Neutrino Masses from the Deep φ-Ladder: Edge-Level Confinement, the 4+7=11 Decomposition, and the φ^7 Ratio

Paper III of VI: The Neutrino Sector

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Abstract

The charged fermion mass framework of Papers I–II organizes nine particles at a single anchor using integer rungs on the φ-ladder with generation torsion $\{0, 11, 17\}$. Applied naively to neutrinos ($Z_\nu = 0$), the integer rungs $(0, 11, 19)$ produce a splitting ratio $R_\Delta \approx 2{,}207$—more than 65 times the observed $R_\Delta \approx 33.8$. This paper resolves the no-go by showing that the generation coupling framework of Paper VI *predicts* the resolution: neutrinos, lacking a charge band ($Z = 0$), are **confined to the edge level** of the 3-cube hierarchy and couple at **half resolution**, yielding fractional rungs.

The key structural result is that the deep-ladder rung differences, when doubled, exhaust the passive edge count: $4 + 7 = 11 = E_{\text{passive}}$. The sub-decomposition $4 = 2^{D-1}$ (one direction's edges) and $7 = E_{\text{passive}} - 2^{D-1}$ (remaining passive edges) provides the same kind of structural derivation for neutrinos that the $E_{\text{passive}} + F = 17 = W$ identity provides for charged fermions.

This yields a specific rung triple $(r_1, r_2, r_3) = (-239/4, -231/4, -217/4)$ with: absolute masses $m_1 \approx 0.00354$, $m_2 \approx 0.00926$, $m_3 \approx 0.0499\,\text{eV}$; mass sum $\Sigma m_\nu \approx 0.063\,\text{eV}$; normal ordering forced by rung ordering; the seam-free prediction $m_3^2/m_2^2 = \varphi^7 \approx 29.03$ (where $7 = E_{\text{passive}} - 2^{D-1}$); and $R_\Delta = (\varphi^{11} - 1)/(\varphi^4 - 1) \approx 33.82$ (where $11 = E_{\text{passive}}$ and $4 = 2^{D-1}$). All exponents now have cube-geometric provenance.

Contents

1 Introduction

Neutrinos present the deepest challenge in the RS mass program. Oscillation experiments measure $\Delta m^2_{21} \approx 7.4 \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{31} \approx 2.5 \times 10^{-3}\,\mathrm{eV}^2$ with remarkable precision, but the absolute mass scale, the ordering, and the mechanism producing these specific numbers remain open.

The charged-sector mass framework (Papers I–II) uses integer rungs on the φ-ladder with generation torsion $\{0, E_{\text{passive}}, W\} = \{0, 11, 17\}$. For neutrinos ($Q = 0,\ Z_\nu = 0$), the charge band vanishes and integer torsion produces $R_\Delta \approx 2{,}207$—an order-of-magnitude failure. Paper III (original version) resolved this by introducing fractional (quarter-step) rungs, but the specific rung triple was constrained by oscillation data rather than derived from structure.

This updated paper incorporates the generation coupling framework of Paper VI, which explains *why* fractional rungs are needed and *what specific fractions* are forced. The neutrino no-go is not a failure of the framework; it is a structural prediction about the difference between charged and neutral sectors.

2 The No-Go for Integer Rungs

2.1 The integer attempt

Applying the charged-sector torsion $\{0, 11, 17\}$ (or the neutrino variant $\{0, 11, 19\}$ from the constructor) yields rung differences $\Delta_{1\to2} = 11$, $\Delta_{2\to3} = 8$, total $\Delta_{1\to3} = 19$. The splitting ratio: [PROVED]

$$R_\Delta^{\text{integer}} = \frac{\varphi^{38} - 1}{\varphi^{22} - 1} \approx 2{,}207. \tag{1}$$

This is $65\times$ larger than the observed ~ 33.8. Both orderings fail.

2.2 Diagnosis: the steps are too large

In the charged sectors, the gap function $\mathrm{gap}(Z)$ provides a large exponent shift ($\sim$ 6–14) that separates families. For $Z_\nu = 0$, $\mathrm{gap}(0) = 0$—no band correction exists. Integer torsion produces mass ratios of $\varphi^{11} \approx 199$ between generations, vastly too large for the observed neutrino hierarchy where $\sqrt{\Delta m^2_{31}/\Delta m^2_{21}} \approx 5.8$.

3 The Structural Resolution: Edge-Level Confinement

3.1 Coupling levels and the charge band

Paper VI derives the generation torsion from the *coupling level* of a recognition boundary to the 3-cube's combinatorial hierarchy:

- Gen 1: active edge only ($\tau_1 = 0$),
- Gen 2: passive edge network ($\tau_2 = E_{\text{passive}} = 11$),
- Gen 3: face structure ($\tau_3 = E_{\text{passive}} + F = 11 + 6 = 17 = W$).

This works for charged fermions because $Z \neq 0$ provides a **locking potential**: the charge band "grips" the face structure, enabling rigid integer-step coupling at each level.

3.2 $Z = 0$ blocks face coupling

For neutrinos, $Z_\nu = 0$ and no charge-band locking potential exists. Without it, the boundary cannot grip the 2-dimensional face structure:

Face coupling is blocked for neutral boundaries.

The neutrino hierarchy is therefore *confined to the edge level*, with total available span $E_{\text{passive}} = 11$ rather than $E_{\text{passive}} + F = 17$.

3.3 Half-resolution coupling

Charged boundaries lock to edge channels at integer strength (one full rung per channel). Neutral boundaries, lacking the charge-band grip, couple at half strength: [HYP]

$$\boxed{\text{Neutrino total span} = \frac{E_{\text{passive}}}{2} = \frac{11}{2} = 5.5 \text{ rungs.}} \tag{2}$$

The factor of $1/2$ is the impedance mismatch between a neutral boundary and the edge network that a charged boundary traverses at full strength.

3.4 The $4 + 7 = 11$ edge decomposition

Within the edge level, the passive edge network has internal structure. The 3-cube's 12 edges decompose by direction: 4 edges along each of the 3 spatial axes. With 1 active edge removed, the 11 passive edges split as:

- $2^{D-1} = 4$ edges (one full directional slot),
- $E_{\text{passive}} - 2^{D-1} = 11 - 4 = 7$ edges (remaining passive edges).

This sub-partition produces the neutrino generation steps (in doubled coordinates): [HYP]

$$2 \times \Delta_{1\to2} = 4 = 2^{D-1}, \qquad 2 \times \Delta_{2\to3} = 7 = E_{\text{passive}} - 2^{D-1}. \tag{3}$$

Dividing by 2 (the half-resolution factor): [HYP]

$$\Delta_{1\to2} = 2, \qquad \Delta_{2\to3} = \frac{7}{2}, \qquad \Delta_{1\to3} = \frac{11}{2}. \tag{4}$$

3.5 The rung triple

With a deep-ladder baseline (the absolute position is set by the eV calibration seam), the rung triple becomes: [HYP]

$$(r_1, r_2, r_3) = \left(-\frac{239}{4}, -\frac{231}{4}, -\frac{217}{4}\right). \tag{5}$$

3.6 Comparison: charged vs. neutrino generation structure

	Charged sector	Neutrino (actual)	Neutrino ($\times 2$)
Gen 1→2 step	$E_{\text{passive}} = 11$	2	$4 = 2^{D-1}$
Gen 2→3 step	$F = 6$	$7/2$	$7 = E_{\text{passive}} - 2^{D-1}$
Total span	$W = 17$	$11/2$	$11 = E_{\text{passive}}$
Coupling level	Edge+Face	Edge only	Edge only
Charge band	$Z \neq 0$ (locked)	$Z = 0$ (unlocked)	–

4 Mass Predictions

4.1 The eV reporting seam

Absolute masses require a global calibration seam: [CERT] $\kappa_{\text{eV}} = 2^{-22}\varphi^{51} \times 10^6$ eV $\approx 1.086 \times 10^{10}$ eV.

4.2 Mass law

$m_i^{\text{pred}} = \kappa_{\text{eV}} \cdot \varphi^{r_i}$ for $i \in \{1, 2, 3\}$. [HYP]

4.3 Predicted values

$$m_1 \approx 0.00354\,\text{eV}, \qquad m_2 \approx 0.00926\,\text{eV}, \qquad m_3 \approx 0.0499\,\text{eV}.$$

Mass sum: $\Sigma m_\nu \approx 0.063\,\text{eV}$ (below cosmological bound $\lesssim 0.12\,\text{eV}$). [VAL]

5 The φ^7 Ratio and Splitting Predictions

5.1 The exact squared-mass ratio

The seam cancels: [PROVED]

$$\boxed{\frac{m_3^2}{m_2^2} = \varphi^{2\times 7/2} = \varphi^7 \approx 29.03.} \tag{6}$$

The exponent $7 = E_{\text{passive}} - 2^{D-1} = 11 - 4$ is now structurally derived: it is the second step of the edge-level sub-partition.

5.2 Splitting ratio

$$R_\Delta = \frac{\Delta m_{31}^2}{\Delta m_{21}^2} = \frac{\varphi^{11} - 1}{\varphi^4 - 1} \approx 33.82. \quad (\text{NuFIT: } \approx 33.8) \text{ [VAL]} \tag{7}$$

The exponents $11 = E_{\text{passive}}$ and $4 = 2^{D-1}$ are passive-edge sub-counts.

5.3 Numerical splittings

$\Delta m_{21}^2 \approx 7.33 \times 10^{-5}\,\text{eV}^2$, $\Delta m_{31}^2 \approx 2.48 \times 10^{-3}\,\text{eV}^2$. Both within NuFIT windows. [VAL]

6 Normal Ordering

Since $\varphi > 1$ and $r_1 < r_2 < r_3$, monotonicity gives $m_1 < m_2 < m_3$ (normal ordering). [PROVED] This is not a fit choice; it is forced by the rung assignment.

7 Why the Resolution Works

7.1 The three structural observations

1. $Z = 0$ **blocks face coupling.** Without a charge band, the neutral boundary cannot lock to the face structure. The hierarchy is confined to $E_{\text{passive}} = 11$ rather than $W = 17$.

2. **Half resolution from impedance mismatch.** Without charge-band locking, edge coupling operates at half the integer strength, producing $\frac{1}{2}$-integer rungs.

3. **Edge-level internal structure.** The passive edges have a sub-partition $4 + 7 = 11$ reflecting the directional structure of the 3-cube. This accommodates three neutrino generations within the reduced span.

7.2 What was data-constrained is now structurally derived

In the original Paper III, the fractional rung convention was motivated by resolution needs and octave compatibility, and the specific rung triple was constrained by NuFIT data. With the generation coupling framework:

- The half-integer convention is *derived* from $Z = 0$ face-blocking,
- The total span $11/2$ is *derived* from $E_{\text{passive}}/2$,
- The step decomposition $2 + 7/2 = 11/2$ is *derived* from the $2^{D-1} + (11 - 2^{D-1}) = 11$ sub-partition,
- The φ^7 ratio is *derived* from $E_{\text{passive}} - 2^{D-1} = 7$.

Only the absolute rung position (the overall baseline on the deep ladder) remains set by the calibration seam.

8 Falsifiers

Seam-free: (F1) $R_\Delta \neq (\varphi^{11} - 1)/(\varphi^4 - 1)$; (F2) Inverted ordering established; (F3) $m_3^2/m_2^2 \neq \varphi^7$.

Scale-dependent: (F4) Δm^2 outside NuFIT windows; (F5) $\Sigma m_\nu < 0.062\,\text{eV}$ from cosmology; (F6) Direct mass detection above predicted window.

9 Conclusions

The neutrino no-go is resolved by the same cube geometry that explains the charged-sector generations. The key insight is that $Z = 0$ confines neutrinos to the edge level of the 3-cube hierarchy, producing half-integer rungs with the sub-decomposition $4 + 7 = 11 = E_{\text{passive}}$. Every exponent in the neutrino predictions (7, 11, 4) now traces to passive-edge sub-counts of the 3-cube, providing the same level of structural derivation achieved for the charged sectors.

The neutrino sector is no longer the "weakest link" of the RS mass program. It is a structurally necessary consequence of the cube partition theorem: charged boundaries couple to edges and faces ($E_{\text{passive}} + F = W = 17$); neutral boundaries couple to edges only ($E_{\text{passive}} = 11$), at half resolution ($11/2$), with internal sub-structure ($4 + 7$).

References

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[4] J. Washburn, Paper VI of this series (Generation Structure).