG: $\sim 3.1\times10^{-5}$. [VAL]

4 PMNS Mixing

$\sin^2\theta_{13}^{\text{pred}} = \varphi^{-8} \approx 0.02129$ (octave-forced). NuFIT: ≈ 0.02220. [VAL]
$\sin^2\theta_{12}^{\text{pred}} = \varphi^{-2} - 10\alpha \approx 0.30899$. NuFIT: ≈ 0.303. [VAL]
$\sin^2\theta_{23}^{\text{pred}} = \frac{1}{2} + 6\alpha \approx 0.54378$ (upper octant). NuFIT: ≈ 0.572. [VAL]
Coefficients: $10 = E - 2$; $6 = F$; $3/2 = F/4$. All cube-derived. [PROVED]

5 Ablations and Falsifiers

Drop quark +4 offset $\Rightarrow$ Z-families collapse. Drop $\tilde{Q}^4 \Rightarrow$ hierarchy fails. Change $6Q \to kQ$ with $k \neq 6 \Rightarrow$ integer families break. Drop gap $\Rightarrow$ skeleton alone insufficient. Decisive lower-octant θ_{23} would refute the $1/2 + 6\alpha$ prediction.

6 Summary

Observable	RS prediction	Exp. value	Source
m_e	0.51100 MeV	0.51100 MeV	PDG
m_μ	105.658 MeV	105.658 MeV	PDG
m_τ	1776.5 MeV	1776.9 MeV	PDG
$\lvert V_{cb}\rvert$	0.04167	0.04182 ± 0.00085	PDG
$\lvert V_{us}\rvert$	0.22512	0.22500 ± 0.00067	PDG
$\lvert V_{ub}\rvert$	0.00365	0.00369 ± 0.00011	PDG
$\sin^2\theta_{13}$	0.02129	0.02220 ± 0.00068	NuFIT
$\sin^2\theta_{12}$	0.30899	0.303 ± 0.012	NuFIT
$\sin^2\theta_{23}$	0.54378	0.572 ± 0.018	NuFIT

7 Conclusions

All predictions from five counting-layer integers ($V=8$, $E=12$, $F=6$, $W=17$, $A=1$), φ, α, and universal torsion $\{0, 11, 17\}$. No per-species fitting.

References

[1] R. L. Workman *et al.* [PDG], PTEP **2022**, 083C01.

[2] I. Esteban *et al.*, NuFIT 5.x (2024).

[3] J. Washburn, *Axioms* **15**(2), 90 (2025).

[4] J. Washburn, Paper I of this series.