# The Exclusion Theorem: The Unique Impossibility Certificate Forced by Canonical Cost

Inevitability of the Obstruction–Sensor–Cayley–Schur Pipeline

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**Abstract**

We prove that the four-step impossibility pipeline—obstruction encoding, reciprocal sensing, Cayley transform, Schur certification—is the *unique optimal* strategy for excluding candidate configurations from a zero-defect structured set, given the canonical cost $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, finite local resolution, and a conservation constraint.

The main result (**Exclusion Master Theorem**, §7) is:

> *Every correct, finite-data, complete exclusion procedure on the rational class factors* ***uniquely*** *as* $\Psi^* = \mathcal{S} \circ \mathcal{C} \circ \mathcal{R} \circ \mathcal{O}$*, where* $\mathcal{O}$ *is the obstruction encoding,* $\mathcal{R}$ *is the reciprocal sensor,* $\mathcal{C}$ *is the Cayley transform, and* $\mathcal{S}$ *is Schur certification. The factorisation order is forced. The four steps are independent. The Cayley transform is unique up to Möbius equivalence.*

Combined with the Coercive Projection Theorem (the membership side), the two procedures form the **unique complete two-sided audit** for zero-defect certification from finite data:

$$\Phi^* \text{ (membership, CPT)} + \Psi^* \text{ (exclusion, this paper)} \;=\; \text{complete decision on the rational class.}$$

No domain-specific input enters the exclusion pipeline. Like CPT, it is a theorem forced by the cost functional, not a method one selects.

## Contents

# 1 Introduction

## 1.1 The problem

Given a unique cost functional $J$ and only finite observational access, determine that a candidate configuration $\mathbf{x}$ does *not* lie in the structured set $S = \{\mathbf{x} : J(\mathbf{x}) = 0\}$.

The companion paper (CPT) addressed the membership question: "is $\mathbf{x}$ in $S$?" This paper addresses the exclusion question: "is $\mathbf{x}$ definitely *not* in $S$?"

We prove there is exactly one way to answer it.

## 1.2 Why exclusion is harder than membership

Membership is local: to certify $J(\mathbf{x}) = 0$, it suffices to verify that the defect vanishes at $\mathbf{x}$ itself. Exclusion is global: to certify $J(\mathbf{x}) > 0$, one must rule out the possibility that the measured deviation is an artifact of finite sampling. The fundamental obstruction is:

**Proposition 1.1** (Finite sampling is insufficient for exclusion)**.** *For any finite sample set* $\{z_1, \ldots, z_m\}$ *and values* $\{w_1, \ldots, w_m\}$ *with all* $w_k \neq 0$*, there exists a holomorphic function* $f$ *matching all samples that nevertheless has a zero at any prescribed point* $a \notin \{z_k\}$*.*

*Proof.* $f(z) = p(z) - p(a) \prod_k (z - z_k) / \prod_k (a - z_k)$ where $p$ is the Lagrange interpolant through $(z_k, w_k)$. Then $f(z_k) = w_k$ and $f(a) = 0$. □

Therefore exclusion from finite data requires *additional structure* beyond point sampling. We show this structure is uniquely determined by $J$.

## 1.3 The forcing chain

$$\underbrace{J\text{ analytic}}_{\text{cosh is entire}} \to \underbrace{\text{holomorphic obstruction}}_{\mathcal{O}} \to \underbrace{\text{reciprocal sensor}}_{\mathcal{R}} \to \underbrace{\text{Cayley transform}}_{\mathcal{C}} \to \underbrace{\text{Schur certification}}_{\mathcal{S}}$$

Each arrow is a theorem proved below, and each step is the unique option at its stage.

# 2 Axioms (Same as CPT)

We use the same axiom set as the Coercive Projection Theorem:

**Definition 2.1** (Axiom set $\mathfrak{A}$)**.** (A1) **Cost uniqueness.** $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, uniquely forced by composition, normalization, calibration.

(A2) **Conservation.** $\sigma(\mathbf{x}) := \sum_i \ln x_i = 0$ on admissible states.

(A3) **Finite resolution.** Window length $W = 8$; only $W$ consecutive values are accessible per cycle.

Write $\phi(t) := J(e^t) = \cosh(t) - 1$. Key property for this paper: $\phi$ extends to an **entire function** on $\mathbb{C}$ (since cosh is entire).

# 3 Step $\mathcal{O}$: Obstruction Encoding Is Forced by Analyticity

## 3.1 The defect-to-zero principle

In the RS ontology, "$\mathbf{x}$ has property $P$" means "the defect of $\mathbf{x}$ with respect to $P$ vanishes." This is not a convention; it is forced by $J$:

**Theorem 3.1** (Defect is the unique existence test)**.** *Under* (A1)*, the only function* $\Delta : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$ *that is zero exactly at* $x = 1$ *(the identity) and respects the composition law is* $\Delta = J$ *itself (up to positive scaling).*

*Proof.* Any such $\Delta$ must satisfy $\Delta(1) = 0$ and $\Delta(x) > 0$ for $x \neq 1$. If $\Delta$ also satisfies the composition law (axiom (A1)), then by cost uniqueness $\Delta = c\,J$ for some $c > 0$, and by the calibration $J''(1) = 1$ one may normalise $c = 1$. $\square$

**Corollary 3.2** (Obstruction encoding is canonical)**.** *For any candidate property "$\mathbf{x} \in S$," the holomorphic obstruction is $G_S(z) := J(z)$ (or a composition with $J$ in multi-component cases). There is no freedom in the choice of obstruction: it is $J$ or a function with the same zero set.*

### 3.2 Analyticity of the obstruction

**Proposition 3.3** (The obstruction is entire)**.** *$J(e^z) = \cosh(z) - 1$ extends to an entire function on $\mathbb{C}$. In particular, $G_S$ is holomorphic on any domain $\Omega \subseteq \mathbb{C}$, and its zeros are isolated.*

*Proof.* $\cosh(z) = \sum_{n=0}^{\infty} z^{2n}/(2n)!$ converges for all $z \in \mathbb{C}$. $\square$

*Remark* 3.4 (Why analyticity is essential). If $J$ were merely continuous (not analytic), zeros could accumulate, and the exclusion problem would be undecidable even in principle. The composition law forces cosh, which forces analyticity, which forces isolated zeros—making exclusion a well-posed problem.

## 4 Step $\mathcal{R}$: Reciprocal Sensing Is the Unique Amplification

To detect a zero of $G_S$ from finite data, one needs an *amplification*: a mechanism that turns a small value of $G_S$ near a zero into a large, detectable signal.

**Theorem 4.1** (The reciprocal is the unique holomorphic amplifier)**.** *Let $f : \Omega \to \mathbb{C}$ be holomorphic with a simple zero at $z_0$. Up to bounded holomorphic factors, the only meromorphic function on $\Omega$ that diverges at $z_0$ and is holomorphic elsewhere is $1/f$ (times a nonvanishing holomorphic function).*

*Proof.* Near $z_0$, write $f(z) = (z - z_0)\,g(z)$ with $g(z_0) \neq 0$. Any meromorphic $h$ with a pole only at $z_0$ and holomorphic elsewhere has Laurent expansion $h(z) = a_{-1}/(z - z_0) + (\text{holomorphic})$. Thus $h(z) = a_{-1}\,[f(z)]^{-1}\,g(z) + (\text{holomorphic}) = a_{-1}\,g(z_0)/f(z) + (\text{holomorphic near } z_0)$. The polar part is proportional to $1/f$. $\square$

**Definition 4.2** (Canonical sensor)**.** The *sensor* for obstruction $G_S$ is

$$\mathcal{J}_S(z) \;:=\; \frac{1}{G_S(z)}. \tag{1}$$

**Corollary 4.3** (Sensor correctness)**.** *If the candidate property $S$ holds at $z_\star$ (i.e., $G_S(z_\star) = 0$), then $\mathcal{J}_S$ has a pole at $z_\star$. Conversely, $\mathcal{J}_S$ is holomorphic at $z$ iff $G_S(z) \neq 0$.*

*Remark* 4.4 (No alternative exists). $\log(1/|G_S|)$ diverges at zeros but is not holomorphic. $|G_S|^{-\alpha}$ for $\alpha > 0$ diverges but is not meromorphic. The reciprocal $1/G_S$ is the *only* amplification mechanism that preserves the analytic (holomorphic/meromorphic) category.

## 5 Step $\mathcal{C}$: The Cayley Transform Is the Unique Conformal Map

A pole of $\mathcal{J}_S$ is a singularity—hard to certify directly. We convert "pole exclusion" into "boundedness," which is amenable to finite certification.

**Theorem 5.1** (Cayley is the unique normalised conformal map)**.** *Up to Möbius equivalence, the Cayley transform*

$$\Xi(w) := \frac{2w-1}{2w+1} \tag{2}$$

*is the unique conformal bijection from* $\{\mathrm{Re}(w) > 0\}$ *onto the open unit disk* $\mathbb{D}$ *satisfying* $\Xi(\infty) = 1$ *and* $\Xi(1/2) = 0$*.*

*Proof.* The general Möbius map from the right half-plane to $\mathbb{D}$ is $\Xi(w) = e^{i\theta}(w - w_0)/(w + \bar{w}_0)$ for $\mathrm{Re}(w_0) > 0$. The normalisation $\Xi(\infty) = e^{i\theta} = 1$ forces $\theta = 0$. Then $\Xi(w_0) = 0$, so the normalisation $\Xi(1/2) = 0$ gives $w_0 = 1/2$. Substituting: $\Xi(w) = (w - 1/2)/(w + 1/2) = (2w-1)/(2w+1)$. □

**Definition 5.2** (Cayley field)**.** The *Cayley field* of sensor $\mathcal{J}_S$ is

$$\Xi_S(z) \;:=\; \frac{2\mathcal{J}_S(z) - 1}{2\mathcal{J}_S(z) + 1}. \tag{3}$$

**Lemma 5.3** (Pole-to-boundary correspondence)**.**

1. *If* $\mathcal{J}_S(z) \to \infty$ *(pole), then* $\Xi_S(z) \to 1$ *(boundary of* $\mathbb{D}$*).*
2. *If* $\mathrm{Re}(\mathcal{J}_S(z)) > 0$*, then* $|\Xi_S(z)| < 1$ *(interior of* $\mathbb{D}$*).*

*Proof.* (1): $\Xi_S - 1 = -2/(2\mathcal{J}_S + 1) \to 0$. (2): $|\Xi_S| < 1$ iff $|2w - 1| < |2w + 1|$ iff $\mathrm{Re}(w) > 0$. □

*Remark* 5.4 (Why the Cayley transform is forced)*.* The problem is: convert "sensor has no pole" into a property certifiable on the unit disk. Lemma 5.3 shows that poles map to boundary hits and non-poles map to interior points. The Cayley transform is the *unique* normalised map with this property (Theorem 5.1). No alternative conformal map achieves this without introducing free parameters.

**Lemma 5.5** (Cayley preserves rationality)**.** *If* $\mathcal{J}_S$ *is rational, so is* $\Xi_S$*. Conversely, if* $\Xi_S$ *is rational and not identically* 1*, then* $\mathcal{J}_S$ *is rational.*

*Proof.* The Cayley transform and its inverse are rational operations (additions, multiplications, divisions). □

# 6 Step $\mathcal{S}$: Schur Certification Is the Unique Boundedness Test

## 6.1 The Schur pinch: boundedness kills poles

**Theorem 6.1** (Removable singularity under a Schur bound)**.** *Let* $D \subset \mathbb{C}$ *be a disk centred at* $\rho$*. If* $\Xi$ *is holomorphic on* $D \setminus \{\rho\}$ *and* $|\Xi| \le 1$ *there, then* $\Xi$ *extends holomorphically to all of* $D$*.*

*Proof.* $\Xi$ is bounded on the punctured disk; by Riemann's removable singularity theorem, the singularity is removable. □

**Theorem 6.2** (The Schur pinch)**.** *Let* $\Omega \subseteq \mathbb{C}$ *be a domain. If* $\Xi_S$ *is meromorphic on* $\Omega$ *with* $|\Xi_S| \le 1$ *away from poles, and* $\Xi_S \not\equiv 1$*, then:*

1. $\Xi_S$ *extends holomorphically to all of* $\Omega$ *(no poles).*
2. *The Cayley inverse* $2\mathcal{J}_S = (1 + \Xi_S)/(1 - \Xi_S)$ *is holomorphic on* $\Omega$*.*
3. $\mathcal{J}_S$ *has no poles in* $\Omega$*.*
4. *The candidate property* $S$ *does not hold at any point of* $\Omega$*.*

*Proof.* (1): Poles of $\Xi_S$ are isolated. Around each pole, $|\Xi_S| \le 1$ on the punctured disk, so Theorem 6.1 removes it.

(2): If $\Xi_S \not\equiv 1$ and $|\Xi_S| \le 1$ on $\Omega$, then $\Xi_S \ne 1$ on $\Omega$ (otherwise the Maximum Modulus Principle forces $\Xi_S \equiv 1$, contradiction). Hence $1 - \Xi_S \ne 0$ and $(1 + \Xi_S)/(1 - \Xi_S)$ is holomorphic.

(3): Follows from (2).

(4): If $S$ held at $z_\star$, then $G_S(z_\star) = 0$, so $\mathcal{J}_S = 1/G_S$ has a pole at $z_\star$, contradicting (3). □

### 6.2 The Pick criterion: the unique Schur test

**Theorem 6.3** (Nevanlinna–Pick criterion). *A holomorphic $\theta : \mathbb{D} \to \mathbb{C}$ satisfies $|\theta| \le 1$ on $\mathbb{D}$ if and only if the Pick kernel*

$$K_\theta(z, w) \; := \; \frac{1 - \theta(z)\overline{\theta(w)}}{1 - z\bar{w}}$$

*is positive semidefinite: every finite Pick matrix $[K_\theta(z_i, z_j)]_{i,j}$ is $\succeq 0$.*

*Remark* 6.4. This is the classical Nevanlinna–Pick theorem. It is the *unique* necessary-and-sufficient characterisation of the Schur class: no alternative criterion exists that tests fewer conditions.

### 6.3 Finite certification in the rational class

**Theorem 6.5** (Finite-state $\Rightarrow$ rational). *Under finite local branching (branching bound $b$) and recognition-respecting dynamics, the quotient output generating function $\theta(z) = \sum_{n \ge 0} y_n z^n$ is rational of degree $\le d$, where $d = |S|$ is the size of the quotient state space.*

*Proof.* Same as CPT Theorem 4.5: $y_n = u^* A^n v$, so $\theta(z) = u^*(I - zA)^{-1} v$ is a ratio of polynomials of degree $\le d$. □

**Corollary 6.6** (Schur certification is finite in the rational class). *For rational $\theta$ of degree $d$, the Schur property $|\theta| \le 1$ on $\mathbb{D}$ is equivalent to:*

1. (State-space route): *Existence of $P \succ 0$ satisfying the bounded-real LMI for a $d$-dimensional realisation.*
2. (Coefficient route): *Positivity of the $(d + 1) \times (d + 1)$ principal minor of the coefficient Pick matrix (no tail to control).*

*Both are finite-dimensional semidefinite feasibility problems.*

*Remark* 6.7 (No tail risk in the rational class). For rational functions, there is no "tail" beyond degree $d$: all coefficients $a_n = 0$ for $n$ beyond the numerator degree (after partial fractions). The tail bound that plagues general holomorphic functions is *automatically zero* in the rational class. This is why finite resolution + finite branching *solves* the exclusion problem completely.

## 7 The Exclusion Master Theorem

**Definition 7.1** (Exclusion procedure). An *exclusion procedure* is a map

$$\Psi : (\mathbb{R}_{>0})^n \longrightarrow \{\texttt{EXCLUDED}, \texttt{INCONCLUSIVE}\}$$

satisfying:

(E1) **Soundness:** $\Psi(\mathbf{x}) = \texttt{EXCLUDED} \implies \mathbf{x} \notin S$.

(E2) **Finite data:** $\Psi$ depends on at most $W = 8$ evaluations per cycle of a finite-state representation.

$\Psi$ is *complete on the rational class* if, for every rational-class $\mathbf{x} \notin S$, $\Psi(\mathbf{x}) = \texttt{EXCLUDED}$.

**Theorem 7.2** (Exclusion Master Theorem). *Under axioms* (A1)–(A3)*, define*

$$\Psi^*(\mathbf{x}) := \begin{cases} \textit{EXCLUDED} & \text{if } \mathcal{S} \circ \mathcal{C} \circ \mathcal{R} \circ \mathcal{O} \text{ certifies a Schur bound and } \Xi \not\equiv 1, \\ \textit{INCONCLUSIVE} & \text{otherwise}, \end{cases} \tag{4}$$

*where $\mathcal{O}$ is the obstruction encoding (Theorem 3.1), $\mathcal{R}$ is the reciprocal sensor (Definition 4.2), $\mathcal{C}$ is the Cayley transform (Definition, eq.* (3)*), and $\mathcal{S}$ is Schur certification (Corollary 6.6).*

*Then:*

(I) ***Soundness:*** *$\Psi^*$ satisfies* (E1).

(II) ***Completeness:*** *$\Psi^*$ is complete on the rational class.*

(III) ***Optimality:*** *$\Psi^*$ resolves every case that any sound, finite-data procedure resolves.*

(IV) ***Uniqueness:*** *$\Psi^*$ is the unique optimal exclusion procedure.*

(V) ***Forced factorisation:*** *$\Psi^* = \mathcal{S} \circ \mathcal{C} \circ \mathcal{R} \circ \mathcal{O}$ is the unique order; no permutation of the four steps is sound.*

*Proof.* **(I) (Soundness).** Suppose $\Psi^*(\mathbf{x}) = \texttt{EXCLUDED}$. Then $\mathcal{S}$ has certified $|\Xi_S| \leq 1$ on the audited region and $\Xi_S \not\equiv 1$. By the Schur pinch (Theorem 6.2), $\mathcal{J}_S$ has no poles in the region. If $\mathbf{x}$ were in $S$, then $G_S(\mathbf{x}) = 0$ and $\mathcal{J}_S = 1/G_S$ would have a pole—contradiction. Therefore $\mathbf{x} \notin S$.

**(II) (Completeness).** Let $\mathbf{x} \notin S$ with $\mathbf{x}$ in the rational class of degree $d$. Then $G_S(\mathbf{x}) \neq 0$, so $\mathcal{J}_S$ is holomorphic near $\mathbf{x}$. The Cayley field $\Xi_S$ is rational of degree $\leq d$ (Lemma 5.5 + Theorem 6.5). In the rational class, Schur certification is a finite semidefinite problem (Corollary 6.6) that terminates. If $|\Xi_S| \leq 1$, the pinch excludes $\mathbf{x}$. If $|\Xi_S| > 1$ somewhere, $\mathcal{J}_S$ has a pole, which means $G_S$ has a zero—but $\mathbf{x} \notin S$ means $G_S(\mathbf{x}) \neq 0$, so the pole is at a different point. In the rational class, the location of all poles/zeros is decidable by exact root-finding. Either way, the procedure terminates with a definite answer.

**(III) (Optimality).** Let $\Psi$ be any sound, finite-data exclusion procedure. Each step of $\Psi^*$ uses the sharpest available tool:

- $\mathcal{O}$: the obstruction is $J$ itself, which is the unique existence test (Theorem 3.1). Any other obstruction has a strictly larger zero set, producing false positives.
- $\mathcal{R}$: the reciprocal is the unique holomorphic amplifier (Theorem 4.1). Any other divergence mechanism leaves the analytic category.
- $\mathcal{C}$: the Cayley transform is the unique normalised conformal map (Theorem 5.1). Any other map introduces free parameters.
- $\mathcal{S}$: the Pick criterion is the unique necessary-and-sufficient Schur test (Theorem 6.3). Any weaker test has a larger inconclusive zone.

Therefore every case $\Psi$ resolves, $\Psi^*$ also resolves.

**(IV) (Uniqueness).** If $\Psi^{**}$ is also optimal, then $\Psi^* \succeq \Psi^{**}$ and $\Psi^{**} \succeq \Psi^*$, so they agree on all resolved cases. Completeness on the rational class forces agreement there. Outside the rational class, both return `INCONCLUSIVE` (Proposition 1.1). Hence $\Psi^* = \Psi^{**}$.

**(V) (Forced order).** *$\mathcal{O}$ must come first:* Without an obstruction, there is no function to analyse—$\mathcal{R}$, $\mathcal{C}$, $\mathcal{S}$ have no input.

*$\mathcal{R}$ must follow $\mathcal{O}$:* The Cayley transform operates on a function with poles (the sensor). Without first forming the reciprocal, there are no poles to convert into boundary behaviour.

*$\mathcal{C}$ must follow $\mathcal{R}$:* Schur certification operates on the unit disk. Without the Cayley map, the sensor lives in the right half-plane where the Pick criterion does not apply.

*$\mathcal{S}$ must come last:* It is the only step that produces a verdict (EXCLUDED vs. INCONCLUSIVE). All prior steps are preparatory transformations. □

# 8 Independence of the Four Steps

**Theorem 8.1** (Independence)**.** *No step of $(\mathcal{O}, \mathcal{R}, \mathcal{C}, \mathcal{S})$ is derivable from the other three.*

*Proof.* **(a) Without $\mathcal{O}$ (no obstruction):** $\mathcal{R}$, $\mathcal{C}$, $\mathcal{S}$ have no function to process. The procedure has no input.

**(b) Without $\mathcal{R}$ (no reciprocal):** The obstruction $G_S$ is holomorphic everywhere (no poles), so the Cayley transform produces a bounded function trivially. The Schur test passes vacuously, certifying nothing.

**(c) Without $\mathcal{C}$ (no Cayley transform):** The sensor $1/G_S$ is meromorphic. To certify "no poles," one would need to test $|1/G_S| < \infty$ everywhere—an infinite check. The Schur test on $\mathbb{D}$ is inapplicable without the conformal mapping.

**(d) Without $\mathcal{S}$ (no Schur certification):** The Cayley field $\Xi_S$ is constructed but never tested. No verdict is produced. □

# 9 Uniqueness of the Cayley Transform

**Theorem 9.1** (Möbius uniqueness)**.** *The Cayley transform* (2) *is the unique conformal bijection* $\{\mathrm{Re}(w) > 0\} \to \mathbb{D}$ *satisfying:*

1. $\Xi(\infty) = 1$ *(poles map to the boundary).*
2. $\Xi(1/2) = 0$ *(the calibration point maps to the origin).*

*Any other Möbius map from the right half-plane to $\mathbb{D}$ differs by a rotation $e^{i\theta}$, and the normalisation $\Xi(\infty) = 1$ fixes $\theta = 0$.*

*Proof.* See Theorem 5.1 in §5. □

*Remark* 9.2. The normalisation point $w_0 = 1/2$ maps to $\Xi = 0$ (the centre of the disk). Under a different normalisation, the Cayley transform changes by a Möbius automorphism of $\mathbb{D}$, which preserves the Schur class and the Pick criterion. Thus the exclusion verdict is *Möbius-invariant*: it does not depend on the choice of normalisation. The specific form (2) is canonical but not essential; what matters is the conformal class.

# 10 The Two-Sided Audit: CPT + Exclusion = Complete Decision

**Theorem 10.1** (Complete two-sided decision)**.** *For configurations in the rational class of known degree $d$ with $n \geq 8(2d + 1)$:*

1. $\Phi^*$ *(CPT, membership) decides* $\mathbf{x} \in S$*.*
2. $\Psi^*$ *(this paper, exclusion) decides* $\mathbf{x} \notin S$*.*
3. *Together,* $\Phi^*$ *and* $\Psi^*$ *resolve* every *configuration: no rational-class input returns* `INCONCLUSIVE` *from both.*

*Proof.* For $\mathbf{x}$ in the rational class, either $J(\mathbf{x}) = 0$ (and $\Phi^*$ certifies membership) or $J(\mathbf{x}) > 0$ (and $\Psi^*$ certifies exclusion). These are exhaustive and mutually exclusive. □

**Corollary 10.2** (The unique complete audit)**.** *The pair* $(\Phi^*, \Psi^*)$ *is the unique optimal two-sided certification system for zero-defect membership in the rational class.*

*Proof.* $\Phi^*$ is the unique optimal membership procedure (CPT Master Theorem). $\Psi^*$ is the unique optimal exclusion procedure (Theorem 7.2). Their union resolves every case (Theorem 10.1). Any alternative pair must agree with $(\Phi^*, \Psi^*)$ on all resolved cases, hence is identical. □

# 11 Discussion

## 11.1 RSA is not an audit; it is a theorem

The original presentation of the Recognition Stability Audit described it as a "compiler" or "audit machine." This paper shows it is neither. The four-step exclusion pipeline is forced by three properties of $J$:

1. **Analyticity** (forces holomorphic obstruction).
2. **Unique zero** (forces reciprocal as the unique amplifier).
3. **Strict convexity** (forces the Schur pinch to be conclusive).

Combined with finite resolution (which forces the rational class), the pipeline is the unique optimal exclusion strategy.

### 11.2 The complete picture

| | **Membership (CPT)** | **Exclusion (this paper)** |
|---|---|---|
| Question | Is $J(\mathbf{x}) = 0$? | Is $J(\mathbf{x}) > 0$? |
| Steps | $\mathcal{P} \to \mathcal{B} \to \mathcal{A}$ (3 steps) | $\mathcal{O} \to \mathcal{R} \to \mathcal{C} \to \mathcal{S}$ (4 steps) |
| Key property | Strict convexity | Analyticity |
| Certificate type | Defect vanishes | Poles excluded |
| Forced by | $J'' > 0$ | cosh is entire |

Together they form the unique complete two-sided audit. Both are theorems, not methods.

### 11.3 The engineering boundary

As with CPT, everything in this paper is foundation. Domain instantiations—identifying the obstruction $G_S$, computing the Cayley field, running the Schur certification—are engineering. The pipeline itself requires no domain-specific input.

## 12 Conclusions

1. $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ is uniquely forced, entire (via cosh), strictly convex, and has a unique zero at $x = 1$.
2. The exclusion pipeline $\Psi^* = \mathcal{S} \circ \mathcal{C} \circ \mathcal{R} \circ \mathcal{O}$ is the **unique optimal** procedure for certifying $\mathbf{x} \notin S$ from finite data (Exclusion Master Theorem 7.2).
3. The factorisation into four steps is **forced**: no reordering is sound.
4. The four steps are **independent**: none is derivable from the other three (Theorem 8.1).
5. The Cayley transform is **unique** up to Möbius equivalence (Theorem 5.1), and the verdict is Möbius-invariant.
6. $\Psi^*$ is **complete** on the rational class.
7. $\Phi^*$ (CPT) + $\Psi^*$ (this paper) = the **unique complete two-sided audit** (Theorem 10.1).

The exclusion pipeline is not an audit one designs. It is a theorem about the analytic structure of the canonical recognition cost.

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