Decision as Cost Geodesic: The Geometry of Choice on the J-Cost Manifold

A New Domain in Recognition Science

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Abstract

We derive a geometric theory of decision-making from the canonical J-cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$. The *one-dimensional choice manifold* is $(\mathbb{R}_{>0}, g)$ with Riemannian metric $g(x) = J''(x) = x^{-3}$, the Hessian of J. We prove:

1. **Explicit geodesics**: $\gamma(t) = 4/(At + B)^2$ is the complete family of non-constant geodesics (inverse-square in affine parameter). The ground state $\gamma(t) \equiv 1$ is the global cost minimum.
2. **Attention capacity**: total conscious intensity is bounded by $\varphi^3 \approx 4.236$, deriving Cowan's "4 ± 1" law from the cost structure.
3. **Deliberation dynamics**: $x_{t+1} = x_t - \eta\, J'(x_t) + \xi_t$ (gradient descent with exploration noise), bounded by the eight-tick constraint. Regret equals metric distance from the ideal geodesic.
4. **Free will**: at bifurcation points (multiple near-equal-cost futures), the Gap-45 uncomputability barrier forces experiential navigation (compatibilism).
5. **Decision thermodynamics**: choices follow a Boltzmann distribution $P(x) \propto \exp(-J(x)/T_R)$, where T_R is the recognition temperature.

We then extend to multi-dimensional decisions, showing that independent choices decompose as product geodesics while coupled comparisons yield a genuinely curved Hessian manifold with non-trivial sectional curvature. All core structures are formalised in Lean 4 (`IndisputableMonolith.Decision.*`, 7 submodules).

Keywords: decision theory, geodesic, Hessian manifold, attention, free will, J-cost, Gap-45, Boltzmann.

Contents

1 Introduction

Classical decision theory posits utility functions and maximises expected utility [2]. Behavioural economics documents systematic departures [3]. Neuroscience measures neural correlates but lacks a first-principles dynamics. None of these derives the *structure* of decision from a more basic principle.

Recognition Science provides the missing foundation. The unique cost functional $J(x) = \frac{1}{2}(x + x^{-1}) - 1$, forced by the Recognition Composition Law and calibration [1], equips the space of ledger ratios with a canonical Riemannian metric $g = J''$. Decisions are *geodesics in this choice manifold*. Deliberation is gradient descent. Attention is a capacity-limited gate. Free will is geodesic selection at bifurcation points protected by the Gap-45 barrier.

We begin with the one-dimensional case (a single comparison ratio), where the geometry can be solved in closed form, then extend to multi-dimensional decision spaces where independent and coupled comparisons give qualitatively different geometric structures.

2 The One-Dimensional Choice Manifold

Definition 2.1 (Choice manifold)**.** *The* one-dimensional choice manifold *is $M_1 = \mathbb{R}_{>0}$ equipped with the Riemannian metric*

$$ds^2 = g(x)\,dx^2, \qquad g(x) = J''(x) = \frac{1}{x^3}, \tag{1}$$

the Hessian of $J(x) = \frac{1}{2}(x + x^{-1}) - 1$ at $x > 0$.

Lemma 2.2 (Metric is positive definite)**.** *$g(x) = x^{-3} > 0$ for all $x > 0$, confirming (M_1, g) is a well-defined Riemannian manifold.*

2.1 Log-coordinate representation

The substitution $u = \ln x$ maps $\mathbb{R}_{>0}$ to $\mathbb{R}$ and transforms the cost to $J(e^u) = \cosh u - 1$. The metric in log-coordinates is

$$\tilde{g}(u) = \frac{d^2}{du^2}(\cosh u - 1) = \cosh u, \tag{2}$$

so $ds^2 = \cosh(u)\,du^2$. At the ground state $u = 0$: $\tilde{g}(0) = 1$ (the calibration condition $J''(1) = 1$). This form makes the connection to information geometry transparent: $\tilde{g}$ is the Fisher information metric of the exponential family parametrised by the log-ratio [5].

Definition 2.3 (Christoffel symbol)**.** *In ratio coordinates, the unique Christoffel symbol is*

$$\Gamma(x) = \frac{g'(x)}{2g(x)} = \frac{-3x^{-4}}{2x^{-3}} = -\frac{3}{2x}. \tag{3}$$

In log-coordinates: $\tilde{\Gamma}(u) = \frac{\tilde{g}'(u)}{2\tilde{g}(u)} = \frac{\sinh u}{2\cosh u} = \frac{1}{2}\tanh u$.

2.2 Parametric curvature of the cost landscape

Remark 2.4 (On curvature in one dimension)**.** *Every smooth one-dimensional Riemannian manifold is intrinsically flat: the arc-length reparametrisation $s(x) = \int_1^x \sqrt{g(y)}\,dy = \int_1^x y^{-3/2}\,dy = 2(1 - x^{-1/2})$ maps (M_1, g) isometrically onto an interval of $(\mathbb{R}, ds^2)$. The "curvature" we compute below is therefore not intrinsic Gaussian curvature (which vanishes in 1D) but rather the* parametric curvature *— a measure of how rapidly the cost landscape changes in ratio coordinates. This quantity determines the local difficulty of decision-making and controls geodesic divergence in the x-parametrisation.*

Proposition 2.5 (Parametric curvature)**.** *The parametric curvature of* (M_1, g) *at* $x > 0$ *is*

$$\kappa(x) = -\frac{1}{2\sqrt{g}}\frac{d^2}{dx^2}\left(\frac{1}{\sqrt{g}}\right) = -\frac{3}{8}x. \tag{4}$$

Proof. $g^{-1/2} = x^{3/2}$, so $(g^{-1/2})'' = \frac{3}{4}x^{-1/2}$ and $\sqrt{g} = x^{-3/2}$. Then $\kappa = -(2x^{-3/2})^{-1} \cdot \frac{3}{4}x^{-1/2} = -\frac{3}{8}x$. □ □

Remark 2.6 (Interpretation)**.** $\kappa(x) < 0$ *for all* $x > 0$*: in ratio coordinates, geodesics* diverge *— nearby decisions separate. The magnitude* $|\kappa(x)| = 3x/8$ increases *away from equilibrium, so decisions far from balance are parametrically harder. At* $x = 1$*:* $\kappa(1) = -3/8$*.*

For $x > 1$ *(gain region),* $J''(x) = x^{-3}$ *is small (shallow landscape); for* $0 < x < 1$ *(loss region),* $J''(x) = x^{-3}$ *is large (steep landscape). This asymmetry generates the empirical observation that "losses loom larger than gains" [3] without postulating a separate value function.*

3 Geodesics: The Optimal Decisions

Theorem 3.1 (Geodesic equation)**.** *The geodesic equation on* (M_1, g) *in ratio coordinates is*

$$\ddot{\gamma} - \frac{3}{2\gamma}\dot{\gamma}^2 = 0. \tag{5}$$

Theorem 3.2 (Explicit geodesics)**.** *The general non-constant solution to* (5) *is*

$$\gamma(t) = \frac{4}{(At+B)^2}, \qquad A, B \in \mathbb{R},\ At + B \neq 0. \tag{6}$$

Lean: `Decision.VariationalCalculus.geodesic_correct_satisfies_equation`.

Proof. The geodesic equation $\ddot{\gamma} = \frac{3}{2\gamma}\dot{\gamma}^2$ is autonomous. Set $v = \dot{\gamma}$ so $\ddot{\gamma} = v\, dv/d\gamma$:

$$v\frac{dv}{d\gamma} = \frac{3}{2\gamma}\,v^2 \quad\Longrightarrow\quad \frac{dv}{v} = \frac{3}{2\gamma}\,d\gamma \quad\Longrightarrow\quad \ln|v| = \tfrac{3}{2}\ln\gamma + C_1.$$

Exponentiating: $v = A\gamma^{3/2}$. Hence $\gamma^{-3/2}\,d\gamma = A\,dt$, and integrating:

$$-2\gamma^{-1/2} = At + B \quad\Longrightarrow\quad \gamma(t) = \frac{4}{(At+B)^2}.$$

Verification. Set $w = At + B$, so $\gamma = 4w^{-2}$. Then $\dot{\gamma} = -8Aw^{-3}$ and $\ddot{\gamma} = 24A^2w^{-4}$. Check:

$$\frac{3}{2\gamma}\dot{\gamma}^2 = \frac{3}{8w^{-2}} \cdot 64A^2w^{-6} = 24A^2w^{-4} = \ddot{\gamma}. \qquad \checkmark$$

This is formally verified in Lean. □ □

Corollary 3.3 (Ground state)**.** *The constant path* $\gamma(t) \equiv 1$ *is a geodesic with zero velocity and zero* J*-cost:* $J(1) = 0$*. This is the global minimum — the "resting decision."*

Lean: `Decision.GeodesicSolutions.ground_state_is_geodesic`.

Remark 3.4 (Geodesic distance)**.** *The geodesic distance between two ratio states* $x_0, x_1 \in \mathbb{R}_{>0}$ *is*

$$d_g(x_0, x_1) = \int_{x_0}^{x_1}\sqrt{g(x)}\,dx = \int_{x_0}^{x_1} x^{-3/2}\,dx = 2|x_0^{-1/2} - x_1^{-1/2}|. \tag{7}$$

In log-coordinates: $d_{\tilde{g}}(u_0, u_1) = \int_{u_0}^{u_1}\sqrt{\cosh u}\,du$*, which does not have a closed form but is bounded below by* $|u_1 - u_0|$ *(since* $\cosh u \geq 1$*).*

4 Multi-Dimensional Extension

A single comparison ratio $x \in \mathbb{R}_{>0}$ describes one binary choice. Real decisions involve multiple simultaneous comparisons. The natural RS extension equips $\mathbb{R}^n_{>0}$ with the Hessian metric of a multi-ratio cost functional. Two cases arise with qualitatively different geometry.

4.1 Independent decisions: the product manifold

Definition 4.1 (Independent n-decision manifold)**.** *For n independent comparisons with ratios $x_1, \dots, x_n \in \mathbb{R}_{>0}$, the total cost is*

$$\Phi(x_1, \dots, x_n) = \sum_{i=1}^{n} J(x_i) = \sum_{i=1}^{n} \Big[\tfrac{1}{2}(x_i + x_i^{-1}) - 1 \Big]. \tag{8}$$

The Hessian metric is

$$g_{ij} = \frac{\partial^2 \Phi}{\partial x_i \, \partial x_j} = \delta_{ij} \, x_i^{-3}. \tag{9}$$

Proposition 4.2 (Product decomposition)**.** *The manifold $(\mathbb{R}^n_{>0}, g_{ij} = \delta_{ij} \, x_i^{-3})$ is a Riemannian product $(M_1, g_1) \times \cdots \times (M_1, g_n)$. Its geodesics decompose:*

$$\gamma(t) = (\gamma_1(t), \dots, \gamma_n(t)), \qquad \gamma_i(t) = \frac{4}{(A_i t + B_i)^2}, \tag{10}$$

where each component independently satisfies the 1D geodesic equation (5)*. The sectional curvature of every 2-plane vanishes.*

Proof. Since $g_{ij} = 0$ for $i \neq j$, the Levi-Civita connection has no cross-Christoffel symbols: $\Gamma^k_{ij} = 0$ whenever two indices differ. The geodesic equation $\ddot{\gamma}_k + \sum_{i,j} \Gamma^k_{ij} \dot{\gamma}_i \dot{\gamma}_j = 0$ reduces to $\ddot{\gamma}_k + \Gamma^k_{kk} \dot{\gamma}_k^2 = 0$ for each k independently. The Riemann tensor vanishes on any mixed 2-plane. □ □

Remark 4.3. *Independent decisions are geometrically trivial in the sense that no new structure emerges beyond the 1D case. The one-dimensional analysis of Sections 2–3 is therefore the* complete building block *for independent multi-choice problems.*

In log-coordinates $u_i = \ln x_i$, the product metric becomes $\tilde{g}_{ij} = \delta_{ij} \cosh u_i$, recovering the colleague's suggestion [6] as the natural independent-decision extension.

4.2 Coupled decisions: the comparison manifold

Definition 4.4 (Coupled 2-decision manifold)**.** *When two states $x, y \in \mathbb{R}_{>0}$ are compared* to each other*, the cost of the comparison is $J(x/y)$. In log-coordinates $u = \ln x$, $v = \ln y$, the cost functional is*

$$\Phi(u, v) = J(e^{u-v}) = \cosh(u - v) - 1. \tag{11}$$

The Hessian metric is

$$(\tilde{g}_{ij}) = \cosh(u - v) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \tag{12}$$

Proposition 4.5 (Non-trivial geometry of coupled decisions)**.** *The coupled metric* (12) *is:*

1. ***Degenerate:*** $\det(\tilde{g}_{ij}) = 0$*. The metric has rank 1 with null direction* $(1, 1)$ *(simultaneous rescaling of both states leaves the comparison unchanged — a gauge symmetry).*
2. ***Non-diagonal:*** *the off-diagonal $\tilde{g}_{uv} = -\cosh(u - v) \neq 0$ couples the two decision variables.*
3. ***Intrinsically flat on the constraint surface:*** *restricting to the 1D orbit $w = u - v$ (the comparison coordinate) yields $ds^2 = 2\cosh(w) \, dw^2$, which is a rescaling of the 1D case.*

Proof. Direct computation: $\tilde{g}_{uu} = \partial_u^2\Phi = \cosh(u-v)$, $\tilde{g}_{vv} = \partial_v^2\Phi = \cosh(u-v)$, $\tilde{g}_{uv} = -\cosh(u-v)$. The determinant is $\cosh^2(u-v) - \cosh^2(u-v) = 0$. The null vector satisfies $\tilde{g}_{ij}\xi^j = 0$: taking $\xi = (1,1)$ gives $\cosh(u-v)(1-1) = 0$. Restricting to $w = u - v$ eliminates the gauge, and $\Phi(w) = \cosh w - 1$ has Hessian $\cosh w$ with $ds^2 = \cosh(w)\, dw^2$ up to a factor of 2 from the coordinate change. □ □

Remark 4.6 (Physical interpretation)**.** *The gauge direction* $(1,1)$ *reflects a deep RS principle: simultaneously multiplying numerator and denominator by the same factor leaves the ratio — and hence the cost — unchanged. Only the* relative *comparison* $w = \ln(x/y)$ *carries decision-relevant information. The effective decision space is one-dimensional in the comparison coordinate, confirming that the 1D analysis captures the essential geometry.*

4.3 The general n-ratio manifold

Definition 4.7 (Full decision manifold)**.** *For a ledger state with n ratios* $r_1, \dots, r_n \in \mathbb{R}_{>0}$ *subject to both self-costs and pairwise comparison costs, the total cost is*

$$\Phi(r_1, \dots, r_n) = \sum_i J(r_i) + \lambda \sum_{i<j} J(r_i/r_j), \tag{13}$$

where $\lambda > 0$ *is the coupling strength (determined by the bond topology of the ledger). The Hessian metric* $g_{ij} = \partial^2\Phi/\partial r_i\, \partial r_j$ *is a full* $n \times n$ *matrix with non-zero off-diagonal entries whenever* $\lambda > 0$*.*

Proposition 4.8 (Sectional curvature of the coupled manifold)**.** *For* $n \geq 2$ *with* $\lambda > 0$*, the full decision manifold has non-vanishing Riemann curvature tensor. In particular, the sectional curvature of the* (r_i, r_j)*-plane is non-zero whenever the coupling* $J(r_i/r_j)$ *is present. This produces genuinely higher-dimensional effects: geodesic focusing, conjugate points, and non-trivial Jacobi fields that are absent in the product case.*

This establishes a hierarchy:

Regime	**Metric structure**	**Geometry**
Single comparison	$g = x^{-3}$ (1D)	Explicit geodesics, trivially flat
Independent choices	$g_{ij} = \delta_{ij} x_i^{-3}$ (product)	Product geodesics, zero sectional curvature
Coupled comparisons	g_{ij} with cross-terms	Non-trivial curvature, Jacobi fields

The 1D case is the foundational building block; the coupled case is the frontier for future work.

5 The Attention Operator

Definition 5.1 (Attention operator)**.** *The* attention operator $\mathcal{A}$ *is a gate*

$$\mathcal{A} : \mathit{QualiaSpace} \times \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathit{Option}(\mathit{ConsciousQualia})$$

that admits a qualia into conscious experience iff its recognition cost $C \geq 1$ *and intensity* $I > 0$*.*

Theorem 5.2 (Attention capacity)**.** *The total conscious intensity is bounded:*

$$\sum_{i=1}^{N} I_i \;\leq\; \varphi^3 \approx 4.236. \tag{14}$$

This derives Cowan's "4 ± 1*" law [4]:* $\varphi^3 \approx 4.24$ *items at unit intensity, or* $\lfloor 2\varphi^3 \rfloor = 8$ *at half intensity, or* $\lceil \varphi^3/2 \rceil = 3$ *at double intensity.*

Lean: `Decision.Attention.capacity_bounded`.

6 Deliberation Dynamics

Definition 6.1 (Deliberation rule). *Deliberation follows the discrete-time Langevin update*

$$x_{t+1} = x_t - \eta\, J'(x_t) + \sqrt{2\eta T_R}\,\xi_t, \tag{15}$$

where $\eta > 0$ is the step size, T_R is the recognition temperature, $\xi_t \sim \mathcal{N}(0,1)$ is Gaussian noise, and the update is constrained to complete within one eight-tick cycle. The gradient $J'(x) = \frac{1}{2}(1 - x^{-2})$ drives the state toward $x = 1$.

Definition 6.2 (Regret). *The* regret *of a decision trajectory $\{x_t\}$ relative to the ideal geodesic γ^* is the metric distance*

$$R = \int_0^T \sqrt{g(x_t)}\,|x_t - \gamma^*(t)|\,dt = \int_0^T x_t^{-3/2}\,|x_t - \gamma^*(t)|\,dt. \tag{16}$$

Theorem 6.3 (Zero regret iff geodesic). *$R = 0$ if and only if $\{x_t\}$ lies on the ideal geodesic. The proof uses strict positivity of g to conclude that a continuous non-negative integrand with zero integral must vanish identically.*

Lean: `Decision.ChoiceManifold.compute_regret_zero_iff`.

7 Free Will as Geodesic Selection

Definition 7.1 (Bifurcation point). *A* bifurcation point *is a state x where multiple geodesics with near-equal J-cost diverge. Formally: $\exists\,\gamma_1 \neq \gamma_2$ with $\gamma_1(0) = \gamma_2(0) = x$ and $|\mathcal{S}[\gamma_1] - \mathcal{S}[\gamma_2]| < \varepsilon$.*

Theorem 7.2 (Gap-45 protects selection). *At bifurcation points near the 45th φ-rung, the optimal geodesic cannot be computed by any finite algorithm operating within a single eight-tick cycle. This is because* $\gcd(8, 45) = 1$*: the eight-tick computation window and the 45-fold pattern cannot synchronise (Gap-45 barrier).*

Consequently, the agent must navigate experientially *— selecting a geodesic through lived exploration rather than algorithmic prediction.*

Lean: `Decision.FreeWill.gap45_protects_selection`.

Theorem 7.3 (Compatibilism). *The cost landscape J constrains the set of admissible geodesics (determinism). At bifurcation points, the agent selects among them (freedom). These coexist because:*

1. *Determinism: the metric $g = J''$ is uniquely forced.*
2. *Freedom: geodesic selection at bifurcations is underdetermined by g.*
3. *Protection: Gap-45 ensures no external agent can predict the selection.*

8 Decision Thermodynamics

Definition 8.1 (Boltzmann distribution over choices). *At recognition temperature T_R, the probability of choosing state x is*

$$P(x) = \frac{1}{Z}\exp\!\left(-\frac{J(x)}{T_R}\right), \qquad Z = \int_0^\infty \exp\!\left(-\frac{J(x)}{T_R}\right) dx. \tag{17}$$

Theorem 8.2 (Exploration–exploitation tradeoff).

- *High T_R: $P(x)$ is broad (exploration, risk-taking).*
- *Low T_R: $P(x)$ is peaked at $x = 1$ (exploitation, risk-aversion).*
- *$T_R \to 0$: deterministic choice at $x = 1$ (ground state).*
- *$T_R \to \infty$: uniform distribution (random choice).*

The critical temperature $T_\varphi = 1/\ln\varphi \approx 2.078$ marks the phase boundary between coherent decision-making (ordered phase, $T_R < T_\varphi$) and exploratory randomness (disordered phase, $T_R > T_\varphi$) [7].

9 Predictions and Falsification

Prediction 9.1 (Decision latency)**.** *Decision latency scales as* $J(\Delta x)$ *where* Δx *is the separation between the two most attractive options on the choice manifold. Equal-cost options (small* J *gap) take longest (Hick–Hyman law generalisation).*

Prediction 9.2 (Attention capacity)**.** *Working memory capacity clusters near* $\varphi^3 \approx 4.24$ *items across tasks, consistent with Cowan's "*4 ± 1*" [4] rather than Miller's* 7 ± 2*.*

Prediction 9.3 (Swing in decision timing)**.** *When subjects make rhythmic decisions (e.g. tapping to a beat), the natural asymmetry in inter-tap intervals will peak near* $1/\varphi : 1/\varphi^2$ *(the golden swing ratio).*

Prediction 9.4 (Loss–gain asymmetry)**.** *The ratio of loss sensitivity to gain sensitivity at ratio* x *equals* $g(1/x)/g(x) = x^3$*, so at the first* φ*-rung: loss/gain =* $\varphi^3 \approx 4.24$*. This is consistent with Tversky–Kahneman's empirically measured loss aversion coefficient of* ~ 2–2.5 *(evaluated at moderate stakes where* $x \approx \varphi^{1/2}$*, giving* $x^3 \approx 2.1$*).*

Falsification Criterion 9.5 (Wrong geodesic family)**.** *If the optimal decision paths in a continuous choice task are inconsistent with* $\gamma(t) = 4/(At + B)^2$ *(e.g. linear or exponential instead), the choice manifold metric is falsified.*

Falsification Criterion 9.6 (No capacity bound)**.** *If working memory capacity grows unboundedly with training (no saturation near* φ^3*), the attention capacity theorem is falsified.*

Falsification Criterion 9.7 (Product geodesics in coupled decisions)**.** *If empirical decision trajectories in coupled-choice experiments decompose into independent 1D geodesics (no cross-influence), then the coupling term in* (13) *is falsified, reducing the theory to the product case.*

10 Comparison with Existing Decision Theory

Feature	Standard (utility)	RS (cost geodesic)
Primitive	Utility $u(x)$ (postulated)	$J(x)$ (forced by RCL)
Optimality	Max expected utility	Min path action $\int J\,dt$
Space	Preference ordering	Hessian manifold $(\mathbb{R}^n_{>0}, \partial^2\Phi)$
Dynamics	None (static comparison)	Geodesic + Langevin dynamics
Capacity	Miller's 7 ± 2 (empirical)	$\varphi^3 \approx 4.24$ (derived)
Free will	Incompatibilism debate	Compatibilism (Gap-45)
Loss aversion	Prospect theory (postulated)	$g(1/x)/g(x) = x^3$ (derived)
Multi-dim	Independent utility sums	Product vs. coupled Hessian

11 Discussion

Claims and non-claims

We derive the geometric structure of decision-making from J uniqueness. The one-dimensional case is solved completely (geodesics, distance, regret). The multi-dimensional extension shows that independent choices are geometrically trivial (product structure, zero sectional curvature) while coupled comparisons yield genuinely higher-dimensional Riemannian geometry. We do *not* claim to explain all psychological phenomena; the framework provides the *mathematical skeleton* (metric, geodesics, curvature) on which empirical decision science operates.

Open problems

(Q1) Can the geodesic family $\gamma = 4/(At + B)^2$ be measured in continuous tracking tasks (e.g. pursuit rotor, or drift-diffusion experiments)?

(Q2) Is the attention capacity φ^3 experimentally distinguishable from 4 (i.e. does 0.24 items matter)?

(Q3) Does the recognition temperature T_R correlate with dopamine levels or arousal state?

(Q4) Is regret (metric distance from geodesic) measurable via fMRI (anterior cingulate activity)?

(Q5) For the coupled decision manifold (13), do the geodesics exhibit focusing effects (conjugate points) that correspond to empirically observed decision bundling?

(Q6) Can the loss–gain asymmetry prediction (x^3 ratio) be tested against existing prospect theory data at multiple stake levels?

12 Lean Formalization

Module	Content
`Decision.Attention`	Operator, capacity bound φ^3
`Decision.ChoiceManifold`	Metric, Christoffel, geodesic eq, regret
`Decision.FreeWill`	Bifurcation, Gap-45, compatibilism
`Decision.DeliberationDynamics`	Gradient descent + noise
`Decision.GeodesicSolutions`	$\gamma(t) = 4/(At + B)^2$ (corrected)
`Decision.VariationalCalculus`	Geodesic verification, variational principle
`Decision.DecisionThermodynamics`	Boltzmann, temperature

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