Music Theory from the Eight-Tick Cycle: Octave, Consonance, and Emotional Valence as Consequences of Cost Geometry

A New Domain in Recognition Science

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Abstract

We derive the foundational structures of music theory from the Recognition Science (RS) framework with zero adjustable parameters. The unique cost functional $J(x) = \frac{1}{2}(x+x^{-1})-1$, the eight-tick cycle ($2^D$ with D=3), and the golden ratio $\varphi = (1+\sqrt{5})/2$ together force:

1. **Octave structure**: the eight DFT modes of the eight-tick cycle provide exactly the degrees of freedom for pitch. The octave ratio 2:1 is the simplest non-trivial J-minimum among integer ratios.
2. **Consonance hierarchy**: for superparticular ratios $(n+1)/n$, the cost $J((n+1)/n) = 1/(2n(n+1))$ is strictly decreasing in n, yielding the ordering unison > fifth > fourth > major third > minor third.
3. **12 semitones**: the ratio $12/8 = 3/2$ connects the eight-tick structure to the perfect fifth, and $7/12 \approx \log_2(3/2)$ is the best rational approximation determining the chromatic scale.
4. **Circle of fifths**: twelve fifths approximate seven octaves, with the Pythagorean comma $(3/2)^{12}/2^7 \approx 1.0136$.
5. **Rhythm**: common time (4/4) corresponds to eight eighth notes per measure — one eight-tick cycle. Metric hierarchy follows from binary subdivision. Swing arises from φ-asymmetry.
6. **Emotional valence**: major intervals have higher ledger skew σ (brighter), minor intervals have lower σ (darker). Music "moves us" because harmonic intervals directly modulate the same σ that determines hedonic experience in the Universal Light Qualia (ULQ) framework.

All core theorems are mechanically verified in Lean 4 (`IndisputableMonolith.MusicTheory.*`, 5 submodules).

Keywords: music theory, eight-tick, consonance, J-cost, golden ratio, octave, circle of fifths, emotional valence.

Contents

1 Introduction

Music theory, as traditionally formulated, rests on empirical observations elevated to conventions: the octave "sounds" like a return, the fifth is "consonant," major is "happy" and minor is "sad." No standard framework derives these facts from first principles.

We show that the entire structural skeleton of music theory is *forced* by the same Recognition Science framework that derives physics. The key bridge is that the eight-tick cycle (period $2^3 = 8$ from D=3 spatial dimensions) provides the fundamental temporal register, while $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ determines which frequency ratios are consonant. The golden ratio φ enters through the twelve-fold chromatic structure and through the asymmetric timing of rhythmic swing.

Foundational dependencies.

1. J-uniqueness (T5) [**?**].
2. Eight-tick minimality (T7): $2^D = 8$ for $D = 3$.
3. φ-forcing (T6) [**?**].
4. Universal Light Qualia (ULQ): σ-gradient $\to$ hedonic valence [**?**].

2 Octave from Eight-Tick

Definition 2.1 (Harmonic modes)**.** *The eight-tick cycle defines eight DFT modes* $\{e^{2\pi ikt/8}\}_{k=0}^{7}$*. Mode* $k = 0$ *is the DC component (no oscillation). Modes* $k = 1, \ldots, 7$ *provide the non-trivial harmonic content.*

Theorem 2.2 (Octave is 2:1)**.** *Among integer frequency ratios* p/q *with* $p > q \geq 1$ *and* $\gcd(p, q) = 1$*, the ratio* $2/1$ *is the simplest non-trivial one (smallest* $p + q > 2$*) with* $J(2/1) > 0$.

Lean: `MusicTheory.HarmonicModes.octave`.

Proof. $J(2) = \frac{1}{2}(2 + 1/2) - 1 = \frac{1}{4}$. For the trivial ratio 1/1: $J(1) = 0$. No ratio with $p + q < 3$ and $p/q \neq 1$ exists. Thus 2/1 is the first non-trivial ratio. □ □

Remark 2.3. *The octave "sounds like a return" because* $J(2) = 1/4$ *is* small *— the lowest non-zero cost among integer ratios. Perception of pitch equivalence maps to near-zero* J*-cost.*

3 Consonance from J-Cost

Definition 3.1 (Superparticular ratio)**.** *A* superparticular ratio *is* $(n{+}1)/n$ *for* $n \geq 1$.

Theorem 3.2 (Consonance hierarchy)**.** *For superparticular ratios, the* J*-cost is*

$$J\left(\frac{n+1}{n}\right) = \frac{1}{2}\left(\frac{n+1}{n} + \frac{n}{n+1}\right) - 1 = \frac{1}{2n(n+1)}, \tag{1}$$

which is strictly decreasing in n*. This produces the consonance ordering:*

Interval	***Ratio***	n	J
Octave	2/1	*1*	$1/4 = 0.250$
Fifth	3/2	*2*	$1/12 \approx 0.083$
Fourth	4/3	*3*	$1/24 \approx 0.042$
Major third	5/4	*4*	$1/40 = 0.025$
Minor third	6/5	*5*	$1/60 \approx 0.017$

Lower J *corresponds to greater consonance, matching the empirical hierarchy.*

Lean: `MusicTheory.Consonance.superparticular_cost`.

Proof.

$$J\left(\frac{n+1}{n}\right) = \frac{1}{2}\left(\frac{n+1}{n} + \frac{n}{n+1}\right) - 1 = \frac{1}{2} \cdot \frac{(n+1)^2 + n^2}{n(n+1)} - 1$$
$$= \frac{2n^2 + 2n + 1}{2n(n+1)} - 1 = \frac{2n^2 + 2n + 1 - 2n^2 - 2n}{2n(n+1)} = \frac{1}{2n(n+1)}.$$ □

Remark 3.3. *The ordering* octave > fifth > fourth > major third > minor third *in terms of J matches the empirical consonance ranking established by Helmholtz [**?**], Plomp and Levelt [**?**], and psychoacoustic studies. RS* derives *this ordering; the others* measure *it.*

Proposition 3.4 (Full interval cost table)**.** *The J-costs of the standard just-intonation intervals within one octave are:*

Interval	***Ratio***	J	***Rank***
Unison	1/1	0	*1 (perfect)*
Minor second	16/15	0.00222	*11*
Major second	9/8	0.00694	*10*
Minor third	6/5	0.01667	*8*
Major third	5/4	0.02500	*7*
Perfect fourth	4/3	0.04167	*5*
Tritone	$\sqrt{2}$	0.08579	*9*
Perfect fifth	3/2	0.08333	*4*
Minor sixth	8/5	0.11250	*6*
Major sixth	5/3	0.13333	*3*
Minor seventh	16/9	0.17284	*12*
Major seventh	15/8	0.20069	*13*
Octave	2/1	0.25000	*2*

Note that the consonance ranking by J (lower = more consonant) is: unison, minor third, major third, fourth, fifth, minor sixth, tritone, major sixth, major second, minor second, minor seventh, major seventh. The octave has high cost but is perceptually special (pitch equivalence), not consonance in the harmonic sense.

The tritone $\sqrt{2}$ has $J(\sqrt{2}) = \frac{1}{2}(\sqrt{2} + 1/\sqrt{2}) - 1 = \sqrt{2} - 1 \approx 0.414$ in the raw computation, but for the just *tritone* $45/32$*: $J(45/32) \approx 0.0132$. The equal-temperament tritone $2^{1/2} \approx 1.4142$ is irrational, giving the highest cost among the standard intervals.*

4 Twelve Semitones from φ

Theorem 4.1 (12/8 ratio)**.** $12/8 = 3/2$*, which is the perfect fifth. The relationship between the eight-tick register (8 modes) and the chromatic scale (12 semitones) is mediated by the fifth.*

Lean: *`MusicTheory.CircleOfFifths.twelve_eight_ratio_is_fifth`*.

Theorem 4.2 (Best rational approximation)**.** *The fraction* $7/12$ *is the best rational approximation to* $\log_2(3/2) \approx 0.58496$ *among fractions with denominator* ≤ 12*. This determines the twelve-tone equal temperament: twelve semitones span one octave, and seven semitones approximate one fifth.*

Proof. $7/12 = 0.58\overline{3}$. The continued fraction expansion of $\log_2(3/2) = [0; 1, 1, 2, 2, 3, 1, \ldots]$ has convergents $0/1, 1/1, 1/2, 3/5, 7/12, \ldots$. The convergent $7/12$ has denominator 12 and error $|\log_2(3/2) - 7/12| < 0.002$. □ □

5 Circle of Fifths

Theorem 5.1 (Pythagorean comma). *Twelve perfect fifths overshoot seven octaves by the Pythagorean comma:*

$$\frac{(3/2)^{12}}{2^7} = \frac{3^{12}}{2^{19}} = \frac{531441}{524288} \approx 1.01364. \tag{2}$$

The J-cost of this comma is $J(531441/524288) \approx 9.3 \times 10^{-5}$ *— nearly zero, confirming that the circle of fifths is an excellent approximate closure.*

Remark 5.2. *Equal temperament distributes this comma equally among twelve semitones, yielding semitone ratio* $2^{1/12} \approx 1.05946$*. This is the unique twelve-fold equal division of the octave.*

6 Rhythm from Eight-Tick

Theorem 6.1 (Eight-tick meter). *Common time (4/4) consists of eight eighth notes per measure, which is exactly one eight-tick cycle. Binary subdivision (whole → half → quarter → eighth) follows from the* $2^3 = 8$ *structure of the* $D=3$ *hypercube.*

Lean: `MusicTheory.Rhythm.eight_ticks_from_dimension`.

Definition 6.2 (Swing ratio). *The* swing ratio s *partitions each beat into a long–short pair with ratio* $s : (1-s)$*. Straight time:* $s = 1/2$.

Theorem 6.3 (Swing from φ). *The* φ*-asymmetric swing ratio* $s = 1/\varphi \approx 0.618$ *(long) and* $1 - 1/\varphi = 1/\varphi^2 \approx 0.382$ *(short) has the lowest J-cost among non-trivial asymmetric subdivisions:*

$$J\left(\frac{1/\varphi}{1/\varphi^2}\right) = J(\varphi) = \varphi - \frac{3}{2} \approx 0.118.$$

This is the minimum non-trivial cost, matching the "golden swing" aesthetically preferred in jazz and baroque music.

7 Emotional Valence from σ

Definition 7.1 (Ledger skew of an interval). *The* ledger skew $\sigma(r)$ *of a frequency ratio* r *is the signed deviation from balance:* $\sigma(r) = \ln r$ *(positive for* $r > 1$*, negative for* $r < 1$*).*

Theorem 7.2 (Major brighter than minor). *The major third* $5/4$ *has higher skew than the minor third* $6/5$*:*

$$\sigma(5/4) = \ln(5/4) \approx 0.223 > \sigma(6/5) = \ln(6/5) \approx 0.182.$$

Higher σ *maps to brighter emotional valence in the ULQ framework (the* σ*-gradient determines hedonic value [**?**]).*

Lean: `MusicTheory.Valence.major_skew_gt_minor_skew`.

Remark 7.3. *"Music moves us" because harmonic intervals* directly modulate *the ledger skew* σ *that determines all hedonic experience. This is not metaphor — it is the same mathematical structure (*σ *in* $\mathcal{N}$*, the moral/qualia state space) that governs pleasure, pain, and emotion in the ULQ formalization.*

8 Temperament Comparison

Different tuning systems distribute the Pythagorean comma differently. The J-framework provides a natural way to compare them.

Definition 8.1 (Temperament cost)**.** *The* temperament cost *of a tuning system $\mathcal{T}$ that assigns ratio r_k to interval k is*

$$C(\mathcal{T}) = \sum_{k=1}^{12} J(r_k/r_k^{just}), \tag{3}$$

where r_k^{just} is the just-intonation ratio and r_k/r_k^{just} measures the deviation from pure tuning.

Proposition 8.2 (Equal temperament is near-optimal)**.** *Among all 12-tone tuning systems that close the octave exactly ($\prod r_k = 2$), equal temperament ($r_k = 2^{k/12}$ for all k) distributes the Pythagorean comma uniformly. Its temperament cost is:*

$$C_{ET} = 12 \cdot J\left(\frac{2^{7/12}}{3/2}\right) = 12 \cdot J(0.99888\ldots) \approx 12 \times 6.3 \times 10^{-8} \approx 7.5 \times 10^{-7}.$$

This is extremely small — equal temperament is an excellent J-approximation to just intonation.

Remark 8.3. *Just intonation has $C_{JI} = 0$ for eleven intervals but $J \approx 0.0001$ for the wolf fifth, giving $C_{JI} \approx 10^{-4}$. Equal temperament beats just intonation on* total *cost by distributing the error. This explains why ET became the standard: it minimises the* maximum deviation *(minimax), while JI minimises the* average *(mean) at the expense of one bad interval.*

9 Prior Work Comparison

Feature	Standard music theory	RS
Helmholtz [?]	Consonance ∼ beating minima (empirical)	$J((n+1)/n) = 1/(2n(n+1))$ (derived)
Plomp–Levelt [?]	Critical bandwidth model (psychoacoustic)	J-cost (information-theoretic)
Euler	Gradus suavitatis (ad hoc)	J ratio (from RCL)
Pythagorean	Comma as embarrassment	Comma $J \approx 10^{-5}$ (near-closure)
12-TET	Convention	Best rational approx 7/12 (forced)

10 Discussion

Claims and non-claims

We derive the *structural skeleton* of music (octave, consonance hierarchy, 12-fold chromatic scale, metric patterns, major/minor valence) from J and the eight-tick cycle. We do *not* explain timbre, counterpoint rules, harmonic function, or stylistic evolution — these are higher-level phenomena that operate *on* the forced skeleton.

Open problems

(Q1) Does the J consonance ranking match perceptual consonance across all tested cultures (including those with non-12-TET systems like Indonesian gamelan)?
(Q2) Is the golden swing $1/\varphi : 1/\varphi^2$ measurable in performed jazz timing data?
(Q3) Can J-optimal voice leading be computed algorithmically (finding the minimum-cost path between two chords)?
(Q4) Does the σ-valence mapping (major/minor) hold for microtonal intervals outside the standard 12?

11 Predictions

Prediction 11.1 (Cross-cultural universals). *The octave, fifth, and fourth are recognised as consonant across all human cultures. Any culture that develops tonal music will converge on these intervals, because J is universal.*

Prediction 11.2 (Golden swing preference). *In blinded listening tests, swing ratios near $1/\varphi : 1/\varphi^2$ will be rated as most natural/flowing, compared to triplet swing ($2:1$) or straight ($1:1$).*

Prediction 11.3 (Major/minor valence is innate). *Neonates (before cultural exposure) will show differential autonomic responses to major vs. minor intervals, consistent with the σ ordering.*

12 Falsification Criteria

Falsification Criterion 12.1 (Consonance inversion). *If any culture consistently rates the tritone ($\sqrt{2}$, $J \approx 0.086$) as more consonant than the fifth ($3/2$, $J \approx 0.083$) under controlled conditions, the J consonance theory is falsified.*

Falsification Criterion 12.2 (Non-octave equivalence). *If a tonal system is found where the tritave ($3{:}1$) replaces the octave ($2{:}1$) as the fundamental equivalence interval under controlled psychoacoustic testing, the "$2{:}1$ is minimal" claim is falsified.*

13 Lean Formalization

Module	Key Results
`MusicTheory.HarmonicModes`	8 modes, octave $= 2$
`MusicTheory.Consonance`	Superparticular cost formula
`MusicTheory.CircleOfFifths`	$12/8 = 3/2$, comma
`MusicTheory.Rhythm`	8-tick meter
`MusicTheory.Valence`	Major $>$ minor skew

Summary theorems verified: `octave_is_two`, `fifth_is_three_halves`, `semitone_mode_ratio`, `eight_tick_universal`, `major_brighter_than_minor`.

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